SUMMARY OF PROFESSIONAL ACCOMPLISHMENTS

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1 General Information

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2 Education and previous employment

POST-DOC:	2010-2014: University of Amsterdam, the Netherlands Mentor: Jan de Boer
PH.D. STUDIES:	2010: Jagiellonian University, Poland Dissertation subject: theoretical physics Dissertation title: "Various aspects of non-perturbative gauge theory dynamics and AdS/CFT correspondence" Advisor: Romuald A. Janik Summa cum laude.
MS.C. STUDIES:	2007: Jagiellonian University, Poland Studies in Mathematics and Natural Sciences, main subject: theoretical physics Thesis title: "Quark-gluon plasma and AdS/CFT correspondence" Advisor: Romuald A. Janik Summa cum laude Recognized with the 2$^{\text{nd}}$ prize in the nationwide competition for the best M.Sc. thesis in physics organized by the Polish Physical Society,
MATRICULATION:	2003: The August Witkowski 5$^{\text{th}}$ High School in Cracow, Poland

3 Summary of the scientific achievement

The scientific achievement presented here is the series of 10 scientific articles studying equilibration processes in strongly-coupled quantum field theories using novel tools from string theory. The series is titled

"Quantum fields out of equilibrium: a holographic approach."

The series consists of the following publications:

1. M. P. Heller, D. Mateos, W. van der Schee, D. Trancanelli, "Strong Coupling Isotropization of Non-Abelian Plasmas Simplified," Phys. Rev. Lett. 108, 191601 (2012), arXiv:1202.0981 [hep-th].

2. M. P. Heller, D. Mateos, W. van der Schee and M. Triana, "Holographic isotropization linearized," JHEP 1309, 026 (2013), arXiv:1304.5172 [hep-th].

3. M. P. Heller, R. A. Janik and P. Witaszczyk, "Characteristics of Thermalization of Boost-Invariant Plasma from Holography," Phys. Rev. Lett. 108, 201602 (2012), arXiv:1103.3452 [hep-th].

4. M. P. Heller, R. A. Janik and P. Witaszczyk, "A numerical relativity approach to the initial value problem in asymptotically Anti-de Sitter spacetime for plasma thermalization - an ADM formulation," Phys. Rev. D 85, 126002 (2012), arXiv:1203.0755 [hep-th].

5. M. P. Heller, R. A. Janik and P. Witaszczyk, "Hydrodynamic Gradient Expansion in Gauge Theory Plasmas," Phys. Rev. Lett. 110, 211602 (2013), highlighted with *Editors' suggestion*, arXiv:1302.0697 [hep-th].

6. J. Casalderrey-Solana, M. P. Heller, D. Mateos and W. van der Schee, "From full stopping to transparency in a holographic model of heavy ion collisions," Phys. Rev. Lett. 111, 181601 (2013), arXiv:1305.4919 [hep-th].

7. J. Casalderrey-Solana, M. P. Heller, D. Mateos and W. van der Schee, "Longitudinal Coherence in a Holographic Model of Asymmetric Collisions," Phys. Rev. Lett. 112, 221602 (2014), arXiv:1312.2956 [hep-th].

8. M. P. Heller, R. A. Janik, M. Spaliński and P. Witaszczyk, "Coupling hydrodynamics to nonequilibrium degrees of freedom in strongly interacting quark-gluon plasma," Phys. Rev. Lett. 113, 261601 (2014), arXiv:1409.5087 [hep-th].

9. A. Buchel, M. P. Heller and R. C. Myers, "Equilibration rates in a strongly coupled nonconformal quark-gluon plasma," Phys. Rev. Lett. 114, 251601 (2015), arXiv:1503.07114 [hep-th].

10. M. P. Heller and M. Spalinski, "Hydrodynamics Beyond the Gradient Expansion: Resurgence and Resummation," Phys. Rev. Lett. 115, 072501 (2015), highlighted with *Editors' suggestion*, arXiv:1503.07514 [hep-th].

The cumulative Impact Factor of the series is equal to 73.165. The publications from the series were cited more than 500 times according to the INSPIRE HEP database, which is the standard bibliometric tool in theoretical high energy physics. These articles constitute the first 10 positions in the bibliography.

4 Description of the scientific achievement

4.1 Introduction

Modern theoretical physics is based on two pillars: quantum field theory being the language of the particle physics as well as quantum-many body systems and general relativity describing the gravitational force at a classical level. Truly outstanding progress made in the field of string theory in the course of the last 30 years is based on the promise of extending the Standard Model of particle physics by a consistent quantum mechanical description of gravity. This resulted in the natural domain to seek phenomenological implications being either extensions of the Standard Model or cosmology of the early Universe.

Within the last 15 years there appeared a new way in which string theory delivered phenomenologically relevant results. This novel possibility is the gauge-gravity duality, also known as the AdS/CFT correspondence or holography, which became a novel ab initio tool allowing to solve a class of strongly-coupled quantum field theories. This very active research area is the subject of the publication series [1–10].

There are 2 well-established families of ab initio approaches to quantum fields: weak-coupling methods assuming the smallness of the coupling constant and Monte Carlo simulations in which path integrals are evaluated numerically in the Euclidean signature. Apart from their complexity when applied to situations of interest for modern physics, these approaches have natural limitations originating from:

- The actual size of the coupling constant in a given process for the weak-coupling methods.
- Convergence of the Euclidean path integral in the presence of chemical potential for fermionic degrees of freedom as well as the Minkowski path integral itself (the so-called sign problem).
- Ambiguous analytic continuation of numerically-computed Euclidean correlation functions to Lorentzian signature

Phenomena evading satisfactory ab initio solution in the quantum field theory language can be found both in high energy physics, especially in the context of quantum chromodynamics (QCD), as well as in condensed matter physics.

Holography maps a certain class of strongly-couple quantum field theories, in particular non-Abelian gauge theories in 3+1 dimensions sharing many features of the QCD, to solutions of classical gravity with negative cosmological constant and matter fields. Although none of these theories seems to be fully realized in Nature, some of the complicated questions in microscopically motivated setups for holographic quantum field theories reduce to finding solutions of a coupled set of nonlinear partial differential equations and analyzing their properties. Although solving Einstein's equations can be technically rather involved, it does not pose a conceptual difficulty as opposed to the aforementioned issues. Thus, the phenomenological role of holography stems from, first, estimating calculations in quantum field theories which are very hard or even impossible to perform using the standard methods and, second, finding unanticipated phenomena. Whereas this first possibility requires that the considered process is strongly coupled, the second possibility seems much more general since it is based on the main virtue of the ab initio approach, i.e. not making unnecessary assumptions about a given process.

The articles [1–10] are directly motivated by the quest to understand non-equilibrium phenomena in QCD. It is a very active arena of research due to ongoing experimental investigations of the quark-gluon plasma in ultra-relativistic heavy-ion collisions at RHIC and LHC (see, e.g., [29] for a review). The methods of holography are extremely well-rooted in this field, since they played a crucial role in the paradigm change in the way of thinking about experimentally accessible quark-gluon plasma from a weakly-coupled gas of partons to a strongly-coupled fluid with very small shear viscosity [30]. Certainly the most impressive measure of this phenomenological success of holography is the fact that experimental estimates of the shear viscosity η

of the quark-gluon plasma at RHIC and LHC not only look at its ratio to the entropy density s, but also express it in terms of multiples of the holographic result [31]

$$\frac{\eta}{s} = \frac{1}{4\pi}, \tag{1}$$

where in the above expression, as well as in the rest of the text, the natural units are implied.

Motivated by the initial state physics in heavy-ion collisions at RHIC and LHC where the nuclear matter is not described by hydrodynamics, the articles [1–10] apply holographic methods to find answers to the following questions:

1. How much time is needed for the expectation value of the energy-momentum tensor to satisfy hydrodynamic constitutive relations starting from a generic non-equilibrium state? This question is the subject of section 4.3.1 based on the original articles [1–4, 9].

2. Is approximate local isotropy needed for the applicability of hydrodynamics? This problem is analyzed in section 4.3.2 which follows the discussion in the original articles [3, 4, 6, 7].

3. What are the properties, as of classical field theories, of existing formulations of relativistic hydrodynamics and what are their generalizations motivated by holographic results delivered in the context of the RHIC and LHC physics? Partial answer to this question can be found in section 4.3.3 which summarizes original results from the articles [2,4,8,10].

4. What is the radius of convergence of the hydrodynamic gradient expansion and do higher order transport coefficients matter? Section 4.3.4 is devoted to answering this question based on the original results from the articles [5, 10].

5. What are the non-equilibrium features of the expectation value of the energy-momentum tensor after ultra-relativistic collisions of 2 strongly-interacting objects? This aspect of the series is discussed in section 4.3.5 which is based on the original results from [6, 7].

The phenomenological importance of the series lies in the question of the early applicability of viscous hydrodynamics to heavy-ion collisions at RHIC and LHC (questions 1 and 2), possible generalizations of viscous hydrodynamics (questions 3 and 4) and open problems in the initial state physics (question 5). Due to the asymptotic freedom, the use of methods based on the largeness of the coupling constant in the far-from-equilibrium regime where one expects high energy densities / large occupation numbers might be questionable. Thus the results motivated by questions 1 and 2 need to be understood as qualitative estimates in theories in which the coupling constant is large which, contrasted with the weak-coupling approaches to QCD, will allow for the better understanding of the experimental results. When it comes to questions 2 and 3, they seem to be based more on the mere ability of performing ab initio calculations within holography rather than the large value of the coupling constant and hence the expectation that the results delivered in this context are more general. When it comes to question 4, it follows from the better understanding due to holography of a relation between hydrodynamic and non-hydrodynamic degrees of freedom and the results delivered in this context aim at providing phenomenological equations of motion generalizing the existing formulation via inclusion of new non-equilibrium phenomena.

Last but not least, it is important to stress that most of the calculations presented here are based on numerical solution of Einstein's equations with negative cosmological constant which is one of the most active and dynamical trends within the relativists' community. These results are pioneering in this technical aspect since most of the previous works on holography were based on analytic or simple semi-analytic methods. Another innovative aspect of the series is the first use of the trans-series and resurgence methods in the context of the hydrodynamic gradient expansion. These developments have found numerous application to perturbative expansion in quantum mechanical systems and are currently a very active research trend among high energy and mathematical physicists alike.

4.2 State of the field before March 2011

The present section provides a short summary of the state of the field before the appearance of the first publication from the series [3], i.e. before March 2011. The investigations in the series lie at the intersection of nuclear physics and string theory. These fields are characterized by a very different methodology, hence their state as of March 2011 is sketched in separate subsections. The string theory part is larger, since it introduces rather new concepts directly needed for the description of delivered results. A special attention is given to stating research questions which originate from the state of the field before March 2011 and motivated the developments in the series . Questions denoted by the letter **Q**, standing for QCD, follow from phenomenological and theoretical aspects of ultra-relativistic heavy-ion collisions. Problems named by the letter **H**, standing for holography, follow directly from previous applications of string theory methods and their methodology.

4.2.1 Thermalization in ultra-relativistic heavy-ion collisions at RHIC and LHC

The aim of ultra-relativistic heavy-ion collision programs, first at SPS, and now at RHIC and LHC is investigating properties of matter under extreme conditions. Creating the quark-gluon plasma and understanding some of its properties is a great and unquestionable success of this endeavour.

The experimental signal in heavy-ion collisions are the emitted particles characterized, among other properties, by their transversal momentum $\mathbf{p}_T$, i.e. the momentum in the plane transversal to the collision axis. Two essential phenomena observed in ultra-relativistic heavy-ion collisions at RHIC and LHC energies are

- Flow for the particles with low $\mathbf{p}_T$.
- Quenching for the particles with high $\mathbf{p}_T$.

Both phenomena are explained by a model, which assumes the creation of the quark-gluon plasma characterized by very small dissipation of the collective modes of low frequencies and momenta from which originate particles with low $\mathbf{p}_T$ and damping of the excitations responsible for the high-$\mathbf{p}_T$ particles. There is a consensus among the researchers that ultra-relativistic heavy-ion collisions at RHIC and LHC create quark-gluon plasma in the form of strongly-coupled liquid rather than a weakly-coupled gas of partons [30].

In the remaining part of this summary, the emphasis will be put on these aspects of the hydrodynamical model which directly motivate the results delivered in the series. This phenomenological model provides a successful description of the spectrum of the low-$\mathbf{p}_T$ particles under several conditions [32] which include

- The applicability of hydrodynamics already as early as about 0.5 fm after the collision.
- The shear viscosity of the quark-gluon plasma is small, of the order of the holographic result expressed by Eq. (1).

The early initialization of hydrodynamics is needed to guarantee long-enough time for the collective evolution necessary for exchanging pressure anisotropy in the transversal plane for the anisotropy of the plasma velocity, whereas the smallness of shear viscosity is needed in order to prevent velocity isotropization. Furthermore, some groups use the early applicability of hydrodynamics as an argument supporting the energy density profile in the transversal plane remaining almost intact since the collision [32]. Finally, the article [33] suggest that the hydrodynamic expansion can be initiated at later times, about 1 fm after the collision, provided it is preceded by a non-trivial non-equilibrium dynamics, in this case free-streaming. This discussion leads to the following question:

> **Q1:** What can be learnt from holography about the evolution of the energy-momentum tensor from the initial state till the applicability of hydrodynamics?

