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## Report on Viktor Svensson's PhD thesis

An intriguing and important line of research in contemporary theoretical physics deals with the understanding of the place taken by hydrodynamic description in the wider perspective of fully nonlinear and quantum dynamics of a fundamental microscopic theory. This question is especially acute from the point of view of heavy ion collisions where the produced quark-gluon plasma quite soon after the collision becomes describable by hydrodynamics. It is clear, nevertheless, that the hydrodynamic modes represent only a subset of all degrees of freedom, and a full description would certainly involve also so-called non-hydrodynamic modes, which are especially important in the initial far from equilibrium regime.

It has been uncovered, however, that once one considers hydrodynamics up to large orders in the gradient expansion, there appear nontrivial interrelations between hydrodynamics and the non-hydrodynamic modes, and one can discover even the existence of the latter from the high order behaviour of the former.

The PhD thesis entitled *Relativistic Hydrodynamics Beyond the Second Order*, submitted by Viktor Svensson deals with the above circle of ideas. It is based on 6 papers (one published in *Physical Review Letters*, three in *Physical Review D*, one in *JHEP* and one in *SciPost Physics*). It comprises a 26-page overview together with a 12 page bibliography as well as reprints of the 6 papers.

The key contributions of the PhD thesis are the following:

1. An analysis of the high order behaviour of kinetic theory (Boltzmann equation) in the RTA approximation and the determination and interpretation of the singularities in the Borel plane (papers A and B).

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2. Determination of the relationship between the convergence properties of the linear response theory in the real space and momentum domain and their connection with initial conditions and complex singularities of the dispersion relation (paper F).
3. An interesting dimensional reduction approach to the understanding of the hydrodynamic attractor (paper C).

In addition, the thesis also includes a detailed analysis of transseries for the telegraphers equation, describing shear fluctuations in the BRSSS theory, including an analysis of Stokes lines and their interrelation with initial condition data (paper D). Finally, paper E contains a study of the possible analytic singularity structure of a family of modes described collectively by a constraint  $P(\omega, k) = 0$ . Below I will comment in more detail on the key contributions of the PhD thesis.

The analysis of the high order behaviour of Boltzmann equations in the boost-invariant setup is especially interesting, as prior investigations of similar issues were performed only in the strong coupling regime using the holographic AdS/CFT correspondence. The outcome turns out to be very subtle. Two prominent complex singularities turn out not to correspond to physical degrees of freedom, while the physical real pole turns out to be a composite of many transient modes differing in their (subleading) power-law behaviour. Both of these results are highly non-trivial both technically and conceptually.

The issue of convergence in linear response theory is interesting for various reasons. On the one hand, it is much more tractable than the nonlinear analysis done so far (exhibiting non-convergence) allowing thus for a better intrinsic understanding. On the other hand, the dispersion relations of the linearized modes have a finite radius of convergence leading to a tension with the non-convergence of the nonlinear analysis. Paper F demonstrated that convergence in the linear response theory in real space of the hydrodynamic energy-momentum tensor is linked with the support of the initial data in momentum space and its relation to the location of singularities in the relevant dispersion relations. This is a very nice concise result showing, in particular, the relevance of the form of initial data for the convergence properties.

The concept of a hydrodynamic attractor is an interesting way of looking at the transition from non-hydrodynamic to hydrodynamic behaviour which singles out particular solution(s) already deep in the non-equilibrium regime. It was mostly studied in low-dimensional hydrodynamic models (i.e. with lot of symmetry). It is a quite nontrivial problem, however, how to look for an attractor in a setup with a higher dimensional phase space. The tool proposed in paper C (published in *Physical Review Letters*) was to use PCA (Principal Component Analysis) and identify an effective lowering of the dimension as a signature of an attractor.

The conceptual overview is well written and clear. I would perhaps have preferred a slightly more detailed description of the results of the relevant papers but the way it is written is a valid choice. I have some questions and comments related to the overall presentation:

- In the discussion of convergence in linearized approximation, a criterion is formulated in terms of a sharp cut-off  $k_{max}$  for the support of initial conditions in momentum space. Is the sharp cut-off necessary? Can it be substituted by some fall-off requirement at large  $k$ ?
- There is a lot of color-coded structure shown in Fig. 7.2. It should be either explained in the caption or a simplified figure made specially for the overview
- The discussion of the slow roll approximation in the overview is too sketchy. As the slow roll approximation was used in the attractor context in ref. [14] (as is cited in the overview), it is not clear what is the concrete contribution of paper C in this respect, like what does its generalization amount to etc.

Despite these minor shortcomings, I view it as an excellent PhD thesis. It contains a number of nontrivial and important results and I judge its overall scientific level to be indeed very high. One of the papers have been published in *Physical Review Letters*, and the overall number of citations is **204** (according to the specialized INSPIRE-HEP bibliography database in high energy physics as of 15.11.2021).

To summarize, I believe that the submitted thesis is a fitting basis for further steps in the procedure of awarding the PhD degree.

Yours sincerely,

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