Summary of the professional accomplishments

Paweł Zin

29 march 2019

1 First name and surname:

Paweł Zin

2 Diploma.

Doctor degree

Date: 23st April 2007

Thesis title: Properties of Bose Einstein Condensates of interacting atoms by quantum field theory

Supervisor: prof. dr hab. Marek Trippenbach

Research unit: Faculty of Physics, University of Warsaw

Master degree

Date: 29th September 2001

Thesis title: Influence of the thermal cloud on the Bose-Einstein condensate lifetime

Supervisor: prof. dr hab. Marek Trippenbach

Research unit: Faculty of Physics, University of Warsaw

3 Academic and reserch career

2013– Assistant professor, National Center for Nuclear Research, Warsaw

2011–2013 Post-doc, Universite Paris-Sud, Orsay near Paris.

2008–2011 Assistant professor, National Center for Nuclear Research, Warsaw

2007–2008 Physicist, National Center for Nuclear Research, Warsaw

4 Scientific achievement (According to Polish law: " art. 16 ust. 2 ustawy z dnia 14 marca 2003 r. o stopniach naukowych i tytule naukowym oraz o stopniach i tytule w zakresie sztuki (Dz. U. 2017 r. poz. 1789)".

4.1 Title of the scientific achievement

Group of publications: *Theoretical description of phenomena caused by quantum and thermal fluctuation in ultracold atom systems*

4.2 List of publications of the scientific achievement

In 2017 I changed my surname from 'Ziń' to 'Zin'. That is why in most of the below publications surname 'Ziń' is present.

[h1] P. Ziń, B. Oleś, M. Trippenbach, K. Sacha, *Second -order quantum phase transition of a homogeneous Bose gas with attractive interactions*, Phys. Rev. A **78**, 023620 (2008).

[h2] P. Ziń, J. Chwedeńczuk, B. Oleś, K. Sacha, M. Trippenbach, *Critical fluctuations of an attractive Bose gas in a double-well potential*, Euro Phys. Lett. **83**, 64007 (2008).

[h3] P. Ziń, B. Oleś, K. Sacha, *Quantum particle number fluctuations in a two-component Bose gas in a double-well potential*, Phys. Rev. A **84**, 033614 (2011).

[h4] J. Chwedeńczuk, P. Ziń, M. Trippenbach, A. Perrin, V. Leung, D. Boiron, C.I. Westbrook, *Pair correlations of scattered atoms from two colliding Bose-Einstein Condensates: Perturbative Approach*, Phys. Rev. A **78**, 053605 (2008).

[h5] Paweł Ziń and Tomasz Wasak, *Properties of atomic pairs produced in the collision of Bose-Einstein condensates*, Phys. Rev. A **97**, 043620 (2018).

[h6] Paweł Ziń and Maciej Pylak, *The influence of the interaction between quasiparticles on parametric resonance in Bose-Einstein condensates*, J. Phys. B **50**, 085301 (2017).

[h7] Maciej Pylak and Paweł Zin, *Influence of the interaction between quasiparticles on parametric resonance in Bose-Einstein quasicondensates*, Phys. Rev. A **98**, 043603 (2018).

[h8] Paweł Zin, Maciej Pylak, Tomasz Wasak, Mariusz Gajda, Zbigniew Idziaszek, *Quantum Bose-Bose droplets at a dimensional crossover*, Phys. Rev. A **98**, 051603(R) (2018).

[h9] Paweł Zin, *Quantum dynamics of Bose-Fermi mixtures via the stochastic-wave-function approach*, Phys. Rev. A **98**, 043608 (2018) .

5 Description of the scientific goal, achieved results and applications

In the theoretical description of ultracold atomic gases one uses the description invented in statistical physics. First we introduce approximate description using mean-field method. It is an effective, most approximate description but the simplest one. In the atomic physics it resembles the Hartree-Fock approximation, where we solve single electron Schrodinger equation. There the interaction of the chosen electron with all the others is approximated by the presence of additional potential - mean field potential. In the case of dilute gas of bosons the description is even simpler. In such system the Bose-Einstein condensate is present - most of the atoms occupy the same mode and are described by the single particle wave-function. To describe the evolution of this single particle wave-function one introduced nonlinear Schrodinger equation called Gross-Pitaevskii equation (GP). There the interaction between the atoms is described by the nonlinear term. This equation describes many phenomena observed in the ultracold dilute bosonic gases. On the other hand there are a number of phenomena where such description turns out to be inadequate. Then, we say, that such phenomena are caused by the fluctuations. We divide the fluctuation present in the system into thermal one, caused by the nonzero temperature, and quantum fluctuations, present even in the ground state of the system. The series of articles described below presents investigations of phenomena caused by above mentioned fluctuations.

The works [h1]-[h3] present the investigations of quantum phase transitions. These are phase transitions taking place at zero temperature where the transition is caused by the change of one of the parameters of the system (for example atomic interaction). In these works the fluctuation of the order parameter is determined while crossing the phase transition.

The works [h4]-[h7] are devoted to the study of correlated atomic pairs. These works are divided into two parts. [h4] and [h5] are devoted to the study of properties of atoms scattered in the collision of Bose-Einstein condensate, where as [h6] and [h7] concern the properties of atomic pairs produced by the parametric resonance process. These phenomena cannot be described within mean-field theory - one needs to take fluctuations into account.

Another system which cannot be described by mean field theory is the subject of [h8]. There the system of *quantum droplet* was investigated. This existence and properties of this droplet is due to quantum fluctuations. The last work of the presented series [h9] describes the stochastic method used in investigation of properties of bose-fermi mixtures. The goal of this method to describe above mentioned systems where the application of the mean field method is inadequate.

5.1 Fluctuations in quantum phase transitions.

This series of works [h1]-[h3] descries quantum phase transitions. First two manuscripts are connected with the phenomena of spontaneous symmetry breaking taking place in the mean-field theory. It turns out, that the solution of the Gross-Pitaevskii equation, which respects the symmetry of the trapping potential, breaks this symmetry when the attractive atomic interaction exceeds the critical value. It is possible since GP equation is a nonlinear equation and its stationary solution do not need to respect the symmetry of the trapping potential. It happens in the case of two symmetric wells potential, where for small attractive interaction the atoms equally populate both wells. However for larger interaction the atoms tend to gather in one, randomly chosen, well. In the described system one can introduce the order parameter as the population difference between both wells. Than the mean field theory predicts the value of the order parameter to be zero, for interaction smaller than the critical value. However after exceeding the critical value the order parameter starts to be nonzero. The important theoretical question is how to describe such system in full quantum treatment. Derivation of such description is the subject of work [h1]. For the system described above the approximate two mode description was applied. For many systems such description is fully justified. In such case the Hilbert space is spanned by the Fock states $|n_1, n_2\rangle$ where n_1 and n_2 denote the number of atoms in the first and second well respectively. If one additionally uses the number conservation i.e. $n_1 + n_2 = N$, then such states are described by the number difference $n = n_1 - n_2$. Therefore the most general quantum state takes the form $|\psi\rangle = \sum_{n=-N/2}^{N/2} c_n |\frac{N+n}{2}, \frac{N-n}{2}\rangle$. When N is of the order of thousands or more, to a good approximation, n can be treated as a continuous parameter. Using such approximation in [h1] the Schrodinger equation