It needs to be stressed on this occasion that obtaining the right multiplicities might require, depending on the model, fitting the amplitude of the energy density at the starting point of the hydrodynamic evolution. The resulting energy densities in the centre of the fireball correspond on average to temperatures T_{ini} of the order of 0.3 GeV (RHIC) - 0.5 GeV (LHC). The natural dimensionless quantity characterizing the timescale of the applicability of hydrodynamics is a product of the initialization time and T_ini which is an order 1 quantity

$$t_{ini} T_{ini} = \mathcal{O}(1). \tag{2}$$

It is not known what is the corresponding value of this quantity in the weakly-coupled quantum field theories in the regime of small but not parametrically-small coupling [34]. Naive extrapolation of the pioneering result of [35] to realistic couplings, which neglects the effects of the Weibel instabilities [36, 37] and concerns the isotropization of the energy-momentum tensor (see also question **Q5**), leads to timescales 5-10 times longer than Eq.(2) [38]. It is then natural to ask the following question:

> **Q2:** Do strongly-coupled systems generically equilibrate over a timescale set by their temperature / energy density?

From the point of view of the effective field theory, relativistic hydrodynamics describes the transport of conserved currents over large distances setups in which microscopic degrees of freedom move with relativistic speeds. As every effective field theory, relativistic hydrodynamics is based on the gradient expansion. When dissipative effects are included, such as in the case of the phenomenological description of the quark-gluon plasma, relativistic hydrodynamics is typically formulated at the level of equations of motion, i.e. conservation equations for the energy-momentum tensor

$$\nabla_\mu \langle T^{\mu\nu} \rangle = 0 \tag{3}$$

and other conserved current. Restricting attention to the transport of energy and momentum, the most general energy-momentum tensor up to the first order in gradients takes the form

$$\langle T^{\mu\nu} \rangle = \{\epsilon(T) + P(T)\}\, u^\mu u^\nu + P(T)\, \eta^{\mu\nu} - \eta(T) \sigma^{\mu\nu} - \zeta(T)\, \{\eta^{\mu\nu} + u^\mu u^\nu\}\, \nabla_\alpha u^\alpha, \tag{4}$$

where $\sigma^{\mu\nu}$ is given by

$$\sigma^{\mu\nu} = \frac{1}{2} \{\eta^{\mu\alpha} + u^\mu u^\alpha\} \left\{\eta^{\nu\beta} + u^\nu u^\beta\right\} \{\nabla_\alpha u_\beta + \nabla_\beta u_\alpha\} - \frac{1}{3} \{\eta^{\mu\nu} + u^\mu u^\nu\}\, \nabla_\alpha u^\alpha, \tag{5}$$

$\epsilon(T)$ and $P(T)$ are the local energy density and pressure related to the local temperature T via the thermodynamic relations and the equation of state and u^μ is the velocity vector containing

3 independent components ($\eta_{\mu\nu}u^\mu u^\nu = -1$). Scalars $\eta(T)$ i $\zeta(T)$ are the shear and the bulk viscosity, which are the functions of local temperature. Dissipation is intrinsically related to these transport coefficients since the divergence of the entropy current evaluated on a formal solution of the equations of hydrodynamics is proportional to these transport coefficients and manifestly non-negative

$$\nabla_\mu \left\{ \frac{\epsilon(T) + P(T)}{T} u^\mu \right\} = \frac{\eta(T)}{2T} \sigma_{\alpha\beta}\sigma^{\alpha\beta} + \frac{\zeta(T)}{T} \left\{ \nabla_\alpha u^\alpha \right\}^2 \ldots \tag{6}$$

In the equation above terms containing 3 and more derivates were neglected. Including viscosity in the phenomenological description of the quark-gluon plasma is important, since it modifies the evolution of the medium through the inclusion of dissipation. The magnitude of dissipative processes opens then a window on the microscopic properties of the quark-gluon plasma. From the effective field theory point of view, there seems to be no obstruction in including terms higher order in derivatives and such an expansion was performed (for conformal field theories) up to the second order in [39, 40]. A rather natural question arising from those considerations is:

> **Q3** What is the radius of convergence of the hydrodynamic gradient expansion?

Including higher derivative terms modifies the equations of motion and includes new dissipative effects. Using the experimental data to estimate the value of the quark-gluon plasma shear viscosity necessarily neglects the cumulated effect of these higher order terms. However, due to early initialization times of hydrodynamic evolution these terms need not to be parametrically smaller from the leading order effects. This leads to the following problem:

> **Q4** Does resummation of the hydrodynamic gradient expansion lead to modified predictions? In particular, does it alter the estimates of the quark-gluon plasma shear viscosity?

This question was originally posed in Ref. [41].

Another aspects of the gradient expansion is the issue of anisotropy of the energy-momentum tensor at the hydrodynamic threshold stemming from the early initialization time of viscous hydrodynamic codes. The perfect fluid energy-momentum tensor, i.e. the one given by Eq. (4) with neglected gradient corrections, is isotropic – its 3 eigenvalues are degenerate and equal to the pressure $P(T)$ and the remaining one is the energy density $\epsilon(T)$. Including the shear viscosity $\eta(T)$ in the description breaks the isotropy and in a general situation leads to 4 distinct eigenvalues. This naturally leads to the following question:

> **Q5** Is the applicability of hydrodynamics limited by the size of gradients in a given process?

In particular, the pioneering work [37] stressing the importance of the plasma instabilities [36] in the evolution of the quark-gluon plasma at weak coupling puts a significant emphasis on the approximate isotropy of the energy-momentum tensor as a necessary condition for the applicability of hydrodynamics. This is a truly fundamental question, since in the experimental context it seems that it is important for the collective models to be applicable quickly rather than in the isotropic setup.

From the point of view of the effective field theory, the equations of hydrodynamics of the form (3) and (4) seem to be correct. However, from the practical point of view they require modifications since corrections to the perfect fluid behaviour break the hyperbolicity and make the initial value problem ill-posed [42–44]. The approach of Müller, Israel and Stewart (MIS)

[42,43] and its modern incarnations following [39] leads to hyperbolic equations, which in the gradient expansion reduce to the effective field theory equations of motion. This approach became the standard tool to describe deconfined nuclear matter using hydrodynamics (see, e.g., [45]). In this framework, the energy-momentum tensor is split into the perfect fluid part and the dissipative part $\Pi^{\mu\nu}$

$$\langle T^{\mu\nu} \rangle = \{\epsilon(T) + P(T)\}\, u^{\mu} u^{\nu} + P(T)\, \eta^{\mu\nu} + \Pi^{\mu\nu}. \tag{7}$$

The dissipative part is promoted to a dynamical field obeying equations of motion forcing its relaxation to the form dictated by the viscous hydrodynamic constitutive relation

$$(\tau_{\Pi}\, u^{\alpha} \nabla_{\alpha} + 1)\, \Pi^{\mu\nu}_{MIS} = -\eta \sigma^{\mu\nu} + \ldots\,, \tag{8}$$

where, for the clarity of presentation, some terms were not displayed (see Ref. [39] for its full form). Solving the initial value problem requires now specifying the initial temperature profile, 3 components of velocity and the tensor $\Pi_{\mu\nu}$. It is also clear that $\Pi_{\mu\nu}$ carries information about the anisotropy of the energy-momentum tensor and, hence, the problem of initial conditions for Eq. (8) is related to the question **Q5**. The anisotropy can be either set directly, as an initial condition, or it can arise in the course of evolution starting with a nontrivial profile of the energy density or velocity field. This issues becomes even more pronounced in the light of the discovery of odd harmonics in the angular distribution of particles at low p_T, which originate from event-by-event fluctuations of the positions of nucleons and the gluon density in the transversal plane (see, e.g., [46]). Question **Q5** should be thus reformulated to take the following form

> **Q5'** Does it makes sense to consider hydrodynamic evolution in the regime of very large gradients?

Eq. (8) is the simplest phenomenological generalization of the perfect fluid hydrodynamics which does have a well-posed initial value problem. This leads to the last question in the section

> **Q6** What are other possible generalizations of relativistic hydrodynamics and what are they good for?

This point is motivated by [47], which discusses supplementing Eq. (8) with the second co-moving derivative of $\Pi_{\mu\nu}$ in the context of the analytic structure of the retarded 2-point function of the energy-momentum tensor in strongly-coupled quantum field theories with gravity duals (see Fig. 1 in section 4.2.2). Also, one should mention here a line of developments called anisotropic hydrodynamics (see, e.g., [48]), which is another generalization of Eq. (8) directly motivated by questions like **Q5** or **Q5'**.

The last point to discuss are the studies of the initial state physics in ultra-relativistic heavy-ion collisions using the weak-coupling methods. Within the so-called color glass condensate framework (CGC) the initial state in ultra-relativistic collisions of atomic nuclei at asymptotically large energies can be established by solving effective Yang-Mills equations describing the dynamics of the system below a fiducial cut-off in the presence of stochastic (when it comes to their transversal structure) colour currents flowing along the lightcone (see, e.g., Ref. [49] for a review). The currents represent partonic degrees of freedom above the cut-off. This description is supplemented by the self-consistency relation between the currents and the Yang-Mills fields dictated by the requirement of cut-off independence. Furthermore, it needs to be stressed that the CGC approach despite assuming small coupling constant needs to resum an infinite number of Feynman diagrams due to parametrically big (in the coupling constant inverse) occupation number for the gluonic degrees of freedom in collided objects . The leading order results are boost-invariant in the sense of Bjorken [50] and lead to a characteristic structure of the expectation value of the energy-momentum tensor after the collision [51] – see the discussion below Eq. (30) in section 4.2.2. Furthermore, in the large-time limit the leading order, in the coupling constant, dynamics leads to the pressure anisotropy proportional (with approximately constant coefficient) to the energy density, which is not the hydrodynamic behaviour. Including the first subleading correction in the weak-coupling expansion destabilizes the systems by introducing exponentially growing modes breaking the boost-invariance [52]. Attempts to overcome this problem within the so-called classical statistical field theory appeared in the literature [53, 54] quite late when the research presented in this summary was in its very advanced stage, which finishes the discussion of the QCD background.

4.2.2 Holography and its applications

Holography was proposed by the Argentinian physicist Juan Maldacena in the late 1997 and postulates equivalence between certain solutions of string theory and certain quantum field theories including non-Abelian gauge theories in 3+1 dimensions [55]. The statement of the duality is the equality of the corresponding partition functions

$$Z_{\text{quantum field theory}}[J] = Z_{\text{string theory}}[J]. \tag{9}$$

Holography is a particular instance of a more general holographic principle [56], which based on the very basic properties of the gravitational force states that its quantum-mechanical degrees of freedom are realized as a non-gravitational theory in one dimension less [57, 58].

The best understood instance of holography is the duality between the type IIB string theory in the $\mathrm{AdS}_5\times\mathrm{S}^5$ geometry and the Yang-Mills theory with $\mathrm{SU}(N_c)$ gauge group and maximal supersymmetry ($\mathcal{N} = 4$) in $3+1$ dimensions and in the planar limit [59]

$$N_c \to \infty \quad \text{oraz} \quad \lambda = g_{YM}^2 N_c = \text{const.} \tag{10}$$

In the equation above, g_{YM} is the coupling constant. The $\mathcal{N} = 4$ super Yang-Mills (SYM) theory, apart from the gluonic fields, contains also 4 fermions and 6 bosons which are all in the adjoint representation of $\mathrm{SU}(N_c)$. This theory is conformal at the quantum level.

In holography, the parameters defining the relevant quantum field theories have natural counterparts in the string theory language. In particular, the number of colours N_c is related to the ratio of the curvature radius of AdS L and the Planck length l_P, whereas the 't Hooft coupling constant λ maps to the ratio of L to the string length l_s. The former measures the strength of quantum effects associated with fluctuations of geometry, whereas the latter governs the magnitude of phenomena related to the extended nature of strings. In particular, the planar limit corresponds to a weakly-coupled string theory, which for the large value of the 't Hooft coupling constant

$$\lambda \to \infty \tag{11}$$

reduces to supergravity. Apart from the $\mathcal{N} = 4$ SYM theory, there are infinitely many other examples of a duality between quantum field theories and solutions of supergravity (see, e.g., [60]). All these theories are characterized by, among other things, large number of microscopic degrees of freedom and strong coupling at all scales [61].

Although holography is a natural consequence of what has been known about gravity and quantum field theories for quite some time by now, it is a relation between two complicated theories that needs to be thoroughly tested. In the planar limit, both the type IIB string theory in $AdS_5 \times S^5$ and the $\mathcal{N} = 4$ SYM are integrable (see, e.g., [62]) and calculating anomalous dimensions of $\mathcal{N} = 4$ SYM operators using this property has led to very strong quantitative arguments in support of the holography (see, e.g., [63]). This very well-tested example of holography is used directly or indirectly in the vast majority of the result presented in the series: [1–8, 10]. Holography applies also to certain theories without the conformal symmetry. The article [9] as the only one in the series considers the effects of the conformal symmetry breaking on equilibration processes at strong coupling using the dual description of the $\mathcal{N} = 2^*$ gauge theory [64, 65]. It is worth stressing that also in this case there are quantitative checks (in this case using supersymmetric localization techniques) of the duality, see [66].

