NATIONAL CENTRE FOR NUCLEAR RESEARCH

Summary of the doctoral dissertation

Relativistic Hydrodynamics Beyond the Second Order

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Ultrarelativistic nuclear collisions, performed at the Relativistic Heavy Ion Collider and the Large Hadron Collider, reach temperatures high enough to melt hadrons and create a quark-gluon plasma. This highly dynamical process is difficult to describe purely using the underlying quantum field theory of quantum chromodynamics. Many different simpler models are employed to study the various stages of this process. In particular, relativistic hydrodynamics has been successfully applied to describe the expanding QGP.

Relativistic hydrodynamics is a theory of close to equilibrium dynamics, but exactly what sets the limit of its applicability is not known. In both experiment and theoretical modeling of nuclear collisions it has been observed that a hydrodynamic description can work surprisingly far from equilibrium. This thesis concerns two different approaches towards understanding the limits of hydrodynamics: constitutive relations at large order and far from equilibrium.

Constitutive relations are central to the formulation of hydrodynamics and they are usually described by a perturbative series in gradients, whose form is highly constrained by the symmetries of the theory. Hydrodynamic models are often constructed by describing constitutive relations as a perturbative series in gradients, and then truncating this series at e.g. second order. However, the hydrodynamic limit of a quantum field theory such as QCD, weakly coupled models such as kinetic theory or strongly coupled holographic quantum field theories give rise to a gradient expansion as an infinite series. To understand the hydrodynamic limit of such models, the properties of this series should be understood. Often, the gradient expansion diverges, and the articles in this thesis contribute towards understanding how and why it does so.

Calculating the large order gradient expansion is technically difficult, and has been done mostly for simple models in very symmetric flows such as the boost-invariant (Bjorken) flow. The research in this thesis shows the divergence in more models and flows. The reason for divergence involves an interplay between hydrodynamic and nonhydrodynamic modes and we deepen this connection. We use tools from the mathematical theory of resurgence to extract information and make sense out of divergent series. This theory offers a way to improve the gradient expansion, by combining hydrodynamic and nonhydrodynamic effects into a series which includes both perturbative and nonperturbative effects, a transseries.

The most detailed calculations are performed in Relaxation Time Approximation Kinetic Theory. This model, a framework for weakly coupled systems, differs in some important ways from earlier analyses performed in strongly coupled cases. We determine the structure of the large order gradient expansion, analyze the hydrodynamic and nonhydrodynamic modes and point out the similarities and differences to the strongly coupled case.

Without restriction to a particular model, we use linear response theory to gain deeper insights and find the criteria for convergence and divergence. We show that the gradient expansion can diverge in less symmetric flows but also that it converges for certain initial conditions which only contain modes with low momentum. In linear response theory we also extend the study of transseries beyond Bjorken flow.

Far from equilibrium, constitutive relations can emerge in the form of hydrodynamic attractors. As with the large order gradient expansion, these were discovered in simple models in symmetric flows, and one main challenge is that of generalizing it to more complicated cases. We present a novel perspective on attractors, based on explicitly quantifying properties such as information loss and universality that are usually left implicit. This is a data-driven approach aiming to cover cases where analytic treatment is too difficult. We test it for models with a known attractor, showing that it works.

This thesis is a cumulative thesis based on five published articles and one under review. An extended introduction that puts the papers into context and presents the necessary background material is provided. The main lessons of the included articles are extracted and open questions are presented.