for probability amplitude c_n (which can be called wave-function) was derived. It turns out that the derived equation has the form of Schrodinger equation of a fictitious particle in a one dimensional space, where the spatial coordinate is a continuous parameter n. In this equation the kinetic energy operator is equal to $\frac{JN}{2}\frac{d^2}{dn^2}$ where one notice that the effective $\hbar^2/m$ is equal to JN where J is the parameter describing the tunneling between the two wells. The external potential that is present in the above mentioned Schrodinger equation is the function of the interaction parameter. Important results of the paper is that the external potential turns out to have the shape of a Landau potential appearing in the phenomenological theory of phase transitions. There, however, it was introduced as a phenomenological description of the phase transition, and here we derive such potential from first principles (full many body quantum treatment). This potential has single minimum for $n = n_1 - n_2 = 0$ for the values of the interaction parameter smaller than the critical value. On the other hand the potential has two symmetric minima $n = \pm n_0$ for $n_0 > 0$ for interaction parameter above the critical value. It is important that the potential has quadratic expansion around the minimum for every value except the critical one, where it has a quartic form. Additionally it turns out that the potential is equal to the mean-field energy, thus the minima of the potential are the stationary solutions of the mean-field theory. The following two system properties follow from the shape of the potential

- the quantum fluctuations are the largest at the critical point
- in the case of two minima, the many body wave-function of the system $|\Psi\rangle$ in a good approximation becomes a superposition of the solutions of the mean-field theory i.e. $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle^{\otimes N} + |\psi_2\rangle^{\otimes N}\right)$ where $|\psi_{1,2}\rangle^{\otimes N}$ denotes the state of N atoms occupying the same mode described by the single particle wave-functions $|\psi_{1,2}\rangle$. These wave functions $|\psi_{1,2}\rangle$ are the solutions of the mean-field theory which break the symmetry of the system - $|\psi_{1,2}\rangle$ denotes the solution with the maximum located at first and second well respectively.

In order to check how universal are the observed phenomena, another system that posses spontaneous symmetry breaking solutions of the mean-field theory, was investigated. This system was a homogeneous, one dimensional bose gas in a finite box with periodic boundary conditions, with the attractive interaction between atoms. As in the previous system, the solution of the GP equation is homogeneous below the critical value of the interaction parameter. After crossing that value, the solution becomes nonuniform with the maximum located in randomly chosen position. When inspecting many body quantum theory, one may find that a three mode approximation of the above system

turns out to be a good one. The three mentioned modes correspond to the wave-vector equal to zero and plus and minus the smallest possible value of the wave-vector. Using the conservation of total particle number one may additionally eliminate the operators of the $k = 0$ mode, effectively obtaining a two mode system. Next step was to introduce the position representation of the annihilation and creation operators of the two modes, i.e.

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x} + x\right) = \frac{\hat{x} + i\hat{p}}{\sqrt{2}} \qquad \hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(-\frac{\partial}{\partial x} + x\right).$$

As a result the Hamiltonian was given by an operator on a two dimensional space. It turned out, that to a good approximation, the Hamiltonian reduces to the one of a fictitious particle in a two dimensional space subject to an external potential. This potential turns out to be the energy of the mean-field theory. One needs to add, that each point in a two dimensional space correspond to a position of a maximum of a solution breaking the symmetry of the system, and additionally gave the information about how much the symmetry was broken. The center of the coordinate system corresponds to a uniform solution that does not breaks the symmetry of the system. The external potential depends on the interaction between the atoms. For the interaction smaller and equal to the critical one, the potential has one minimum at the center of the coordinate system. For larger interactions the potential has a shape of a Mexican hat with the minimum given by the valley of this hat. The degeneration of the minimum corresponds to the degeneration of the solution that breaks the symmetry of the system - the position of the maximum of such solution is a continuous parameter. Additionally, before, the expansion around the minimum is given by the quadratic form apart from the critical point, where is is given by the quartic form. If one takes the order parameter as some measure of the symmetry breaking, than from what is written above one finds, that the largest fluctuations of the order parameter are at the critical point. Additionally the many body wave function of the system $|\Psi\rangle$ turns out to be given by the superposition of the solutions of the mean-field theory i.e. $|\Psi\rangle \propto \int_0^L \mathrm{d}x\, |\psi_x\rangle^{\otimes N}$, where L denotes the length of the system and ψ_x is single particle wave-function that breaks the symmetry and has a maximum at x. The results described above constitute the manuscript [h2]. The phenomena observed here are analogous as the one seen in the previous system. In these two cases, when the mean-field theory signalized second order phase transition, the many body theory gave the description analogous to the phenomenological Landau theory.

As written above, the fluctuations of the order parameter turned out to be be the largest at the critical point. However they were relatively small. Still it was interesting to find a system having large quantum fluctuations. A system, that potentially could have such property, is a two bose gases mixture in double well symmetric potential.

Such a system can be approximately described by two spatial modes, which in the case of two different gases gives four modes. Therefore a single quantum state has a form $|n_a, n_b; N - n_a, N - n_b\rangle$, where n_a, n_b denotes the number of atoms in the first well of a a and b species respectively, while $N - n_a$ and $N - n_b$ are this numbers in the second well. Here for simplicity the same total number of atoms of both species, equal to N, are assumed. In the system we deal with three independent constants U_{aa}, U_{bb} and U_{ab} describing the interaction between the atoms. Here one notice the interaction between the species a-a, b-b and a-b. In the situation $U_{aa} = U_{bb} = U_{ab}$ one finds interesting case of energy of interaction degeneration. Namely the interaction energy has the same value for each of the states of the form $|n_a, n_b; N - n_a, N - n_b\rangle$ where $n_a + n_b = N$. It means that the state has the same interaction energy for the value of n_a changing from 0 to N (on the same time n_b has to change from N to 0). Such a situation gives maximal fluctuations of each of the bose component. In the Hamiltonian, apart from interaction energy, one finds a tunneling term. The investigation of how this term influences such large fluctuation was the subject of manuscript [h3]. There the conditions for the existence of the described fluctuations are presented.