Despite significant differences at the Lagrangian level (i.e. the vacuum) between the $\mathcal{N} = 4$ SYM and QCD, their plasma states share a lot of qualitative features both at strong (see, e.g., [67] and weak coupling [68]. These similarities and limitations of the other ab initio methods in quantum field theories have provided very strong motivation to pioneer the use of holography to model time-dependent processes in the context of heavy-ion collisions at RHIC and LHC. Apart from questions on the initial state and the threshold of the applicability of hydrodynamics which directly motivate the results presented here, this research agenda covers also

- Thermodynamics of QCD (see, e.g., [69–71])[1]
- Hydrodynamic transport (see, e.g., [72]),
- Interactions jet-medium (see, e.g., [73–75]).

Comprehensive overviews of the aforementioned developments can be found, e.g., in the review articles [76–79] and in the book [80]. From a broader perspective, the applicational potential of holography follows from the possibility of computing correlation functions in strongly-coupled quantum field theories via solving Einstein's equations with negative cosmological constant (and often with additional matter fields). Apart from the physics of the QCD plasma and heavy-ion collisions, the applications of holography concern also

- Chiral symmetry breaking in QCD and the spectrum of hadrons (see, e.g., [81–84]).
- Strongly-coupled extensions of the Standard Model (see, e.g., [85]).
- Condensed matter physics, in particular superfluidity and superconductivity (see, e.g., [86]).

The results delivered within these research threads are related to the series mainly via their similar methodology. Accordingly, they will not be reviewed here and were mentioned just for the sake of completeness of the presentation of the basic aspects of holography and its applications.

In order to present the state of the applications of holography to time-dependent processes as of March 2011, one needs to briefly discuss how to compute correlation functions using the dual gravity description. In the regime of interest, string theory reduces to the Einstein gravity (coupled to matter). This leads to

$$Z_{\text{string theory}}[J] \approx e^{iS_{\text{gravity}}[J]}, \tag{12}$$

[1] The aim of these papers was not modelling the equation of state, but rather obtaining gravitational background needed to compute correlations functions in holographic models that are closer to QCD than the $\mathcal{N} = 4$ SYM theory.

where J is a fixed configuration of sources in a dual quantum field theory and S_{gravity} is the on-shell gravity action of the form

$$S_{\text{gravity}} = \frac{1}{2l_P^3}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}\right) + S_{\text{matter}}. \tag{13}$$

In the equation above l_P is the 5-dimensional Planck length, $12/L^2$ stands for negative cosmological constant, whereas S_{matter} hides possible matter fields. The formula (13) implicitly assumes the Kaluza-Klein reduction of the supergravity action to 5-dimensions, where S_{matter} contains contributions from the (remaining) supergravity fields and their Kaluza-Klein modes. In the case of the $\mathcal{N}=4$ SYM theory it is a reduction over a 5-dimensional sphere S^5. In particular, in the case of conformal field theories including the $\mathcal{N}=4$ SYM, it turns out that it is consistent [87] to set all the matter fields to 0 in the action (13) and solve the Einstein's equations with a negative cosmological constant

$$R_{ab} - \frac{1}{2}R\, g_{ab} - \frac{6}{L^2}g_{ab} = 0. \tag{14}$$

Different microscopic theories correspond to different values of the L/l_P ratio, which for the $\mathcal{N}=4$ SYM takes the value

$$\frac{L}{l_P} = \left(\frac{N_c^2}{4\pi^2}\right)^{1/3}. \tag{15}$$

It might be worth stressing here that in the action (13) there is no string length l_s and, as a result, none of the quantities obtainable from it exhibits dependence on the coupling constant when the latter is very large.

Different solutions of Eqs. (14), possibly coupled to matter fields (13), correspond to different states in dual quantum field theories. The most symmetric solution, 5-dimensional anti-de Sitter (AdS) space is given by

$$ds^2 = \frac{L^2}{u^2}\left\{du^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right\} \tag{16}$$

and represents dual quantum field theory's vacuum. In the above formula, $\eta_{\mu\nu}$ is the 4-dimensional Mikowski metric, whereas $u>0$ is the AdS radial coordinate. The u-coordinate has a natural interpretation in terms of the scale (cut-off) in dual quantum field theories. Small values of u represent the UV, whereas its large values correspond to the IR.

The $u=0$ hypersurface needs to be understood as the boundary of the AdS space and is the locus where the bulk metric and matter fields need to be supplemented with the appropriate boundary conditions. These boundary conditions have natural interpretations of sources J in dual quantum field theories. This interpretation together with the equality of the partition functions (9) define the holographic dictionary [88, 89]. The dictionary allows to compute correlation functions in dual quantum field theories from gravity, schematically

$$\langle O_1 \ldots O_n\rangle \sim \frac{\delta^n S_{\text{grawitacji}}}{\delta J_1 \ldots \delta J_n}, \tag{17}$$

and provides the link between the fields propagating in the AdS space and local single trace operators in dual quantum field theories. In the case of Lorentzian signatures, there are additional subtleties related to causal relations between the insertion points (see, e.g., [90]), but they will not be crucial for the results presented here since the series is focused on the physics of one-point functions (i.e. expectation values of local operators).

In the case of gravity (i.e. the bulk metric), the source has the interpretation of the metric $g^{(0)}_{\mu\nu}(x)$ which appears in the Lagrangian of the dual quantum field theory. Since the situation of interest are heavy-ion collisions, the metric here will be flat, $g^{(0)}_{\mu\nu}(x) = \eta_{\mu\nu}$. It is important to stress at this point that metric perturbations

$$g^{(0)}_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu}(x) \tag{18}$$

provide not only a way to create an excited state, but also allow to compute correlation functions for the energy-momentum tensor operator. In the so-called Fefferman-Graham gauge,

$$ds^2 = \frac{L^2}{u^2}\left\{du^2 + g_{\mu\nu}(u,x)dx^\mu dx^\nu\right\}, \tag{19}$$

Einstein's equations (14) allow to determine $g_{\mu\nu}(u,x)$ in the near-boundary expansion (i.e. for $u \approx 0$)

$$g_{\mu\nu}(u,x) = g^{(0)}_{\mu\nu}(x) + \ldots + g^{(4)}_{\mu\nu}(x)u^4 + \ldots \tag{20}$$

up to 2 symmetric 4x4 matrices, $g^{(0)}_{\mu\nu}(x)$ and $g^{(4)}_{\mu\nu}(x)$ [91]. The ellipsis in Eq. (20) denotes terms that are fully determined by $g^{(0)}_{\mu\nu}(x)$, $g^{(4)}_{\mu\nu}(x)$ and their derivatives. Generically, they also contain logarithms of the u variable. When $g^{(0)}_{\mu\nu}(x) = \eta_{\mu\nu}$, i.e. when the dual quantum field theory is formulated in Minkowski space, then Eq. (17) leads to

$$g^{(4)}_{\mu\nu}(x) = \frac{2\pi^2}{N_c^2}\langle T_{\mu\nu}\rangle. \tag{21}$$

In this equation, $T_{\mu\nu}$ is the energy-momentum tensor operator in the dual quantum field theory and the expectation value is taken in the state (pure or mixed) dual to the geometry solving Einstein's equations (14). Gravitational constraints equations ensure that the corresponding expression is traceless and conserved [91].

From the point of view of the application of holography to equilibration processes, Eq. (21) allows to determine the ansatz for the dual metric which encapsulates symmetries of the considered state [92] (for example for the Bjorken flow [50]). Furthermore, having solved Einstein's equations, Eq. (21) allows to obtain the expectation value of the energy-momentum tensor and, e.g., check if it obeys the hydrodynamic relations.

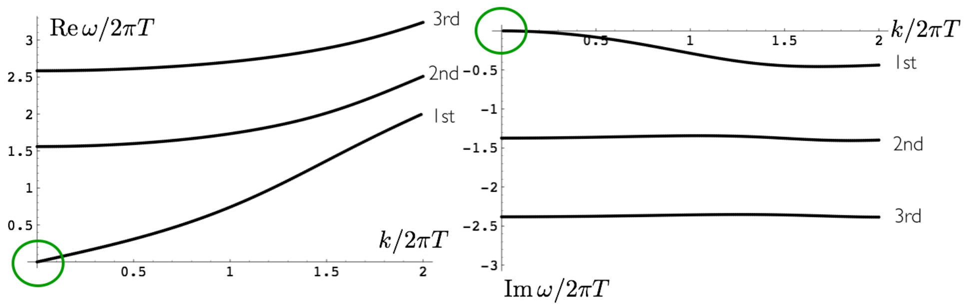
When it comes to the analysis of different states using holography, equilibrium plasma configuration is captured by the black hole spacetime with translationally-invariant event horizon at $u = u_0$

$$ds^2 = \frac{L^2}{u^2}\left\{du^2 - \frac{(1-u^4/u_0^4)^2}{1+u^4/u_0^4}dt^2 + (1+u^4/u_0^4)d\vec{x}^2\right\}. \tag{22}$$

The Hawking temperature of this so-called AdS-Schwarschildt black brane, $T = \frac{\sqrt{2}}{\pi u_0}$, and its Bekenstein-Hawking entropy are associated with the temperature and entropy of the equilibrium state in a dual quantum field theory. The event horizon plays the role of perfectly absorbing membrane and this very property is directly responsible for dissipation in the holographic description of non-equilibrium phenomena in strongly coupled quantum field theories. Combining all the equations we obtain the following expression for the energy density of the $\mathcal{N} = 4$ SYM at finite temperature

$$\mathcal{E} = \frac{3}{8}N_c^2\pi^2T^4. \tag{23}$$

By considering metric perturbations on top of the static black brane one can obtain the retarded Green's function G_R for the energy-momentum tensor in the equilibrium plasma state.



Rysunek 1: Real and imaginary parts of the poles of the retarded 2-point function of the energy-momentum tensor in strongly coupled $\mathcal{N} = 4$ SYM plasma. Considered components of the Green's function correspond to plasma perturbations giving rise to sound waves. The frequencies depend on momentum k and for each value of k there are infinitely many poles. Green circles highlight the dispersion relation for the hydrodynamic sound wave which can survive arbitrary long in the plasma provided the momentum k it carries is small-enough. All the other modes are damped over the timescale of 1/T. The plots are adopted from [95].

The latter describes the evolution of the expectation value of the energy-momentum tensor in a dual quantum field theory by starting with a thermal state and perturbing it according to Eq. (18)

$$\delta\langle T_{\mu\nu}\rangle = \int d\omega\, d^3k\, e^{-i\omega t + i\vec{k}\cdot\vec{x}}\, \left[G_R(\omega,k)\cdot\delta g(\omega,k)\right]_{\mu\nu}. \tag{24}$$

The analytic structure of G_R as a function of complex ω determines the excitations in strongly-coupled plasma by evaluating Eq. (24) using contour integrals, see, e.g., [93]. As explicit calculations have shown, at strong coupling for each value of momentum k there are infinitely many poles in the complex-ω plane as depicted in Fig. 1. The poles have the interpretation of the quasinormal modes of the dual black brane [94]. For conformal field theories imaginary parts of all of them, apart from a single quasinormal mode, is of the order of temperature T [95]. The special mode for which $\omega \to 0$ when $k \to 0$ is the mode whose dispersion relation can be obtained from the linearized equations of relativistic hydrodynamics. Because of this property, this special mode is called the hydrodynamic mode. It is important to stress that naive hydrodynamic equations of the form (4) do not reproduce dispersion relations of the modes with $\mathrm{Im}(\omega) = \mathcal{O}(T)$. As a result, these excitations are often referred to as to non-hydrodynamic modes. The non-hydrodynamic modes are exponentially damped and their effects are physically negligible already for timescales of the order of $1/T$. This implies that in conformal field theories at strong coupling the time after which hydrodynamics become applicable is of the order of $1/T$ provided the amplitude of the deviations from the global thermal equilibrium is small. In the subsequent analysis it will be beneficial to recall the frequency of the lowest non-hydrodynamic mode at $k = 0$ which is approximately equal to

$$\omega_1/T|_{k=0} = \hat{\omega}_1 = \pm 9.800 - 8.629\, i. \tag{25}$$

The results on the analytic structure of retarded Green's functions in holography for the energy-momentum tensor led to the following questions:

H1 Does the inclusion of nonlinear effects modify the relaxation timescale?

H2 Can equilibration times in the absence of conformal symmetry be parametrically different from $1/T$?