5.2 Description of creation and properties of correlated atomic pairs

One of the directions in research in physics is the formation and manipulation of entangled quantum states. It is known, that in interferometry the use of entanglement improves the sensitivity of the measurement of the certain quantities. An example is the use of entangled states to improve the sensitivity of the measurment of gravitational waves in the interferometer located in Hanover. The increase of the sensitivity of the measurement helped to detect this tiny fluctuations of spacetime. The creation and manipulation of entangled states is now the research goal of many experimental groups in cold atomic physics. There are many systems and processes in which such states are generated. In the research presented below, two of such processes, in which states of definite number of pairs of atoms are created, are described. In extreme case we deal with single atomic pair. In the field of quantum optics the state of single pair of photons, were created many years ago, and used in many experiments, showing entanglement of such state. For example in the Hong-Ou-Mandel effect or breaking the Bell inequalities. The obvious idea is to perform such experiments using atoms instead of photons. This is the way undertaken, among other also by Chris Westbrook group from Institute d'Optique located near Paris. They have observed the Hong-Ou-Mandel effect [1] and made a two atom, four mode interferometer, that is to be used in the breaking Bell inequalities experiment [2]. One

of the processes that they used in creation of atomic pair state is the collision of Bose-Einstein condensate [3].

5.2.1 Properties of atomic pairs generated in collision of Bose-Einstein condensates

In this case in counter propagating clouds the collisions of atoms takes place, which leads to scattering of atomic pairs. Due to the energy and momentum conservation, the two atoms constituting a pair have opposite velocities. However, due to the s-wave scattering, the direction of single velocity is random. The experimental measurements performed for such scattered atoms, showed breaking of Cauchy-Schwartz inequality, which proves the existence of entanglement in this system [4]. The longstanding goal of such experiments is to use such quantum state in breaking Bell inequalities. However, before such experiment is to be performed, there is a need of conducting simpler measurements, which check if the quantum state present in the experiment is that what is expected. For example it is necessary to check if indeed the correlated pairs are present. Another example is the measurement of two particle correlation function and comparison of the experimental results against theoretical calculations. It is a very important test of the experimental equipment and in particular the single atom detector, which are essential in breaking the Bell inequalities experiment. The lack of agreement between experiment and theory may suggest problems with the experimental setup. The experimentalists need agreement between experimental measurements and theoretical predictions of the two particle correlation function, as an argument that the experimental conditions are well controlled. It is necessary before conducting much more difficult breaking Bell inequalities experiment. Because of the above, the need of theoretical investigations of properties of atoms scattered in collisions of Bose-Einstein condensate occurred. Investigations of this properties is the content of works [h4] and [h5].

In the case of system under investigation there exact solution of the many-body Schrodinger equation is not known. Therefore it is necessary to use approximate methods. Most of the experiments of Bose-Einstein condensate collisions are performed in so called collision-less regime. In such case the probability of atom scattering from colliding cloud is much smaller than unity. This enables of neglecting secondary collisions (collisions of scattered atoms with the atoms of the condensates) and permits the use of Bogoliubov method. In this method, the colliding Bose-Einstein condensates are described by single particle wave-function (all the atoms are described by the same wave-function) satisfying

Gross-Pitaevskii equation

$$i\hbar\partial_t\psi(\mathbf{r},t) = \left(-\frac{\hbar^2}{2m}\triangle + g|\psi(\mathbf{r},t)|^2\right)\psi(\mathbf{r},t). \tag{1}$$

Thus this is the description within mean-field theory. The scattered atoms halo are described by field operator $\hat{\delta}(\mathbf{r},t)$ satisfying linear Heisenberg equation

$$i\hbar\partial_t\hat{\delta}(\mathbf{r},t) = H_0(\mathbf{r},t)\hat{\delta}(\mathbf{r},t) + B(\mathbf{r},t)\hat{\delta}^\dagger(\mathbf{r},t) \tag{2}$$

where

$$H_0(\mathbf{r},t) = -\frac{\hbar^2}{2m}\triangle + 2g|\psi(\mathbf{r},t)|^2, \tag{3}$$

$$B(\mathbf{r},t) = g\psi^2(\mathbf{r},t). \tag{4}$$

The nonlinear terms in Heisenberg equation (not present above), describe secondary collisions. As we restricted the investigations to systems in which this processes are very unlikely to happen, these nonlinear terms are neglected. The conditions of many experiments additionally allow for perturbative solution of the Heisenberg equation (in the lowest order of perturbation). This enables to obtain the analytic expressions for such observables as single particle density and two particle correlation function of the scattered atoms. The formulas for these expressions are given as integrals over space and time of the integrand, made of time dependent condensate wave-function and single particle propagators of the H_0 Hamiltonian. They described the scattered atoms subject to the potential generated by the condensate. This potential describes the interaction between the scattered atom and the atoms of the condensate. The analytic form of above mentioned single particle propagator is not known. The same happens with the condensate wave-function where the analytic solution of the colliding condensates is, in general, not known. Thus in order to analytically calculate two particle correlation function one needs to perform approximations. The simplest idea is to neglect the nonlinear term in the GP equation and, in the same time, neglect the potential $2g|\psi|^2$ in the single particle propagator (so approximating the H_0 Hamiltonian by the kinetic energy operator). Then the GP equation and single particle propagator equation can be solved analytically. Thus, mentioned above observables are given by analytical formulas. This enables evaluation of the two particle correlation function. In work [h4] using the above approximations the calculation of the two particle correlation function for the parameters of the experiment carried in Palaseau near Paris, was performed. There are two parts of two particle correlation:

- opposite particle correlation (back to back). These are present because the atoms are scattered in pairs of opposite velocities.

- Correlation of velocities close to each other (local correlations). These are present due to the fact, that we deal with bosons and in addition scattering of atoms is an incoherent process. Because of that bunching effects takes place.

In [h4] the theoretical calculation were compared with experimental results. Good comparison was obtained for opposite particle correlation. In the case of local correlations the significant differences were observed between the experiment and the theory. However as written above the theoretical calculations were perfomed upon neglecting the interaction between condensate atoms, and between condensate atoms and scattered atoms. The impact of the neglected interactions on the two particle correlation function was not known. Thus the obtained comparison did not gave much. It was necessary to take into account the neglected interaction.

This was the central point of the work [h5]. In the case of single particle propagator, it was crucial to use the fact, that the characteristic size of the potential coming from the condensate atoms, was much larger that the wavelength of the scattered atoms. This enabled the use of semiclassical approximation, which gave the analytical (approximated) form of the single particle propagator. This enabled to derive the analytical expressions for the two particle correlation function and other observables of interest. All these expressions contained time dependent condensate wave function. Thus to calculate the two particle correlation function the GP equation needed to be solved. As written above, the exact solution of the GP equation in the case of collisions between condensates, are not known. Thus one had two possibilities. One could, for a single given set of parameters, solve GP equation numerically and then, for such parameters, calculate numerically the quantities of interest. Alternatively, one could solve GP equation approximately using variational method - then the solution would be analytical and one would obtain a quantities of interest for wide range of parameters. In work [h5] the second possibility was chosen. Having analytical form of the condensate wave-function a two particle correlation function for a wide range of parameters was investigated. It was important to find a maximum and widths of this correlation function as a function of experimental parameters. This information are crucial for planning breaking Bells inequality experiments. In [h6] the above mentioned properties of the correlation function were obtained in a form showing explicit dependence on experimental parameters. Additionally in this work a semiclassical limit of the above system was investigated. The observables, suggested to have semiclassical limit, were single particle density and opposite part of two particle correlation function. The physics standing behind these observables are a classical process of binary atom collisions. To find this quantities it was enough to use simple classical model, where colliding clouds were described by single particle phase space distribution.

In [h6] it was shown that the best agreement between quantum and semiclassical model were obtained, if the single particle phase space distribution was chosen as the Wigner function calculated using condensate wave-function. This agreement between the models turned out to be excellent for a wide range of experimental parameters. On the other hand the agreement between two models in the case of opposite part of two particle correlation function turned out to be only partly satisfactory.

The two particle correlation function depends on positions of two atoms. The best parametrization of this function is the sum and difference of atoms position. In the opposite correlation case the difference of positions is much larger than its sum. This is because the positions have nearly opposite vectors. It turns out that the correlation function averaged over position difference (than it depends only on sum of positions) has the same form, as this quantity obtained from the semiclassical model, for a wide range of experimental parameters. On the other it was shown in [h6] that in the case of positions difference dependence (averaged over sum of positions) the quantum model gives, in general, different results than the semiclassical one. This shows a *quantum* character the position difference dependence. The analysis of analytical expressions gives some explanation of this observation. The opposite part of two particle correlation function is give by the product of two temporal integrals. On the other hand the semiclassical mode has only one temporal integral - this is a sum of scatterings coming from different moments of time. It turns out, that averaging two particle correlation function over position difference or obtaining single particle density, leads effectively to changing two temporal integrals into single one. This, however does not prove, but *gives a chance*, that the discussed quantities can have semiclassical counterparts. Averaging two particle correlation function over sum of positions still leaves two temporal integrals, which shows that there is no possibility of having semiclassical counterpart.

The situation gets different in the case of local part of two particle correlation function. Within Bogoliubov method this function is equal to the modulus squared of the single particle correlation function. So the quantity to be calculate is single particle correlation function $G^{(1)}$. This function may be represented as fourier transform of scattered atoms single particle Wigner function:

$$G^{(1)}\left(\mathbf{r}+\frac{\Delta\mathbf{r}}{2},\mathbf{r}-\frac{\Delta\mathbf{r}}{2},T\right)=\int \mathrm{d}\mathbf{k}\, e^{-i\mathbf{p}\Delta\mathbf{r}/\hbar}W(\mathbf{r},\mathbf{p};T)$$

where $\mathbf{p}$ denotes atomic momenta and T time of measurement. Using a semiclassical model one obtains the single particle phase space density of scattered particles. Such phase space density can be obtained using Boltzmann like model, where the colliding clouds are described by single particle phase space density. Equating, the above phase

space density of scattered particle with the Wigner function present in the above formula, enables to calculate single particle correlation function, and as a consequence, local part of two particle correlation function, using a semiclassical model. The method described above gives a possibility of calculating local part of the two particle correlation function using a semiclassical model. Looking at the above formula, one clearly sees that this calculation is not purely semiclassical. In the above formula one notices the presence of the Planck constant, However, in this case, one only uses de Broglie'a formula, which gives the relation between momenta and wavelength of the moving atoms.

Additionally the Wigner function present in the above formula was derived within many body quantum model. It turned out, that for wide range of parameters, the Wigner function derived from many body quantum model was almost the same as the one derived from semiclassical model described above. The results of, described above, investigations were presented in large work [h6].

5.2.2 Description of atomic pairs properties generated via parametric resonance process

For some purposes one needs atomic pairs with given direction of emission. One just needs to know where and in what time the pair of atoms shall be present. In such case one needs to use another process than collisions of Bose-Einstein condensates (where atoms are scattered in random direction). One of such processes was realized experimentally in prof. Westbrook group in Palaseiau [5]. In this experiment the ultracold cloud of atoms in cigar shaped trap was obtained. Than, the atom trapping force was periodically varied in time. Looking at this system as quasi-one dimensional, such periodic change causes periodic temporal modulation of one-dimensional interaction constant. It turns out that this causes emission of pair of atoms, with opposite velocities, along elongated direction of trapped cloud. Additionally the energy of emitted atoms is directly connected to the frequency of modulation of the atom trapping force. True to form in the experiment the velocity correlated pairs of atoms were observed.

The above described process is a example of parametric resonance. One of the parameters of the system was changed periodically and pairs of atoms were generated, with frequency (energy of atoms divided by Planck constant), being in resonance with frequency of periodic changes. This process was investigated using Bogoliubov method. The theoretical calculation predicted existence of entanglement in such system. The suggested way of proving entanglement in such system was the measurement of so called number number squeezing parameter. According to the predictions based on Bogoliubov method, the value of number of number squeezing parameter should be below certain critical value,

which would prove existence of entanglement in the system (this connection was proved in work [o20]). Unfortunately, despite the expectations, the value of the number squeezing parameter, turned out to be larger than the critical value. This mean, that the measurement did not prove the existence of entanglement in the system.

In the Bogoliubov method part of the terms present in the Hamiltonian are being neglected. These terms are responsible for the interaction between system quasi-particles (Bogoliubov method defines and describes non-interacting quasi-particles). The observed discrepancy between theoretical predictions and experimental measurements suggested, important influence of the neglected Hamiltonian terms on the system properties. Thus it was obvious to take into account the neglected interaction terms. As the system of interest is out of equilibrium (the time dependent force acts on the system) the Keldysh method was chosen as the suitable one. It is quite standard method of quantum field theory to deal with nonequilibrium systems. Using certain approximation applied to the self energy (which appears in the Dyson equation) the time dependent Dyson equation for the single particle Green's function, was solved analytically. Additionally the higher order correlation function were expressed using single particle Green's function. This enabled to find analytical formulas for the quasiparticles population and number squeezing parameter as a function of time. It turned out that the interaction between quasiparticles manifests itself in the, mentioned above, expressions, through parameter called quasiparticle decay constant. This parameter is an inverse of quasiparticle lifetime determined for equilibrium state of the systems. On the other hand, the speed of quasiparticle generation is given by single parameter (called amplification parameter) present in the Bogoliubov method. The results showed dramatic influence of the quasiparticle interaction on the increase of quasiparticle population as well as the value of the number squeezing parameter. Generally, when the quasiparticle decay constant is larger than the amplification constant, the atomic pair production process is practically frozen - the increase of the pair population in time is very small and tends to a constant at infinite time. Additionally in this case the value of the number squeezing parameter is usually above the critical value. All these clearly shows that in such case the system is not an efficient source of entangled atomic pairs. The situation is different in the other case. If the amplification parameter is significantly larger than the quasparticle decay constant, than the pair production process is exponential in time, and the number squeezing parameter eventually, after some time, drops below the critical value. In such case the system is an efficient source of entangled atomic pairs. The above described results are the content of work [h6].