These questions directly motivated, correspondingly, the articles [1, 2] and [9] from the series. It is also worth pointing out that practically nothing is known about the structure of the retarded Green's function for the energy-momentum tensor in QCD both at weak and strong coupling. In this context, the holographic results are truly pioneering

Having explained the physics of small perturbations of strongly-coupled plasmas in equilibrium, it is time to summarize the status of the studies of nonlinear equilibration processes prior to [3]. In this context, the groundbreaking work is the article [92] which was to first to discuss nonlinear time-dependent process using holography. The authors considered there the boost-invariant flow [50] in the large proper-time limit and demonstrated that the absence naked singularities in the metric of the form

$$ds^2 = \frac{1}{u^2}\left\{-e^{a(\tau,u)}d\tau^2 + \tau^2 e^{b(\tau,u)}dy^2 + e^{c(\tau,u)}(dx_2^2 + dx_3^2)\right\} \tag{26}$$

fixed the behaviour of dual quantum field theory to be of the perfect fluid type

$$\langle T_{\tau\tau}\rangle \sim \frac{\Lambda^4}{(\Lambda\,\tau)^{4/3}}. \tag{27}$$

In the above equation Λ is the only undetermined dimensional constant setting the overall energy scale. From the technical standpoint, the innovative character of [92] follows from the introduction of a scaling variable which reduced Einstein's equations from PDEs in τ and u variables to a set of ODEs in $v \equiv u/\tau^{1/3}$. Power-law corrections to the behaviour at $\tau = \infty$ taking the form

$$a(\tau,u) = a_0\left(\frac{u}{\tau^{1/3}}\right) + \frac{1}{\tau^{2/3}}a_1\left(\frac{u}{\tau^{1/3}}\right) + \frac{1}{\tau^{4/3}}a_2\left(\frac{u}{\tau^{1/3}}\right) + \ldots \tag{28}$$

with analogous expressions for $b(\tau,u)$ and $c(\tau,u)$, originate from the hydrodynamic gradient expansion and were constructed in the first, second and third orders in, respectively, [96, 97], [25] and [19,22]. The result of these considerations is the late-time expansion of the temperature

$$T(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}}\left\{1 - \frac{1}{6\pi\,(\Lambda\tau)^{2/3}} + \frac{-1+\log 2}{36\pi^2\,(\Lambda\tau)^{4/3}} + \frac{-21+2\pi^2+51\log 2-24\log^2 2}{1944\pi^3\,(\Lambda\tau)^2} + \ldots,\right\}, \tag{29}$$

where the ellipsis denotes terms with 4 and more derivatives.

The article [21] focused, on the other hand, on the small-time regime where it was shown that the energy density must necessarily behave as

$$\langle T_{\tau\tau}\rangle = \epsilon_0 + \epsilon_2\tau^2 + \epsilon_4\tau^4 + \ldots \tag{30}$$

with ϵ_0, ϵ_2, ... being a family of infinitely many dimensionful integration constants constrained only by the requirement of the absence of naked singularities in the dual gravitational description. Although the CGC approach in the weakly-coupled QCD [51] predicts $\epsilon_0 \neq 0$, at strong coupling there seem to be no obstructions for the early time expansion (30) to start as τ^2 ($\epsilon_0 = 0$) or τ^4 ($\epsilon_0 = \epsilon_2 = 0$) etc. Assuming the leading order behaviour to be $\langle T_{\tau\tau}\rangle \sim \tau^{2n}$, the conservation and tracelessness of the energy-momentum tensor lead to

$$\langle T_{yy}\rangle/\langle T_{\tau\tau}\rangle = -\tau^2(1+2n) \qquad \text{and} \qquad \langle T_{\perp\perp}\rangle/\langle T_{\tau\tau}\rangle = 1+n. \tag{31}$$

These relations will be one of the points of departure when discussing the results of [6].

Situation encountered in [21] where there are infinitely many free parameters need to be contrasted with Eq. (27) which describes the hydrodynamic expansion and has only 1 free coefficient. The approach to hydrodynamics in this setup can be then understood as an effective decoupling of all the degrees of freedom apart from the ones associated with the single integration constant Λ. The article [21] attempted to observe this transition for several families of coefficients (30) but the radius of convergence of the resulting series was too small. These results led to the following questions

H3 Is there a simplified formulation of the equations of motion for the boost-invariant hydrodynamics?

H4 How does the transition to hydrodynamics occur for the initial data constructed in the article [21]?

Another groundbreaking work in this context was the article [98], which provided the gravity dual to an arbitrary solution of the equations of hydrodynamics in the $\mathcal{N} = 4$ SYM theory. From the technical perspective, the main innovation in this paper came from the use of the generalized ingoing Eddington-Finkelstein coordinates which are regular at the horizon and in which the black brane metric takes the form

$$ds^2 = 2\,dR\,dt - \frac{R^2}{L^2}\left\{1 - \left(\frac{\pi T L^2}{R}\right)^4\right\} dt^2 + \frac{R^2}{L^2}\,d\vec{x}^2. \tag{32}$$

The spacetime dual to relativistic hydrodynamics in the perfect fluid approximation is then given by

$$ds^2 = -2\,dR\,u_\mu dx^\mu - \frac{R^2}{L^2}\left\{1 - \left(\frac{\pi T L^2}{R}\right)^4\right\} u_\mu\,u_\nu\,dx^\mu\,dx^\nu + \frac{R^2}{L^2}\left(\eta_{\mu\nu} + u_\mu u_\nu\right) dx^\mu dx^\nu. \tag{33}$$

The article [98], often referred to as the “fluid-gravity duality”, obtained general corrections to the perfect fluid energy-momentum tensor up to the second order in gradients. As it was shown in the paper [22], the results of the articles [25, 92, 96, 97] from the contemporary perspective are special cases of the more general procedure from [98]. Since holography allows to relatively easy, as compared to other methods, generate transport coefficients in a large class of quantum field theories, it was then natural to ask

H5 Can one shed light on the behaviour of hydrodynamic gradient expansion at large order with the use of holography?

This question is interconnected with the point **Q1** discussed in the previous subsection.

To summarized the status of the field as discussed so far:

- Strongly coupled plasmas in global equilibrium are described by static black branes in AdS.
- Hydrodynamics is then expected to correspond to long-wavelength perturbations along the horizon directions.
- Far-from-equilibrium physics for small-amplitude perturbations of equilibrium correspond to modes carrying large frequency or momentum.

These points make it natural to anticipate that the general far-from-equilibrium phenomena correspond to nonlinear perturbations of black brane with large spatiotemporal variations or even to gravitational collapse in AdS. Shedding light of these fascinating questions required numerical implementation of the Einstein's equations with negative cosmological constant. In the context of asymptotically flat spacetimes solving Einstein's equations on computers is one of the leading trends in relativity, whereas in the case of the AdS spacetime the question of boundary conditions at the AdS boundary seemed to be the main stumbling block for the relativity community. As a result, the first numerical simulations in the context of holography were written by the high energy physics community.

The use of the ingoing Eddington-Finkelstein coordinates led to the discovery of remarkably simple iterative algorithm to solve the Einstein's equations with negative cosmological constant. This method is summarized in the review article [99]. By 2011, this approach had allowed to solve the gravity dual to the homogeneous isotropization [100]

$$\langle T_{\mu\nu}\rangle = \mathrm{diag}\left(\mathcal{E}, \frac{1}{3}\mathcal{E} - \frac{2}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t)\right)_{\mu\nu}. \tag{34}$$

the boost-invariant flow [101] and, in [102], analyzing a collision of 2 strongly-interacting projectiles described by the energy-momentum tensor given by

$$\langle T_{\mu\nu}\rangle dx^\mu dx^\nu = \frac{N_c^2}{2\pi^2}\rho^4 e^{-(x^\pm)^2/2w} dx^\pm dx^\pm. \tag{35}$$

In the equation above $x^\pm$ are null coordinates on the boundary and the solution is parametrized[2] by the dimensionless combination of ρ and w

$$e = \rho\, w. \tag{36}$$

The pioneering results from the articles [100–102] provided preliminary – since based only on a few very special examples of initial states – indications that

- The approach to equilibrium in strongly-coupled quantum field theories occurs over the inverse of the local temperature also in the cases when nonlinear effects seem to play the dominant role.
- In the presence of hydrodynamic modes, the energy-momentum tensor can be described with a satisfactory accuracy (of the order of $10 - 20\%$) by the viscous hydrodynamics despite the fact that the viscous contribution is comparable to the energy density.

The articles [100–102] provided one of the key motivations for the series [1–10]. As opposed to the article [21], these papers circumnavigated the question of specifying initial states by generating non-equilibrium configurations from the vacuum with the use of the boundary metric perturbations [100, 101] and colliding objects which individually are exact solutions of the Einstein's equations [102]. As it turns out, the states created in the first approach are described by hydrodynamic form of the energy-momentum tensor basically as soon as the boundary metric perturbation terminates (see Tabel I in [101]). From this point of view, generating strongly-coupled non-equilibrium states by perturbing the vacuum with the sources does not allow to univocally separate equilibration and forcing processes. This leads to the following question:

[2] In Eqs. (35) and (36) the parametrization from the article [6] is used.

H6 What are the properties of relaxation processes at strong coupling in isolated systems?

An example of such a setup is, of course, expanding nuclear matter created in heavy-ion collisions at RHIC and LHC.

When it comes to the results of [102], it discussed only one of the possible values of the parameter given by Eq. (36): $\rho w \approx 0.64$. For this value of the parameter e the setup seems to be at the hydrodynamics threshold as soon as the projectiles overlap. However, the analytic results of the earlier paper [103] subject to similar limitations to [21] suggest the existence of far-from-equilibrium regime for the early times when $\rho w \to 0$. This leads to the following questions:

H7 What is the imprint of the longitudinal structure of the projectiles on the result of a collision?

and

H8 Are the results boost-invariant in the sense of Bjorken [50]?

To summarize this part of the discussion, it is important to stress that the articles [100–102] were the only results that used numerical methods to understand relaxation phenomena in strongly-coupled systems described via holography and appeared prior to the first article from the series [3]. Most of the other authors in order to circumnavigate the need of numerical implementation of Einstein's equations used (and some are still using) the AdS-Vaidya spacetime describing a collapsing shell of dust. Within this model, some progress has been made towards understanding of the relaxation of non-local observables such as 2-point functions or Wilson loops[3], but these results has not yet led to concrete phenomenological findings and, to a certain degree, are not directly related to the developments presented in the series [1–10]. Finally, it might be useful to refer to another pioneering work in the context of the AdS gravity, [104], which discusses the gravity dual to equilibration phenomena for strongly-coupled quantum field theories on $\mathbb{R} \times S^2$. Subsequent developments generalized [104] to the cases of $\mathbb{R} \times S^1$ and $\mathbb{R} \times S^3$ boundaries. As it turns out, these processes have very different properties from the corresponding phenomena in Minkowski space and the results are not directly related to the series apart from their methodology.

4.3 Description of the delivered results

The main aim of the series [1–10] is utilizing ab initio capacities of holography to perform cutting edge calculations in strongly-coupled gauge theories motivated by theoretical aspects of the applicability of hydrodynamics in ultra-relativistic heavy ion collisions at RHIC and LHC. Delivered results naturally organize themselves into the following interrelated topics:

- Timescale needed for the hydrodynamics to apply [1–4, 6, 7, 9].
- Size of the anisotropy of $\langle T^{\mu\nu} \rangle$ at the hydrodynamic threshold [3, 4, 6, 7].
- Simplified models of non-equilibrium physics [8, 10].
- Hydrodynamic gradient expansion at large orders [5, 10].

[3]Quite curiously, it turned out that in the case of these nonlocal observables the relaxation timescale depends on their characteristic size and can get arbitrarily long as the size gets very large. The latter follows from causality of the microscopic theory.

- Collisions of ultra-relativistic projectiles [6, 7].

The subsequent sections discuss at length each of those topics. The details of the used methodology is outlined in section 4.5. A separate section is devoted to outline the impact of the series [1–10] on the theoretical understanding of transition to hydrodynamics in ultra-relativistic heavy-ion collisions, as well as its influence on numerical holography and related subjects.

4.3.1 Timescale needed for the hydrodynamics to apply

Ultra-relativistic heavy-ion collisions are the only known way to generate experimentally collective excitations of quantum fields responsible for strong interactions. Within QCD, the only method allowing for ab initio predictions are calculations based on the smallness of the coupling constant. These calculations are very complex and thus it was natural to focus most of these efforts on studying non-equilibrium phenomena that result from a collisions of two nuclei[4]. From a purely theoretical standpoint, one can imagine many more possibilities of creating non-equilibrium QCD states, e.g., by exciting QCD quanta using the perturbations of the background metric.

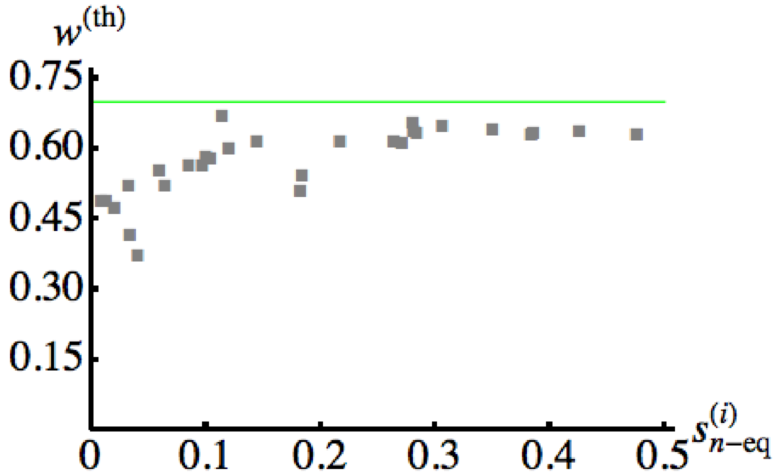
Within holography, it is not clear which initial conditions for the bulk metric and, if considered, bulk matter fields reflect in the best way the initial state at RHIC or LHC (see also section 4.3.5). As a result, scanning over various initial conditions leading to diverse non-equilibrium phenomena in different holographic theories and isolating universalities in the behaviour seems to be the best approach. This was the key motivation to consider in the series:

- 29 different initial states for the Bjorken flow at $\tau = 0$ [3, 4];
- about 2000 different initial states that homogeneously isotropize [1, 2];
- 6 different configurations for ultra-relativistic collisions [6];
- quasinormal modes in the (non-conformal) $\mathcal{N} = 2^*$ gauge theory for 6 different perturbation channels [9].

In all the considered cases, the time needed to reach the hydrodynamic regime turned out to be bounded from above by Eq. (2). In particular, the results for the Bjorken flow are displayed in Fig. 2, for the homogeneous isotropization in Fig. 3, whereas representative quasinormal mode frequencies in $\mathcal{N} = 2^*$ gauge theory are shown in Fig. 4. The results for the actual collisions are discussed at length in section 4.3.5. The fact that all these results agree with Eq. (2) is highly non-trivial because of the following 2 factors:

- Although the articles [1–4, 6] study non-equilibrium processes in strongly-coupled conformal field theories (including the $\mathcal{N} = 4$ SYM theory) in which the only equilibrium scale is set by the temperature T, the initial conditions in the bulk contain infinitely many dimensionful parameters (see also Eq. (30)). Generally, the local temperature at the hydrodynamic threshold as well as the timescale over which hydrodynamics becomes an appropriate description of the $\langle T^{\mu\nu} \rangle$ dynamics depend in a complicated way on the infinitely many parameters specifying the initial state.

[4]It is worth to stress here that phenomenological applications typically require extrapolating the weak-coupling results to a larger values of the coupling constant. As a result of that, most of the benefits of the ab initio weak-coupling approach are lost, since it is used outside its regime of validity.



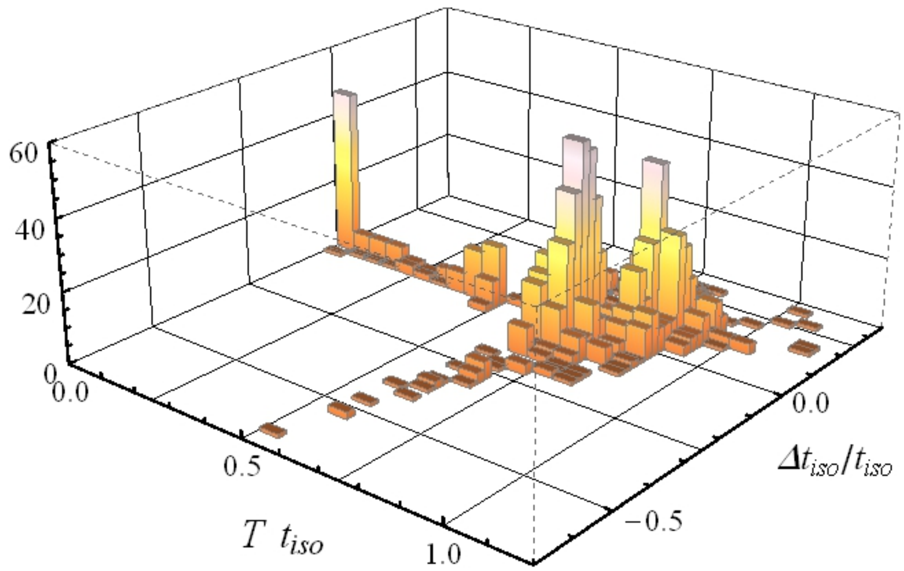
Rysunek 2: Timescale needed for the applicability of viscous hydrodynamics in the units local temperature at the hydrodynamic threshold ($w^{th} = \tau_{hydro} T(\tau_{hydro})$) for 29 different initial conditions undergoing the boost-invariant expansion. The horizontal axis shows the initial entropy. The definition of τ_{hydro} is outlined in section 4.3.2. The results support the applicability of Eq. (2). The plot is adopted from the article [3].

- The article [9] considers non-conformal $\mathcal{N} = 2^*$ gauge theory, which arises as a mass-deformation of $\mathcal{N} = 4$ SYM (see, e.g., [105]). As a result, its equilibrium state is parametrized by 2 dimensionful parameters: T and m (see Fig. 5). The results of [9] (see Fig. 4) demonstrated that despite this feature, the imaginary parts of the frequencies of the least-damped quasinormal modes remain of the order of the temperature, which suggests the applicability of Eq. (2) also in this case.

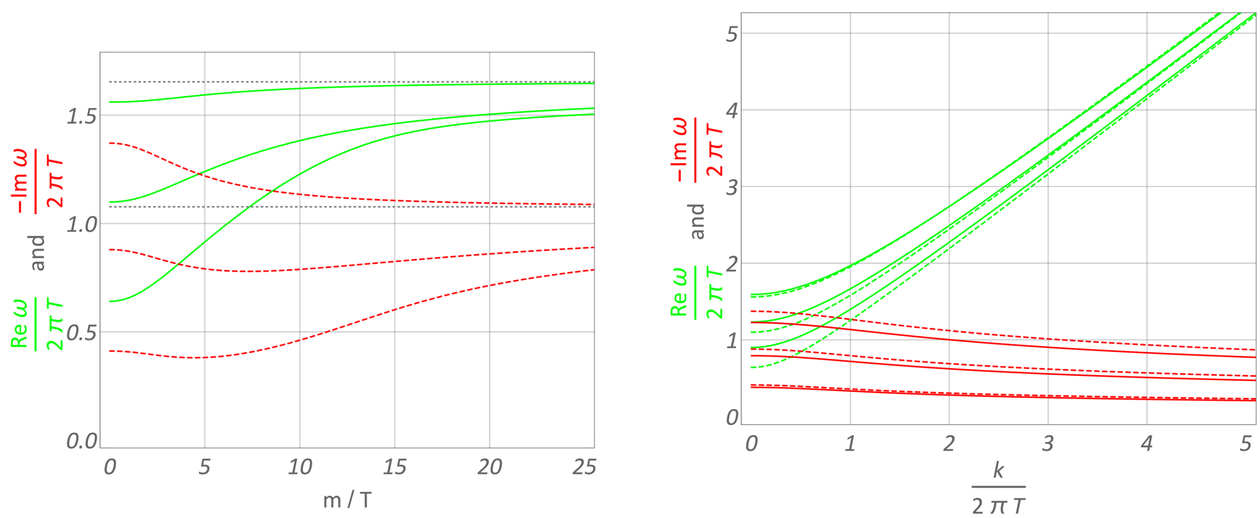
The importance of the results delivered in this part of the series stems from uncovering a novel generic feature of strongly-coupled gauge theories, which is fast applicability of viscous hydrodynamics. Obviously, these results do not show that the processes responsible for the transition to hydrodynamics at RHIC and LHC are governed by the strong coupling physics. However, if it was the case, then the results delivered in this part of the series suggest that fast applicability of viscous hydrodynamics would be then natural [**Q1**, **Q2**, **H1**, **H2**, **H6**].

It is important to note that whereas in the case of the shear viscosity in strongly-coupled gauge theories there is a gravitational understanding of its universality (see, e.g., [106, 107]), it is not entirely clear why Eq. (2) would be so widely applicable. A certain step in this direction were the results of 2 papers from the series, [1,2], which demonstrated that linearized Einstein's equations [**H1**] provide a remarkably good description of the dynamics of $\langle T^{\mu\nu} \rangle$ in the homogeneous isotropization setup (see Fig. 3). This allowed to relate the isotropization time with the properties of the quasinormal modes, which are rather well-understood [95]. Unfortunately, this approach does not generalize to more complicated cases and the question whether there are strongly-coupled quantum field theories in which generic states reach the hydrodynamic regime over a timescale significantly longer than $1/T$ remains open. However, this question might not be phenomenologically relevant, since these hypothetical models can break the conformal symmetry much more drastically than the QCD equation of state [108] or have other unwanted features.

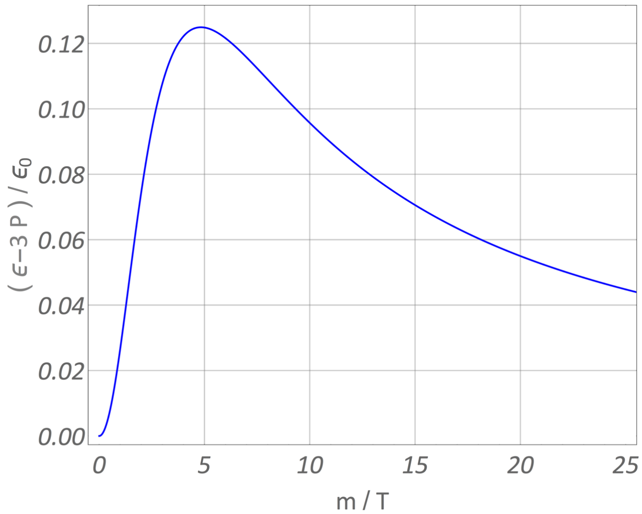
Finally, it is worth stressing that the articles discussed in this section constitute the first examples of a comprehensive analysis of many initial conditions in the context of modelling



Rysunek 3: The histogram depicts equilibration times in the process of homogeneous isotropization for about 1000 different non-equilibrium states. Isotropization time t_{iso} is defined as the latest time at which the pressure anisotropy is equal to 10% of the energy density $\mathcal{E}$, see Eq. (34). The results clearly demonstrate that none of the considered states takes longer to equilibrate than the effective temperature inverse. Furthermore, the quantity Δt_{iso} is a difference between the isotropization time obtained by means of solving the full Einstein's equations and its approximation obtained from the linearized Einstein's equations on the AdS-Schwarzschild background. Quite strikingly, isotropization times obtained in these 2 very distinct ways never differed significantly, see also section 4.3.3. The plot is adopted from the original article [1].



Rysunek 4: Plots depict normalized frequencies of the least damped quasinormal modes for the operators of dimensions $\Delta = 2$, 3 and 4 (from the bottom to the top) in the $\mathcal{N} = 2^*$ non-conformal plasma. Left: The normalized frequency dependence at vanishing momentum k as a function of the parameter measuring the conformal symmetry breaking. Right: The frequency dependence on momentum k for $m/T \approx 4.8$, which corresponds to the maximal deviation from the conformal equation of state as depicted in Fig. 5. Dotted curves denote the corresponding frequencies in the $\mathcal{N} = 4$ SYM theory. The combined results suggest that small perturbations of the equilibrium non-conformal plasma decay over a timescale of the order of the temperature inverse, see Eq. (34). The plots are adopted from the original article [9].



Rysunek 5: Deviation from conformality as seen by the equation of state of the $\mathcal{N} = 2^*$ gauge theory as a function of the conformal symmetry breaking parameter m/T. The normalization is set by $\mathcal{E}_0$ which is the energy density in the $\mathcal{N} = 4$ SYM theory (corresponding to $m = 0$). The plot is adopted from the original article [9].

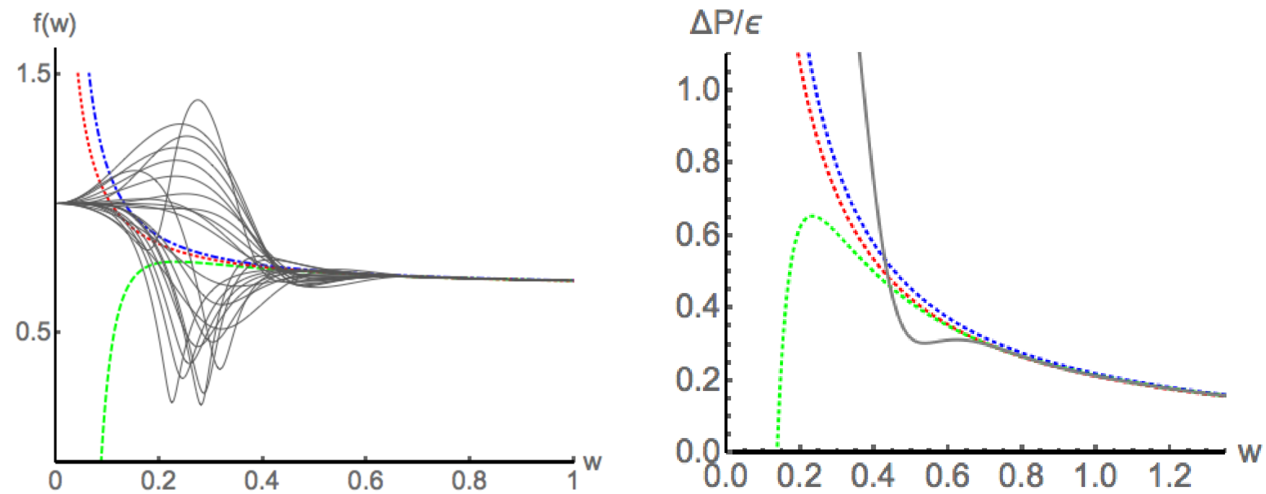
non-equilibrium phenomena using numerical holography. These efforts were continued by many other authors, including [109–112], in all the cases confirming generic applicability of Eq. (2). Furthermore, the article [9] pioneered the studies of relaxation processes in strongly coupled quantum field theories without the conformal symmetry [**Q2**, **H2**]. The results of [9], showing the applicability of Eq. (2) also in this case, were confirmed in several other models [108, 111, 113, 114].