As written above all these results were obtained within certain approximation of the self energy function. This approximation is correct in the case of three dimensional

systems. However, strictly speaking, it cannot be applied to one dimensional or quasi-one dimensional systems. It means, that straightforward use of the above results to the analysis of the experimental system is not well justified.

The literature concerning the one dimensional systems, clearly showed, that the use of Keldysh method in such case would be very challenging task. Thus to investigate this system another method was chosen - it was classical field approximation. This method takes into account thermal fluctuations, neglecting the quantum one. But it turned out, that for experimental parameters the quantum fluctuations can be neglected, which justified the use of classical field approximation.

Using that numerical method for experimental parameters, the number squeezing parameter was calculated. It turned out that it value was significantly larger than the critical value, which was in agreement with experimental measurements. This results clearly showed that the value of the number squeezing parameter measured experimentally (larger than the critical value) was caused by the quasiparticle interaction. This results are the content of [h7].

The works described above, clearly showed the necessity of taking into account the quasiparticle interaction, while designing the source of entangled atomic pairs.

5.3 The description of the properties of quantum droplets.

The dilute, homogeneous, bose gas was the subject of extensive theoretical investigations. In particular its ground state energy was analyzed. It was shown that the first two terms in the ground state energy expansion depends only on the s-wave scattering length of the interaction potential. The first term in this expansion is given by the mean-field theory, where the energy density is proportional to the square of the density of the system multiplied by the s-wave scattering length. The second term in the expansion, called Lee-Huang-Yang (LHY) energy, is caused by the quantum fluctuation [6]. It is equal to the first term multiplied by the attenuation parameter to the power two thirds. This parameter is equal to the ratio of the scattering length to the mean inter-particle distance. In the case of dilute gas this parameter is much smaller than unity, which means that the second term in expansion is much smaller than the first one (that is why it is a correct expansion). In most of the experiments carried so far and its theoretical explanations, the LHY term was neglected, as it was usually much smaller than the mean-field energy.

This situation changed recently, due to the research conducted by Dymitr Petrov [6]. He was investigating the two bose gas mixture. There are three s-wave scattering lengths in such a system: between two atoms of the first component, between two atoms of the second component and between one atom of the first and second atom of the second

component. In his investigations Petrov took two first lengths as positive and the third one as negative. In such a configuration the mean-field theory predicts two possibilities:

- gas fills whole available space in the container uniformly - this happens when the modulus of the negative scattering length is smaller than some critical value.
- the collapse of the mixture takes place - it happens in the opposite case as described above.

Collapse means, that the solution of the mean-field equations tends towards larger localization together with increase of the density to infinity. In fact it means that the mean field theory does not work any more.

Petrov showed, that in such case the LHY energy is positive. Additionally, he showed, that the LHY energy grows faster with density increase, that the mean-field energy. Thus the existence of the LHY energy causes the stabilization of the system - the collapse shall not take place. Petrov showed that in such a case a localized cloud shall be formed, with the density given by the properties of the LHY energy. An interesting state of the system is formed. If one defines the gas state as the one in which the atoms fill the whole available space in the container, than in the above described case we do not deal with gas state. Instead the atoms tend to localize in space with given constant density. When the number of atom increases, the localized state enlarges keeping the same density. Of course it posses also a finite edge, where the density drops to zero on a finite length. This properties usually characterize the liquid state. Therefore this state was called *quantum droplet* state. The adjective *quantum* appears because the droplet is formed due to quantum fluctuations. One needs to add, that in such a state, the mean interparticle distance is still much larger than the scattering lengths. Therefore we deal with weakly interacting system, where the use of perturbative methods (for example Bogoliubov method) is fully justified. The described above droplets were observed in the experiment [7]

Soon after his first work D. Petrov published another one discussing similar systems in two and one dimensional configurations [8]. It turned out, that the quantum droplets exist there as well. In the experiment, the lower dimensional systems are obtained by tightly squeezing the atoms with trapping forces in one or two directions. In his work Petrov did not gave any condition how strong the squeezing has to be in order to obtain quasi-low dimensional system. This problem became recently crucial, when the experimental groups started to work on obtaining quantum droplets in lower dimensional systems. The solution of such problem is the content of work [h8].

In this work the uniform system in a box of size L_x,L_y andL_z z with periodic boundary conditions was analyzed. In order to simulate the tight squeezing leading to two dimen-

sional system the limit $L_x, L_y \to \infty$ was taken with $L_z = L$ being fixed. Similar procedure was adopted to simulate quasi-one dimensional system ($L_x \to \infty$ and $L_y, L_z = L$ being constant). In the rest of the work the LHY energy term was calculated for such systems in the critical point (just before the mean-field theory predicts collapse of the mixture). At this point one deals with uniform system with two kinds of excitations: first are well known Bogoliubov quasiparticles, second the quasiparticles w having spectrum being a quadratic function of the quasiparticle momenta (identically as for non-interacting atoms). It turned out that only first kind of quasiparticles contributes to the LHY energy. Additionally it was shown that it contributes in the same way as the single component bose gas does. This showed a clear connection between the LHY energy of the mixture and single component bose gas. There appeared one dimensionless parameter which was a ration of of characteristic interaction energy per atom to the kinetic energy of the lowest possible excitation in the squeezed direction. Characteristic energy of interaction was in this case proportional to $n_1 a_{11} + n_2 a_{22}$ where $n_{1,2}$ is a density of first and second component while $a_{11,22}$ denotes the scattering length between two atoms of first and second component respectively. In [h8] the dependence of the LHY energy as a function of this dimensionless parameter was computed numerically. Additionally the analytical expansion for small values of the above mentioned parameter was found. It enabled, using local density approximation, for obtaining the density of the quantum droplet in an analytical formula. The relation between three dimensional and lower dimensional scattering lengths, enabled for expressing this density as a function of lower dimensional quantities (lower dimensional scattering length and lower dimensional density). For very small values of the dimensionless parameter the derived expressions for the quantum droplet density turned out to be identical with the ones derived by Petrov in his work [8]. This was expected because very tight squeezing should lead to lower-dimensional systems. But it was surprising that in order to obtain lower dimensional systems the value of the dimensionless parameter needed to be very small - smaller than 3/100. Such a small value shall be very hard to reach in the experiment. Additionally the work showed another astonishing fact. For values of the parameter above 3/10 the LHY energy was practically equal to the one given by the three dimensional formula. Naively thinking such behavior was expected to happen for the values of the dimensionless parameter much larger than unity. In such a case many of the "squeezed" modes are excited and the system acts as being three dimensional. However in our case, when the value of the parameter is 3/10 it is hard to speak about excitation of the first squeezed mode, and still the system adopts a three dimensional value. This also shows how strong one needs to squeeze the atoms to obtain any deviations from the three dimensional physics.

5.4 The description of the bose-fermi mixture via stochastic method approach

This research are the continuation of investigation described in works [o9]-[o12] undertaken during the collaboration with P. Deuar for Institute of Physics Polish Academy of Sciences in Warsaw. Peter Deuar was actively developing a Positive-P method [9]. It is a stochastic method which exactly maps the many body Schrodinger equation on a set of classical field differential equations possessing a stochastic noise. Such mapping is shown to take place in the case of two particle interaction together with the presence of an external potential. However Positive-P method has certain limitations of use. The quantum averages are obtained as an averages many realizations of classical trajectories. Using the performed simulations the uncertainty of the mean value of the given observable is being determined. The value of this uncertainty depends on the number of realizations and goes to zero with the number of realizations tending to infinity. From the literature it is known that this uncertainty becomes very large after some time. This time may be enlarged by increasing the number of realizations, but in practice, it turns out that it does not give substantial improvement. Thus the Positive-P method, in practice, gives the possibility of investigating the evolution of the system up to some characteristic time. It is optimistic that for many systems this time is long enough to observe and analyze interesting physical phenomena. The above shows that the Positive-P method describes quantum fluctuations and their impact on dynamics of the system.

The above described Positive-P method in its original formulation was made to describe the bosonic systems. It fundamentally makes use of bosonic coherent states properties. A short time ago new exact approach to map the many body Schrodinger equation on to the set of stochastic equations was formulated. It was called *stochastic wave-function approach* [10]. It is an analog of the Positive-P method ,which instead of bosonic coherent states (which are states of indefinite number of atoms), uses N-particle coherent states, which are analog of coherent states but with conserved number of particles. Additionally the stochastic wave-function approach has much simpler and shorter derivation of stochastic equations than Positive-P method. Original formulation of stochastic wave-function method was performed for bosons interacting via two particle potential. In the case of fermions the analog of bosonic stochastic wave-function approach was formulated few years after the bosonic one [11]. And the same as for bosons it was formulated for two particle interaction potential. Such situation suggested the possibility of generalization of such stochastic method to bose-fermi mixtures. The derivation of such method is the content of [h9].

In this work the compilation of methods known for bosons and fermions was used.

As written above, the bosonic method uses N-particle coherent states. It is the simplest N-particle bosonic state, where all N particles of the system populate the same single particle mode. General theorem states that any N-particle bosonic state can be represented as a superposition of un-normalized N-particle coherent states with identical, positive probability amplitudes (the un-normalized state means, that its norm does not need to be equal to unity). This theorem enables to introduce stochastic method. Let us notice, that un-normalized N-particle coherent state is defined using un-normalized single particle wave-function (classical field). The stochastic method is defined as an differential equation, for this single particle wave-function, with some additional stochastic noise. Starting from N-particle coherent states, the stochastic method, transforms this state into another N-particle coherent state (un-normalized). In each of the realizations it shall be a different final state, due to different realizations of the stochastic noise. After many realizations, the exact quantum state is approximated, by the sum of realizations of final N-particle coherent states divided by the number of realizations. In the above described way, the stochastic wave-function method for bosons is introduced.

It was not written above, how the stochastic equations are obtained and how it is proved that the above described procedure is a correct one i.e. that the above described sum of N-particle coherent states tens to true quantum state wit the number of realizations tending to infinity. There the crucial fact is that the quantum evolution under single particle operators of N-particle coherent states transforms, N-particle coherent state into another state of the same form. If one would evolve N-particle coherent state with two body operators then the result would not be a N-particle coherent state but more complex state. In work [10] it was shown that the evolution operator $\exp(-i\hat{A}_2\Delta t)$ where $\hat{A}_2$ is a two particle operator can be written as a sum of evolutions under single particle operator, i.e. $\exp(-i\hat{A}_2\Delta t) = \lim_{M\to\infty}\frac{1}{M}\sum_{j=1}^{M}\exp(-i\hat{A}_{1,j}\Delta t)$ where $\hat{A}_{1,j}$ is a single particle operator. The above mentioned operator $\hat{A}_{1,j}$ has stochastic noise inside. As a result we have

$$\exp(-i\hat{A}_2\Delta t)|\psi\rangle_N = \lim_{M\to\infty}\frac{1}{M}\sum_{j=1}^{M}\exp(-i\hat{A}_{1,j}\Delta t)|\psi\rangle_N = \lim_{M\to\infty}\frac{1}{M}\sum_{j=1}^{M}|\psi_j\rangle_N$$

where $|\psi_j\rangle_N = \exp(-i\hat{A}_{1,j}\Delta t)|\psi\rangle_N$. In the above the action of the operator $\exp(-i\hat{A}_2\Delta t)$ on N-particle coherent state $|\psi\rangle_N$ is represented as a sum of N-coherent states $|\psi_j\rangle_N$.

The fermionic stochastic wave-function approach is introduces in an analogous way. Instead of bosonic N-particle coherent states a N-particle Slater state [11] is used. A Slater state is the simplest fermionic N-particle states, which is a Slater determinant of N orthogonal single particle wave-functions. Just as in the case of bosons, the general theorem states, that every N-particle fermionic state can be represented as a superpo-

sition of un-normalized N-particle Slater states with constant and positive probability amplitude. The fermionic stochastic method introduces stochastic equations evolving N-particle Slater states. This reduces to evolution of N un-normalized single particle wave-functions. As written before, such equations were derived in the case of two particle interaction.

Making use of the above results, in the work [h9] which deals with bose-fermi mixtures the state being a product of N_1-particle coherent state and N_2-particle Slater state was introduced (in this system one deals with N_1 bosons and N_2 fermions). In [h9] a successful decomposition of evolution operator $\exp(-i\hat{A}_n\Delta t)$ as a sum of single particle evolutions $\exp(-i\hat{A}_{1,j}\Delta t)$ was made. Here $\hat{A}_n$ is a sum of operator describing any two particle interaction between atoms, and operator describing higher than two particle interaction of special form. Additionally $\hat{A}_{1,j}$ denotes a single particle operator possessing stochastic noise. Using that representation as stochastic equations for single particle wave-function defining N_1-particle coherent state and N_2 single particle wave-functions defining N_2-particle Slater state were derived. In this way a stochastic wave-function approach was generalized to bose-fermi mixtures and higher than two particle interaction.