4.3.2 Large anisotropy at the hydrodynamic threshold

Numerical holography allows to obtain $\langle T^{\mu\nu} \rangle$ for excited states in a class strongly-coupled gauge theories without making any simplifying assumptions about the microscopic dynamics. Having the exact form of $\langle T^{\mu\nu} \rangle$, the applicability of hydrodynamics can be tested by comparing it with the energy-momentum tensor for a perfect fluid, viscous fluid etc. The pioneering role of the articles [3, 4] stems from demonstrating in a clear and unambiguous way earlier partial results of [101, 102] suggesting that the applicability of hydrodynamics does not require approximate isotropy of $\langle T^{\mu\nu} \rangle$. This was possible due to:

- Separating perturbation of the system from the actual relaxation process.
- Analyzing 29 different non-equilibrium states.
- Novel parametrization of $\langle T^{\mu\nu} \rangle$ evolution in the context of the boost-invariant dynamics.

In the articles [3, 4] the gravity dual to the boost-invariant dynamics was solved for proper times ranging from $\tau = 0$ all the way to a late stage of the evolution where the applicability of hydrodynamics is unquestionable. The initial conditions were set by infinitely many parameters, as seen in Eq. (30). The late time behaviour in this case depends only on a single integration constant Λ, see Eq. (29). Different initial conditions (30) lead to different values of Λ, hence it made a lot of sense to compare evolution of different excited states by using



Rysunek 6: Left: Comparing the dynamics of 29 different excited states (continuous grey curves) with the predictions of the hydrodynamic gradient expansion truncated in the first (red), second (blue) and third (green) order. The plot clearly demonstrates that such a truncated gradient expansion does a remarkable job in describing the dynamics of $\langle T^{\mu\nu}\rangle$ for all the considered initial states for $w > 0.7$ and in some cases as early as $w > 0.35$. Right: Normalized pressure anisotropy for sample initial condition and its approximation with the hydrodynamic gradient expansion truncated in the first, second and third orders. Right at the threshold of the applicability of hydrodynamics, i.e., in this case for $w \approx 0.7$, the anisotropy is very significant and equals about 30% of the energy density. It is worth reminding that there are also other initial conditions for which viscous hydrodynamics is applicable earlier and in these cases the anisotropy at the hydrodynamic threshold gets even bigger, of the order 50% of the energy density. The plots are adopted from the original articles [3, 4].

measures independent of Λ. Perhaps the most natural way to achieve it is by introducing a dimensionless clock variable w defined as

$$w \equiv \tau\, T(\tau), \tag{37}$$

where $T(\tau)$ is the effective temperature, i.e. the temperature of the equilibrium state of the same energy density as the local energy density of the plasma at a given instance of its evolution, see also Eq. (23). The formula is also motivated by the dimensionless combination appearing in Eq. (2). In a general situation, $\partial_\tau w$ is independent of the value of the w variable at a given instance of time. However, within the hydrodynamic gradient expansion this is no longer the case, since using Eq. (29) one obtains

$$\partial_{\log\tau} \log w_{hydro} \equiv f_{hydro}(w) = \frac{2}{3} + \frac{1}{9\pi w} + \frac{1-\log 2}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45\log 2 + 24\log^2 2}{972\pi^3 w^3} + \ldots, \tag{38}$$

where the ellipsis denotes terms with 4 and more derivatives[5]. In particular, the RHS of Eq. (38) does not depend on the hydrodynamic integration constant Λ. As a result, having 29 different profiles for the energy density as a function of proper time, it was natural to create a plot of the corresponding $\partial_{\log\tau} \log w$ as functions of w and compare it with Eq. (38). This is depicted in Fig. 6 and clearly demonstrates the applicability of viscous hydrodynamics at latest for the times corresponding to $w \approx 0.7$, see also Fig. 2.

Furthermore, using Eq. (38) and defining in a rather natural way the pressure anisotropy as

$$\Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L, \tag{39}$$

[5]Note the difference between Eq. (38) and its original version in [3, 4].

we are led to the following relation

$$\Delta\mathcal{P} = \Big(6\, f(w) - 4\Big) \times \mathcal{E}. \tag{40}$$

The term $\Big(6\, f(w) - 4\Big)$ reveals itself then to be the normalized pressure anisotropy[6]. Using Eq. (38) one concludes that the applicability of viscous hydrodynamics at $w = 0.7$, see Fig. 6, which is the longest relaxation time recorded in [3, 4], gives

$$\Delta\mathcal{P} \approx 0.3 \times \mathcal{E}\Big|_{w=0.7}. \tag{41}$$

On the other hand, for $w = 0.35$, which corresponds to the earliest relaxation times recorded in [3, 4], one gets

$$\Delta\mathcal{P} \approx 0.5 \times \mathcal{E}\Big|_{w=0.35}, \tag{42}$$

where proportionality constant depends in this case (stronger than for $w = 0.7$) whether we used hydrodynamic gradient expansion truncated at the first or higher orders in derivatives. Taking to account that in equilibrium the pressures are equal to $\mathcal{P} = \frac{1}{3}\mathcal{E}$, the pressure anisotropy at the hydrodynamic threshold turned out to be equal at least to the equilibrium pressure and in several cases turned out to exceed it by about 50%. Using Eq. (40) in the viscous approximation it can be estimated that reaching the approximate isotropy will require time of the order of magnitude longer than $w = O(1)$.

To summarize, the results discussed in [3, 4] provided a particularly clear demonstration that:

- Hydrodynamics with viscosity applies very quickly in the sense of Eq. (2) [**Q1**, **Q2**, **H4**].
- The anisotropy of $\langle T^{\mu\nu}\rangle$ at the hydrodynamic threshold can exceed the equivalent equilibrium pressure by as much as 50% [**Q5**, **Q5'**].
- The leading order contribution to the pressure anisotropy comes from the shear viscosity. However, the higher order terms might be important for smaller values of w [**H3**].
- The setup reaches isotropy for times by an order of magnitudes larger than the timescale needed for the applicability of hydrodynamics.

The most important phenomenological lessons from the discoveries made in [3, 4] seem to be the following:

- The applicability of hydrodynamics needs to be a priori distinguished from the local thermalization (understood as the isotropization of 3 out of 4 eigenvalues of $\langle T^{\mu\nu}\rangle$). In order to make this distinction more apparent it was more recently proposed to refer to the applicability of hydrodynamics as to the "hydrodynamization" [80] or "hydroization" [115]).
- The ab initio methods in QCD should aim at reproducing short hydrodynamization time rather then require approximate isotropization and this should be also the criterium for their success.

[6] Comparing the pressure anisotropy to the energy density seems to be the most meaningful measure since the longitudinal pressure can be very close to 0 with the transversal pressure significantly enhanced and, as a result, incomprehensive.

- Viscous hydrodynamics can provide a remarkably good description of physical systems also in the presence of very large gradients. Although the results delivered in [3,4] concern longitudinal gradients (i.e. along the collision axis), it is natural to expect that the conclusions of these studies apply as well to the situation with large transversal gradients. The latter play a prominent role after including fluctuations in the initial profiles of the energy density (see, e.g., [116]) or in the case of p-A or p-p collisions and the results of [3, 4] can act as a partial justification of the use viscous hydrodynamic modelling also in these extreme cases.

4.3.3 Simplified models of the non-equilibrium evolution at strong coupling

The discovery of the universality of equilibration time in strongly coupled quantum field theories naturally leads to a question about possible relation to dampening rates of quasinormal modes, which are of the same order. This observation provided the main motivation for the articles [1, 2, 8] discussed below.

The articles [1,2] considered the homogeneous isotropization in which $\langle T^{\mu\nu}\rangle$ takes the form given by Eq. (34). The main reason for focusing on this particular class of non-equilibrium processes was the absence of a non-trivial hydrodynamic tail, which allows to entirely focus on the physics of transient excitations. When the pressure anisotropy $\Delta\mathcal{P}(t)$ is small compared to the energy density $\mathcal{E}$, then $\Delta\mathcal{P}(t)$ after a time of about $1/T$ will be given by the least damped quasinormal mode[7]

$$\Delta\mathcal{P}(t) \sim \exp\left(-i\,\hat{\omega}_1 T t\right), \tag{43}$$

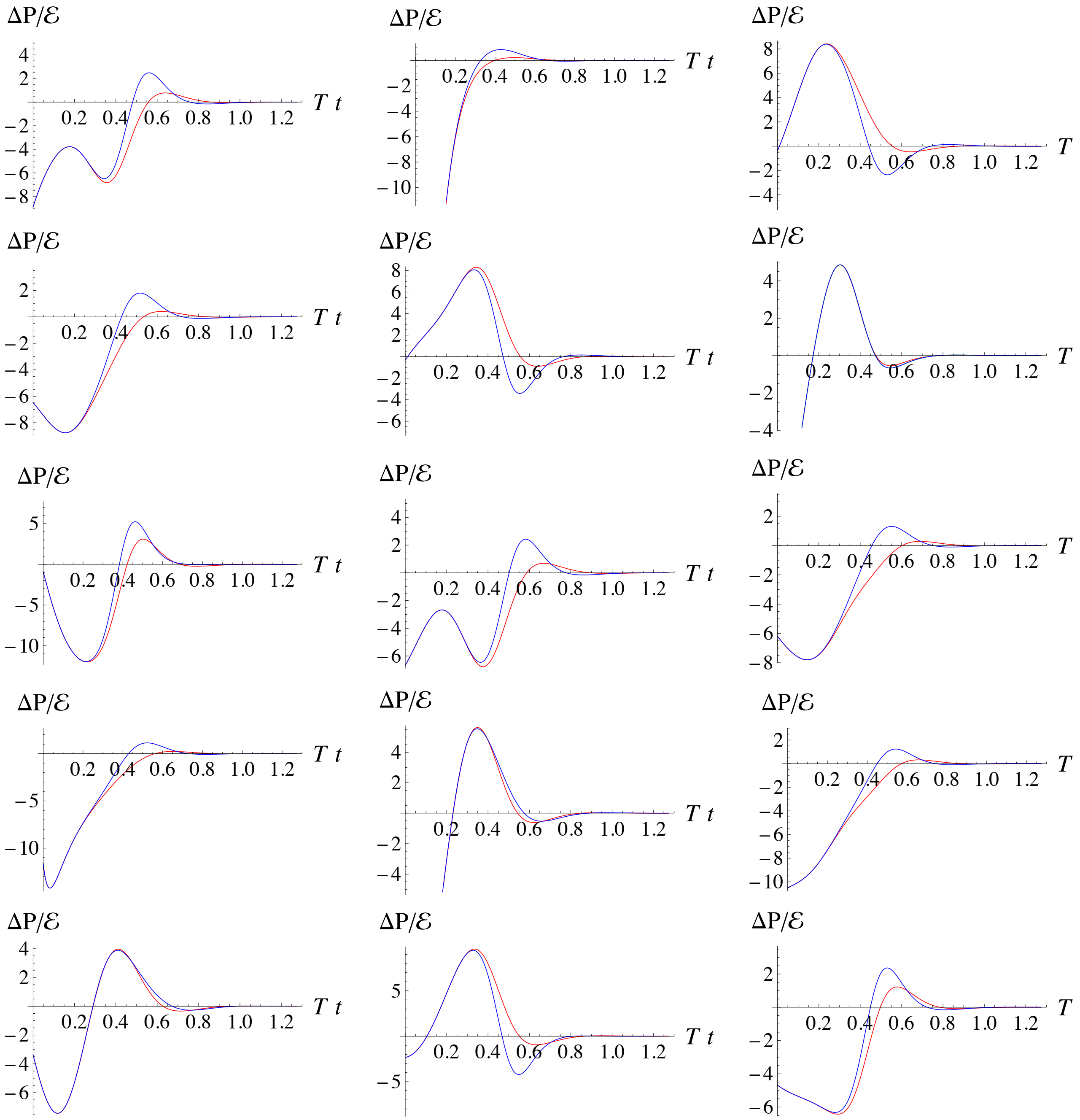
where T is the effective temperature (constant in this case), whereas $\hat{\omega}_1$ is given by Eq. (25). For slightly smaller times, one anticipates that Eq. (43) will contain contributions also from the other quasinormal modes. The articles [1, 2] posed then the question to which extent the evolution of $\langle T^{\mu\nu}\rangle$ can be reproduced by an infinite sum over all the quasinormal modes or a few least damped. From the dual gravitational standpoint, this is a question about the importance of nonlinear effects on gravitational waveforms detected at the boundary of the AdS space and emitted by equilibrating black brane.

As it turned out in all the considered cases (in total, about 2000 different initial states taking into account both articles), nonlinear effects not only did not change the qualitative features of $\Delta P(t)$, but also neglecting them completely still led to a satisfactory quantitative agreement. This finding is illustrated in Fig. 7, which shows 15 typical non-equilibrium states from the analysis in [1, 2]. It can be also seen in Fig. 3 from section 4.3.1: in the vast majority of cases neglecting nonlinear effects did not change significantly equilibration time. Deviations that can be seen in Fig. 3 after a closer analysis turned out to follow form a subtlety in applying the isotropization criterium rather than from the existence of counterexamples. This apparent linearity occurs also in the cases when the ratio of $\Delta\mathcal{P}(t)$ to $\mathcal{E}$ is very large, even of the order of several hundreds. Form the point of view of a quantum field theory, this is a completely surprising results, since in such cases one would be tempted to conclude that the system under consideration is not a small perturbation of the equilibrium configuration by any means.