6 Description of other scientific achievements

6.1 Correlation of atoms scattered in the collisions of Bose-Einstein condensates

These are the investigations undertaken before works [h4],[h5]. There the investigation of atoms scattered in the collisions of *spherical* Bose-Einstein condensates was considered. The main goal of this investigation was the analysis of transition between the regime where the spontaneous process dominates, to a regime of bosonic enhancement domination. In the work [o1] the numerical calculation illustrating this transition were carried. In [o2] the analytical calculation of this passage were presented.

[o1] P. Ziń, J. Chwedeńczuk, A. Perez, K. Rzążewski, M. Trippenbach, *Quantum multi-mode model of elastic scattering from Bose Einstein condensates*, Phys. Rev. Lett **94**, 200401 (2005).

[o2] P. Ziń, J. Chwedeńczuk, M. Trippenbach, *Elastic scattering losses from colliding Bose-Einstein condensates*, Phys. Rev. A **73**, 033602 (2006).

6.2 The simulation of single realization of many body quantum system

This series of works is devoted to the study of the single simultaneous measurement of positions of all the atoms of the system. According to the laws of quantum mechanics, the modulus square of the many-body wavefunction gives the probability density of the measurement of positions of all the atoms. Of course these positions can take very different values depending on the draw. But in case of many quantum states it was expected that the result of *typical* measurement has certain properties. This was the case taking place in [o3]. There the atoms scattered in the collision of Bose-Einstein condensate were investigated. The parameters of the system were chosen such that the scattering was dominated by the bosonic stimulation effects. Then the naive picture of scattering looks in the following way. First pair of atoms is scattered with equal probability in all spatial directions (this is because the s-wave scattering). After that the probability of scattering of next pair to region where the first pair was scattered is magnified due to the presence of bosonic enhancement effect. As the probability of scattering due to the bosonic stimulation effect is proportional to the population of a given quantum state, so the probability that the third pair is scattered in the same place as the two others, is even larger. As a result one expects highly populated regions in space and others weakly populated. These regions should appear in pairs of opposite velocities as the atoms are scattered in such way.

The crucial technical issue of solving such theoretical problem is to perform a single draw from such many dimensional ($3N$ dimensional where N si the number of atoms) probability density. As N is a large number thus the number of dimensions is enormous. In [o3] an approximate method (suitable for the considered quantum state) of performing a single draw was developed and presented. It turned out that a density of atoms in a typical random draw looks as it was predicted above, using a naive model - one observes pairs of spikes of atoms.

In work [o4] another quantum state was investigated. It was a state $|N, N\rangle$ where exactly N atoms populates each of the two modes present. These modes can be for example two plane waves with opposite sign of single wavevector i.e. $\pm\mathbf{k}$. One clearly sees here that we deal with the interference of two Fock states. If instead of two Fock states one would deal with two coherent states than on the screen one would see interference fringes. The position of central fringe is given by the different of phases of two coherent states. In the case of Fock states, the phases are indefinite. Thinking naively in typical realization one would expect to have random phase difference, which would give random central peak position. In [12] it was shown, that the result of typical realization of measurement of

all atomic positions gives interference fringes with random central peak position - so it is as it was expected. This result was based on a numerically performed, random draw from multidimensional probability density performed in a smart way. The goal of the investigations undertaken in [o4] was to show the above described property of typical realization in an analytic way. This goal was obtained and it is the content of manuscript [o4].

[o3] J. Chwedeńczuk, P. Ziń, K. Rzążewski, M. Trippenbach, *Simulation of a single collision of two Bose-Einstein condensates*, Phys Rev. Lett **97**, 170404 (2006).

[o4] A. Dragan, P. Ziń, *Interference of Fock states in a single measurement*, Phys. Rev. A **76**, 042124 (2007).

6.3 Description of the ultracold dilute bosonic gas using mean-field theory

The works described here concern the investigations of the ultracold, dilute bosonic gas using mean-field theory, that is GP equation. The works [o5] and [o6] were devoted to the study of analytical solutions of the GP equation in the case of symmetric double well external potential. The considered system war one dimensional with attractive (work [o5]) and repulsive (work [o6]) atom interaction. These works were mainly devoted to the solutions of the GP equation which spontaneously break the symmetry of the external potential. In [o6], additionally, the dynamics of the system was analyzed within variational method. The solutions, which spontaneously break the symmetry, were considered also in works [o7] and [o8]. In [o7] the two dimensional system was considered, in which the interaction parameter was function of one of the coordinates - it was equal to zero apart from two parallel stripes. It turned out, that in such a system, the localized solutions exists (analogous to solitons) which break the symmetry of the system. Similar system was considered in [o8] where the interaction parameter was constant, but the external potential depended on one of the coordinates - it had a shape of symmetric double well. In two dimensional depedence the potential has a shape of two parallel channels. It turned out that in such system, solutions which break the symmetry exist. In addition in this work, the collision of such localized solutions were analyzed.

[o5] P. Ziń, E. Infeld, M. Matuszewski, G. Rowlands, M. Trippenbach, *Method for obtaining exact solutions of the nonlinear Schrodinger equation for a double-square-well potential*, Phys. Rev. A **73**, 022105 (2006).

[o6] E. Infeld, P. Ziń, J. Gocałek, M. Trippenbach, *Statics and dynamics of Bose-Einstein condensates in double square well potentials*, Phys. Rev. E **74**, 026610 (2006).

[o7] N.V. Hung, P. Ziń, M. Trippenbach, B.A. Malomed, *Two - dimensional solitons in media with the stripe - shaped nonlinearity modulation*, Phys. Rev. E **82**, 046602 (2010).

[o8] N.V. Hung, P. Ziń, E. Infeld, M. Trippenbach, *Symmetry breaking in the collisions of double channel BEC solitons*, Phys. D **269**, 37 (2014)

6.4 Description of atoms scattered in the collisions of Bose-Einstein condensates via stochastic methods.

This series of publication was connected with the use of stochastic methods in investigations of properties of atoms scattered in the collisions of Bose-Einstein condensates. There exist, described in the literature, stochastic method using so called *Positive-P* representation. There any quantum state can be represented as real and positive probability density in certain space. Additionally one my show, that in the case of bosons interacting via two body potential , the Schrodinger equation can be exactly mapped onto set of stochastic equation. However it happens that the derived equations are highly unstable, which causes the simulations being hard to undertake.