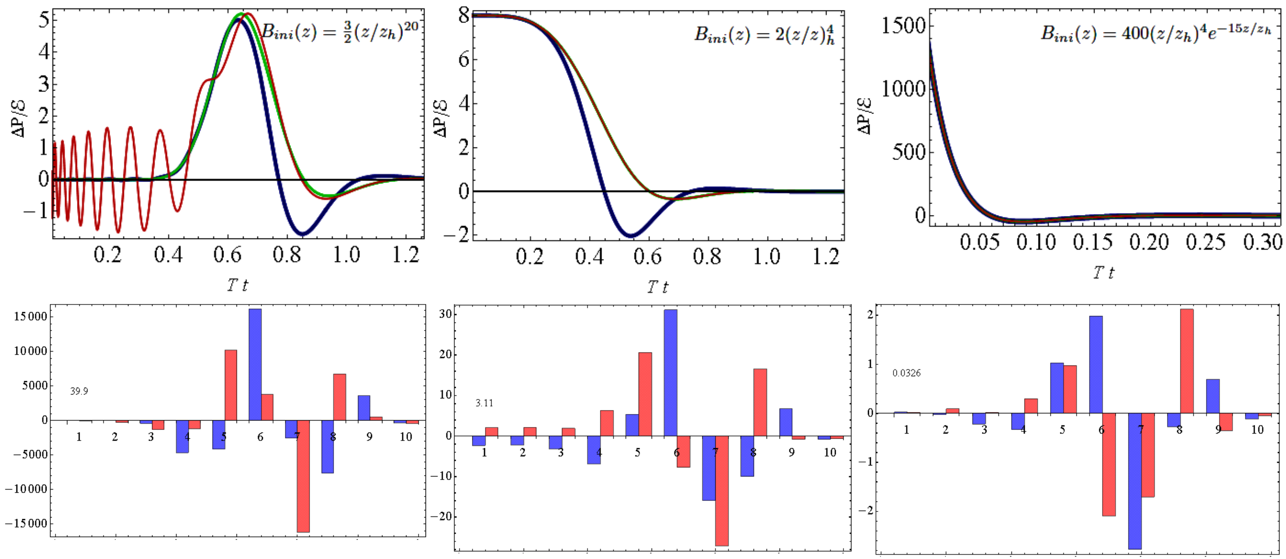
These considerations had naturally led to a question about the decomposition of the initial state onto the quasinormal modes[8]. As it turned out, typically an approximation in which

[7]See Fig. 4 in Ref. [99] for a particularly clear demonstration of this expectation. Also, note that in the homogeneous isotropization setup only the $k = 0$ quasinormal modes contribute due to the translational invariance.

[8]Mathematically speaking, it is a rather subtle issue since quasinormal modes do not form a basis in the standard sense of this word. In practice, we fitted the sum of the 10 least damped quasinormal modes, see the



Rysunek 7: Comparison between the time evolution of the pressure anisotropy obtained without any simplifying assumptions (blue curves) and as a sum over all the quasinormal modes (red curves – in practice, obtained by solving the linearized Einstein's equations in the AdS-Schwarzschild background). The plots document quantitative agreement between the approximate and exact calculations also in the regime which a priori, from a quantum field theory standpoint, does not seem to have anything in common with being a small perturbation of equilibrium. The plot is adopted from the original article [2].



Rysunek 8: Top: Comparison of the normalized pressure anisotropy as a function of time (blue curves) with the predictions given by a sum of infinitely many (green curves) and 10 least damped quasinormal modes (red curve) for 3 different non-equilibrium states. Bottom: Coefficients of a fit of the 10 least damped quasinormal modes (red: real part, blue: imaginary part). Oscillations on the left plot stem from the truncation to the 10 least damped quasinormal modes and disappear if several more modes are included. The plots are adopted from the original article [2].

the profile of $\langle T^{\mu\nu}\rangle$ is expressed as a sum of several least damped quasinormal modes works remarkably well, as can be seen in Fig. 8. This directly leads to an idea to approximate the time evolution of $\langle T^{\mu\nu}\rangle$ for a rather large class of initial states by the means of viscous hydrodynamics[9] coupled to a few least damped quasinormal modes, which directly leads to the article [8].

The starting point for the considerations in [8] is reinterpretation of the equations of motion for relativistic hydrodynamics with viscosity within the MIS theory given by Eqs. (7) and (8). The structure of these equations is dictated by the necessity to have a well-posed initial value problem for a few classical fields including the hydrodynamic variables T and u^μ. In particular, the dispersion relations for small perturbations of equilibrium should have both the group velocity smaller than the speed of light[10] and no growing modes. Furthermore, these equations need to agree with the predictions of hydrodynamic constitutive relations for $\langle T^{\mu\nu}\rangle$ when they are solved in the gradient expansion. As it turns out, the MIS theory apart from the gradient expansion containing terms of an arbitrary order in the number of derivatives of the hydrodynamic fields posses also an exponentially decaying mode with purely imaginary frequency. The easiest way to see the latter is to consider the homogeneous isotropization for which Eq. (8) reduces to

$$\frac{C_{\tau\Pi}}{T}\Delta\mathcal{P}'(t) = -\Delta\mathcal{P}(t), \tag{44}$$

discussion in the original article [2].

[9]The necessity to include viscous effects stems from the phenomenon of hydrodynamization.

[10]In the MIS theory this leads to the condition on the relaxation time [39]. As it happens to be the case, fitting hydrodynamic transport coefficient to the $\mathcal{N} = 4$ SYM theory up to the second order results in the stable and causal MIS theory.

where the relaxation time is given by

$$\tau_\Pi = \frac{C_{\tau\Pi}}{T}. \tag{45}$$

The general solution of Eq. (44) is an exponentially decaying mode,

$$\Delta\mathcal{P}(t) \sim \exp\left(-\frac{1}{C_{\tau_\Pi}} T t\right), \tag{46}$$

which is an analogous property to holographic quantum field theories, see Eq. (43), but does not exhibit oscillations. Following this line of thought one can reinterpret the MIS theory as a phenomenological model of a hypothetic microscopic theory in which the least damped non-hydrodynamic quasinormal mode has a vanishing real part and which accounts for some of the interactions of this mode (encoded in $\Pi^{\mu\nu}$) with the hydrodynamic fields T and u^μ. It is also useful to mention that the standard interpretation of the MIS theory provided by [39] assumes associating the parameters in the theory with transport coefficients in the hydrodynamic gradient expansion performed up to the second order[11]. Within this approach, one also has an exponentially damped mode, but now its frequency is directly related to a particular hydrodynamic transport coefficient and not a putative transient microscopic effect. This situation is, of course, an artifact of the interpretation of the MIS theory along the lines of [39].

The frequencies of the $T^{\mu\nu}$ quasinormal modes in strongly-coupled quantum field theories, in particular for the $\mathcal{N} = 4$ SYM theory in the appropriate regime of parameters, have both real and imaginary parts[12]. As a result, the MIS theory interpreted as above does not describe their lowest non-hydrodynamic quasinormal mode and its interactions with the hydrodynamic degrees of freedom. However, this theory can be generalized to account for it in, as it turned out, a rather simple manner[13], which was the subject and the main outcome of the article [8].

Probably the simplest generalization to account for the quasinormal mode adds a second derivative term to Eq. (8) so that it takes now the form

$$\left((\frac{1}{T}\mathcal{D})^2 + 2\,\mathrm{Im}(\hat{\omega}_1)\frac{1}{T}\mathcal{D} + |\hat{\omega}_1|^2 \right) \Pi^{\mu\nu} = -\eta |\hat{\omega}_1|^2 \sigma^{\mu\nu}, \tag{47}$$

where $\mathcal{D}$ is an appropriately defined co-moving derivative (see the original article [8] for details), $\hat{\omega}_1$ is a dimensionless frequency of the least damped non-hydrodynamic quasinormal mode (for the $\mathcal{N} = \triangle$ SYM theory it is given by Eq. (25)), η is the shear viscosity and for simplicity several other possible terms were neglected. Equations of motion for the theory defined by Eqs. (7) and (47) are second order in time and therefore require providing initial profiles of T, u^μ, $\Pi^{\mu\nu}$ and $\partial_t \Pi^{\mu\nu}$.

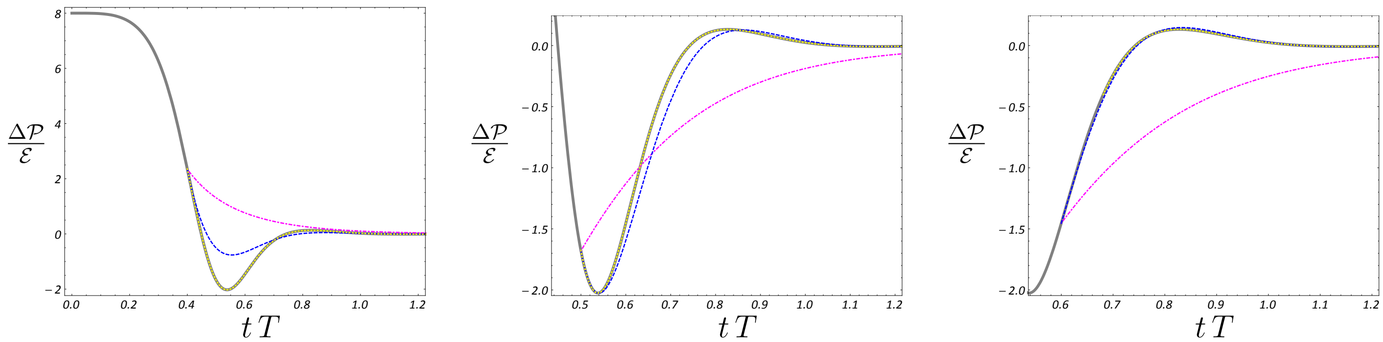
As it turns out and is described in more detail in the original article [8], for the parameters of interest (i.e. η/s given by Eq. (1) and $\hat{\omega}_1$ given by Eq. (25)), Eq. (47) has either unstable modes or breaks causality[14]. This necessitates its generalization. As it turns out, a rather satisfactory

[11] Hence the alternative name for the MIS framework, which is the theory of second order hydrodynamics.

[12] This is a mere empirical observation without any deeper microscopic insight.

[13] It needs to be stressed that quasinormal modes exhibit nontrivial dispersion relations. However, it turns out that for a rather large range of momenta around $k = 0$ the dependence on momentum is weak and can be to a certain degree neglected, see Fig. 1.

[14] Of course, this issue does not have anything to do with the microscopic physics, but rather it follows from the way we chose to construct the effective equations of motion. If such a theory were applied to the phenomenology of heavy-ion collisions, then there would be a whole spectrum of parameters for which the relevant equations of motion are well-behaved.



Rysunek 9: The results of the effective evolution equations coupling the lowest non-hydrodynamic quasinormal mode to viscous hydrodynamics for the homogeneous isotropization. Continuous grey curve denotes the ab initio numerical solution for a generic non-equilibrium state. Dotted yellow curve is its excellent approximation given by a sum of infinitely many quasinormal modes. Dotted magenta curve corresponds to the evolution with the MIS framework, whereas the dotted blue curve represents the relevant solution of the new equations of motion put forward in the article [8]. The initial times for the solutions of the effective equations of motion are set to $t\,T = 0.4$ (left), $t\,T = 0.5$ (centre) and $t\,T = 0.6$ (right). For sufficiently late initialization, the MIS framework can significantly overestimate the isotropization time (centre) and, more generally, it does not take into account visible pressure oscillations. The plots are adopted from the original article [8].

description can be achieve by using the MIS framework to account for the viscous effects and a separate equation to describe the evolution of the lowest non-hydrodynamic quasinormal mode. The non-equilibrium part of the energy momentum tensor is then split as

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{\mathrm{hydro}} + \tilde{\Pi}^{\mu\nu}, \tag{48}$$

where $\Pi^{\mu\nu}_{\mathrm{hydro}}$ obeys Eq. (8), whereas $\tilde{\Pi}^{\mu\nu}$ describes the evolution of the lowest non-hydrodynamic quasinormal mode and obeys a version of Eq. (47) given by

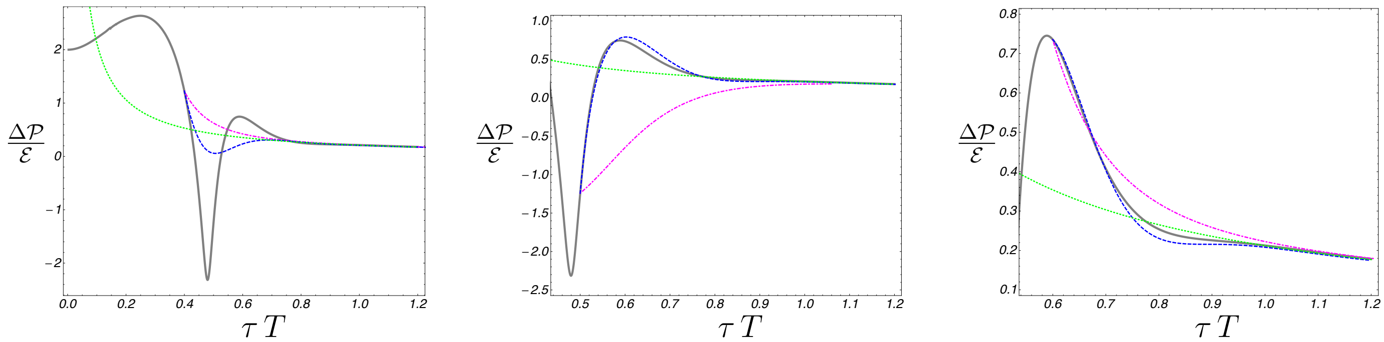
$$\Big((\frac{1}{T}\mathcal{D})^2 + 2\,\mathrm{Im}(\hat{\omega}_1)\frac{1}{T}\mathcal{D} + |\hat{\omega}_1|^2 \Big)\, \tilde{\Pi}^{\mu\nu} = 0. \tag{49}$$

Obtained equations of motion contain now 2 non-hydrodynamic quasinormal modes: the purely damped excitation of the MIS theory and an additional degree of freedom exhibiting damped oscillations. In order to minimize the effects of the unphysical (in holographic theories) MIS mode, it seems sufficient to choose

$$\Pi^{\mu\nu}_{\mathrm{hydro}} = -\eta\,\sigma^{\mu\nu} \tag{50}$$

as an initial condition. Figs. 9 and 10 show successful tests of the new equations coupling hydrodynamics to an oscillating non-hydrodynamic quasinormal mode for the values of parameters corresponding to $\mathcal{N} = 4$ SYM and comparing their predictions to the MIS theory from [39] and ab initio solutions for 2 different non-equilibrium states. New equations work comparably well to the MIS framework when it comes to the hydrodynamic part of the evolution[15] and, as opposed to the MIS framework, they capture some of the transient features of the dynamics of $\langle T^{\mu\nu}\rangle$ in strongly-coupled quantum field theories.