As written before, in many cases it is enough to use Bogoliubov method, in the description of atoms scattered from colliding Bose-Einstein condensates. Due to that fact, the natural idea was to use Positive-P method in the Bogoliubov description. Work [o9] was devoted to this problem. In this work the stochastic equation within Bogoliubov method were derived and used in the calculation of the process of atoms scattering. It was shown that the equations are much more stable and they lead to solutions which correctly describe the system. In the subsequent work [o10] the impact of the mean-field present in the GP and Heisenberg equations for the field operator, was investigated. In this work it was inspected what impact this two terms have on the mean velocity of scattered atoms in the collisions of *spherical*. This work is closely related to [o11] in which the mean-velocity of atoms scattered in the collisions of highly elongated condensates, with collision velocity having direction perpendicular to the long axis of the cloud. This work is written together with the experimental group and deals with the results of the experiment. There it was found that the shape of density of scattered atoms (in the velocity space) has, instead of expected spherical distribution, an ellipsoidal one. According to theoretical findings this sphere deformation was caused by the, mentioned above, mean field terms. In the next work [o12] the entanglement present in the quantum state of

scattered atoms, was investigated. It was shown that the best conditions for obtaining entanglement are present, when the atoms are scattered into localized regions, which is possible in the bosonic enhancement regime.

[o9] P. Deuar, J. Chwedeńczuk, M. Trippenbach, P. Ziń, *Bogoliubov dynamics of condensate collisions using the positive-P representation*, Phys. Rev. A **83**, 063625 (2011).

[o10] P. Deuar, P. Ziń, J. Chwedeńczuk, M. Trippenbach, *Mean field effects on the scattered atoms in condensate collisions*, Eur. Phys. J. D **65**, 19 (2011).

[o11] V. Krachmalnicoff, J.-C. Jaskula, M. Bonneau, V. Leung, G. B. Partridge, D. Boiron, C. I. Westbrook, P. Deuar, P. Ziń, M. Trippenbach, and K. V. Kheruntsyan, *Spontaneous four wave mixing of de Broglie waves: beyond optics*, Phys. Rev. Lett. **104**, 150402 (2010).

[o12] P. Deuar, T. Wasak, P. Ziń, J. Chwedenczuk, M. Trippenbach, *Tradeoffs for number squeezing in collisions of Bose-Einstein condensates*, Phys. Rev. A **88**, 013617 (2013).

6.5 Works on different subjects

Below the works are described, each of which describes different subject.

In [o13] the problem of the decay of Bose-Einstein condensate was considered. Such decay is due to different processes. One of such process is the collision of the condensate atoms with the 'hot' and fast atoms being in the vacuum chamber. Such atoms, present due to the finite pressure in the chamber, are not trapped. They collide with the walls of the chamber, and have room temperature. Thus, they are very fast, with respect to the condensate atoms. The collision of such atom with the condensate atom causes the latter, to leave the condensate. In [o13] the condensate with large thermal cloud was considered. It was shown, that the decay of the thermal cloud (due to the same processes as the condensate decay) lead to another process of the condensate decay - the transfer of atoms from the condensate to the thermal cloud. The results of this theoretical paper were verified experimentally few years after publishing this work, in the experiment [13].

In [o14] the classical system of two interacting atoms, placed in the rotating asymmetric harmonic trap, was considered. It was shown that, for some rotation frequencies and lack of interaction between atoms, the distance between the atoms grows exponentially with time. On the other hand, when the repulsive interaction are present, the interatomic distance stays rather constant.

In [o15] the two particle correlation function, in the ground state of dilute bose gas, was considered. In this work it was calculated in the case of helium atoms being in the metastable state.

In the work [o16] the system of relativistic interacting bosons was considered. It was described within classical fields approximation. This approximation enables to describe the system possessing Bose-Einstein condensate at nonzero temperature. In [o16] many properties of this system were found, for example excitation spectrum as a function of temperature of the system.

In [o17] the rigorous mathematical derivation of the Hartree-Fock-Bogoliubov method was considered. Additionally, in this work, few hypothesis concerning the excitation spectrum of dilute, interacting bose gas, were stated.

In [o18] the evolution of the condensate subject to double well external potential, was considered. In particular the phenomena of oscillation revivals was investigated. It was shown, that the phenomena of revivals of oscillation can be obtained using a semiclassical model.

In [o19] the Raman scattering from Bose-Einstein quasicondensate was considered. There, the impact of the temperature on the width of the density and two particle correlations function of the atoms scatter due the Raman process, was investigated,

In [o20] the relation between breaking the Cauchy-Schwartz inequality and particle entanglement was investigated. It was proved that breaking this inequality proofs the existence of entanglement in the system of indistinguishable bosons.

In [o21] the analog of Hawking radiation in the ultracold bosonic gas was considered. It was show the single particle density and two particle correlation function provides the information about (i) the existence of 'black hole' horizon (ii) the associated acoustic Hawking radiation and (iii) of the quantum nature of the Hawking process. Additionally it was shown, that the considered quantities are measurable using present-day experimental techniques.

In [o22] the same system as in [h2] was considered. However in this work it was investigated using Bogoliubov method.

[o13] Paweł Ziń, Andrzej Dragan, Szymon Charzyński, Norbert Herschbach, Paul Tol, Wim Hogervorst, Wim Vassen, *Effect of atomic transfer on the decay of a Bose-Einstein condensate*, J. Phys B **36**, L149 (2003)

[o14] J. Chwedeńczuk, P. Ziń, M. Trippenbach, B. Dąbrowska, M. Gajda, K. Rzążewski, *Harmonically Trapped Classical Gas under Critical Rotation*, Acta Physica Polonica A **104**, 399 (2003)

[o15] P. Ziń, M. Trippenbach, M. Gajda, *Pair-correlation function of a metastable helium Bose-Einstein condensate*, Phys. Rev. A **69**, 023614 (2004)

[o16] E. Witkowska, P. Ziń, M. Gajda, *Classical fields method for a relativistic interacting Bose gas*, Phys. Rev. D **79**, 025003 (2009).

[o17] H.D. Cornean, J. Dereziński, P. Ziń, *On the infimum of the energy-momentum spectrum of a homogeneous Bose gas*, J. Math. Phys. **50**, 062103 (2009).

[o18] K. Pawłowski, P. Ziń, K. Rzążewski, M. Trippenbach, *Revivals in the attractive BEC in a double-well potential and their decoherence*, Phys. Rev. A **83**, 033606 (2011).

[o19] T. Wasak, J. Chwedeńczuk, P. Ziń, M. Trippenbach, *Raman scattering of atoms from a quasicondensate in a perturbative regime*, Phys. Rev. A **86**, 043621 (2012).

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[o21] D. Boiron, A. Fabbri, P.-E. Larre, N. Pavloff, C.I. Westbrook, and P. Ziń *Quantum Signature of Analog Hawking Radiation in Momentum Space*, Phys. Rev. Lett. **115**, 025301 (2015)

[o22] B. Oles, P. Zin, J. Chwedeńczuk, M. Trippenbach, K. Sacha *Bose-Einstein condensate in a double well potential in the vicinity of a critical point* Laser Physics **20**, 671 (2010).

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Phys. Rev. Lett. **105**, 190402 (2010).

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Paweł Zin