[15] For Eqs. (47) this aspect was also tested for the initial data breaking the translational invariance in the transversal plane



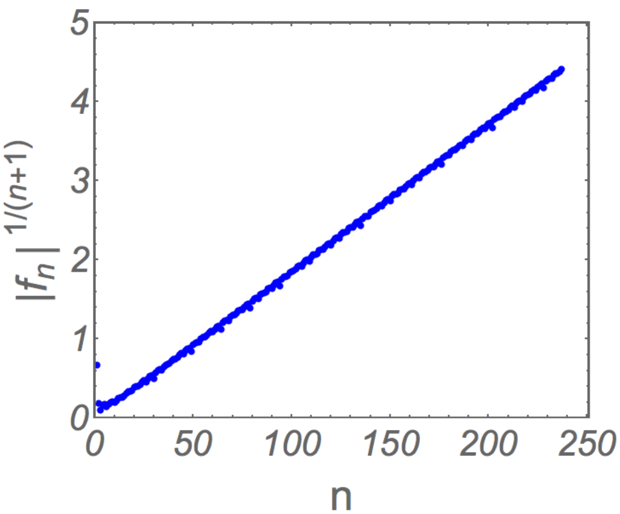
Rysunek 10: The results of the time evolution of the effective equations coupling the lowest non-hydrodynamic quasinormal mode to viscous hydrodynamics in the case of the boost-invariant flow. Continuous grey curve denotes the ab initio results for a representative non-equilibrium state. Dotted magenta curve corresponds to a solution of the MIS framework, whereas dotted blue curve is a prediction of the generalized equations postulated in [8]. For completeness, dotted green curve signals viscous hydrodynamics constitutive relation. All the effective frameworks are initialized at $\tau T = 0.4$ (left), $\tau T = 0.5$ (centre) and $\tau T = 0.6$ (right). As expected on general grounds, both MIS and the new equations converge to a single curve at late times which demonstrates yjr applicability of viscous hydrodynamics. However, for earlier times (but not much earlier since the effects of other quasinormal modes are neglected) new equations give a better qualitative agreement with the ab initio study. The plots are adopted from the original article [8].

Despite their phenomenological potential[16], the importance of the results from [8] lies mostly on the theoretical side. The latter follows from the possibility of constructing more advanced theories including more non-hydrodynamic quasinormal modes. This, according to the earlier results from [1, 2], would most likely allow to achieve satisfactory equations of motion for $\langle T^{\mu\nu}\rangle$ in a large class of strongly-coupled non-equilibrium states without the need for an explicit use of the corresponding (4+1)-dimensional gravitational dual. In this context, one should perhaps think of the (4+1)-dimensional dual geometry as of obeying hypothetic (3+1)-dimensional nonlinear equations of an infinite order, which agrees with Eq. (30). Furthermore, the article [8] is a part of a larger trend of various hybrid approaches to describe the dynamics of the quark-gluon plasma in which holographic calculations, or observations from them, are combined with other methods such as the kinetic theory (see, e.g., [117]) or relativistic hydrodynamics (see, e.g., [118]).

To summarize, from the point of view of simplified models of non-equilibrium physics, the most important results delivered in this part of the series, i.e. in [1, 2, 8], can be recapped as follows:

- Several least damped quasinormal modes seem to play the dominant role in the dynamics of $\langle T^{\mu\nu}\rangle$, which can partly explain the universality of the hydrodynamization time at strong coupling. This idea was investigated in detail in the articles [1,2] discussed in this section [**Q2**, **H3**].
- From a general quantum field theory standpoint, it seems quite surprising that the time-dependence of $\langle T^{\mu\nu}\rangle$ can be well-described by a linear wave equation even in the regime of huge anisotropies as the results of the articles [1, 2] demonstrate.
- The phenomenon of hydrodynamization, combined with the results of [1, 2], lead to

[16] There seem to be no obstacles is using the new equations in the context of heavy-ion collisions with an exception that one needs more initial conditions.



Rysunek 11: The first 241 terms of the large-w (gradient) expansion of Eq. (38). Coefficients alter their signs depending on the order. Approximately from the fifth order the absolute value of the f_n coefficients starts exhibiting factorial growth as a function of n, which translates to the linear behaviour seen on the plot. This result, originally delivered in the article [5] demonstrates that the hydrodynamic gradient expansion in the $\mathcal{N} = 4$ SYM theory has a zero radius of convergence. To the very best of the authors' knowledge, this is the first such result in the literature. The plot is adopted from the original article [5].

natural generalizations of the MIS framework in which viscous hydrodynamics is coupled to the lowest quasinormal mode. Theory of this type was constructed and successfully tested using the ab initio holographic calculations in the article [8] [**H3**].

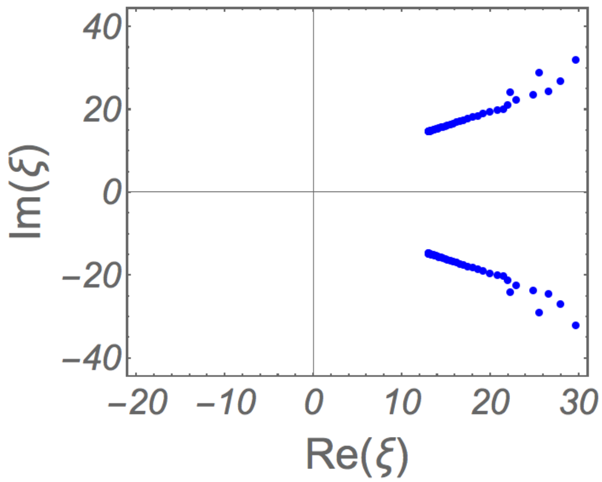
4.3.4 Hydrodynamic gradient expansion at large orders

The direct motivation for the investigations presented in the articles [5,10] is Fig. 6. It suggested that terms higher than the third order in gradients does not seem to play a significant role for $w \approx 0.7$ and higher. This was certainly not true for the lower values of w corresponding to the shortest recorded in this setup hydrodynamization time of $w \approx 0.35$, see Fig. 2, and led to the developments in the articles [5, 10] discussed in the present section.

The article [5] used the fluid-gravity duality to generate the hydrodynamic gradient expansion for the boost-invariant flow up to terms containing 240 derivatives, i.e. the first 241 coefficients of the large-w expansion of $f(w)$ given by Eq. (38) were generated. The result, displayed in Fig. 11, shows that hydrodynamic gradient expansion in the $\mathcal{N} = 4$ SYM theory at strong coupling has a zero radius of convergence. This finding can be taken as a partial explanation of the possibility of the hydrodynamization phenomenon (see also section 4.3.2): since the hydrodynamic gradient expansion does not converge in the standard sense then its applicability is not limited by the size of subsequent contributions to the series. This statement seems to be the most important phenomenological lesson following from the results of delivered in the articles [5, 10] discussed here.

The standard tool in the analysis of divergent series is the Borel transform defined as

$$Bf(\xi) \equiv \sum_{n=0}^{\infty} \frac{f_n}{n!} \xi^n \tag{51}$$



Rysunek 12: Singularities of the analytic continuation of the Borel transform of the hydrodynamic gradient expansion in the $\mathcal{N} = 4$ SYM theory truncated at 240 derivatives and given by Eq. (38). The plot has removed both numerical artifacts, as well as the artifacts of the adopted approximation. The condensations of poles within the Padé approximation correspond to cuts, which in the present case turned out to originate from 2 points associated with the frequency of the lowest non-hydrodynamic quasinormal mode given by Eq. (53). Higher quasinormal modes as well as their integer multiples are not visible due to the limitations of the used analytic continuation. For the visible singularities one can imaging 3 distinct choices of the integration contour C in the inverse Borel transform given by Eq. (52): above the upper cut, between the cuts and below the lower cut. The uniqueness of the result, up to a freedom of choosing initial conditions, turns out to be guaranteed by the hydrodynamic expansion being a part of the bigger trans-series structure (see Eq. (59) written in the context of MIS theory). The plot is adopted from the original article [5].

and its inverse (Borel summation)

$$f(w) = w \int_C d\xi \, e^{-w\xi} \, Bf(\xi), \tag{52}$$

where C is the integration contour connecting the origin with infinity on the complex plane. Singularities of the analytic continuation of the Borel transform are the source of the zero radius of convergence of the original series. Furthermore, the presence of these singularities implies that the inverse transform given by Eq. (52) is not uniquely defined. The latter observations was the main motivation behind the subsequent studies in the article [10].

In the article [5] Padé approximation was used to provide analytic continuation of the Borel transform of the truncated hydrodynamic expansion in the $\mathcal{N} = 4$ SYM theory. Its structure of singularities on the complex plane is depicted on Fig. 12 and one can clearly see there 2 symmetric cuts starting, up to 5 decimal places, at

$$\xi_0 = \frac{3}{2} i \hat{\omega}_1, \tag{53}$$

where $\hat{\omega}_1$ is the frequency[17] of the lowest non-hydrodynamic quasinormal mode for $\langle T^{\mu\nu} \rangle$ at $k = 0$ given by Eq. (25). This highly nontrivial result demonstrates that the hydrodynamic

[17] It needs to be stressed here that the real part of the quasinormal mode frequency is responsible for oscillations and hence can take both the positive and negative values. This is the rational behind the 2 cuts

gradient expansion is not convergent since in the corresponding collective states there are excitations which cannot be approximated by a truncated hydrodynamic gradient expansion.

The results of the article [5] naturally led to a question to which extent the hydrodynamic gradient expansion defines a physical theory – the relativistic hydrodynamics. This question was very hard to address in the case of $\mathcal{N} = 4$ SYM given large complexity of the calculations in [5], but it turned out that interesting observations in this context can be made using a relatively simple model given by the MIS theory defined by Eq. (8). These considerations were the subject of the article [10]. In this context it is crucial to restress in the spirit of [8] that the MIS theory is treated here not as a hydrodynamic model describing the evolution of the nuclear matter in its hydrodynamic regime, but rather as a model of some putative microscopic theory which, apart from the long-lived hydrodynamic excitations, contains also exponentially decaying degrees of freedom.

The equations of motion for the boost-invariant flow in the theory given by Eq. (8) reduce to a set of first order coupled nonlinear differential equations originating from the conservation of the energy-momentum tensor and the phenomenological relation (8). Dynamical variables in these equations are the energy density (or equivalently the effective temperature) and pressure anisotropy and both are functions of proper time τ. Introduction of the w variable given by Eq. (37) and defining $f(w)$ given by Eq. (38) decouple the equation relating $f(w)$, its derivative and the w variable

$$\begin{aligned} w C_{\tau\Pi} f(w) f'(w) \quad &+ \quad f(w)^2 \left(4 C_{\tau\Pi} + \frac{3 w C_{\lambda_1}}{2 C_\eta}\right) + f(w) \left(-\frac{16 C_{\tau\Pi}}{3} - \frac{2 w C_{\lambda_1}}{C_\eta} + w\right) \\ &- \quad \frac{4 C_\eta}{9} + \frac{16 C_{\tau\Pi}}{9} - \frac{2 w \left(C_\eta - C_{\lambda_1}\right)}{3 C_\eta} = 0. \end{aligned} \tag{54}$$

This equations needs to be understood as the evolution equation for the normalized pressure anisotropy, see Eq. (40), where time is measured in the units of the local effective temperature. The advantage of Eq. (8) as a *model* of non-equilibrium physics is its striking simplicity.

In Eq. (54) parameters C_η, $C_{\tau\Pi}$ and C_{λ_1} are numerical coefficients related to the shear viscosity, relaxation time (see Eq. (45)) and the so-called λ_1 transport coefficient from the second order gradient expansion (see [39] for details)

$$\eta = C_\eta \, s \quad \text{and} \quad \lambda_1 = C_{\lambda_1} \frac{\eta}{T}. \tag{55}$$

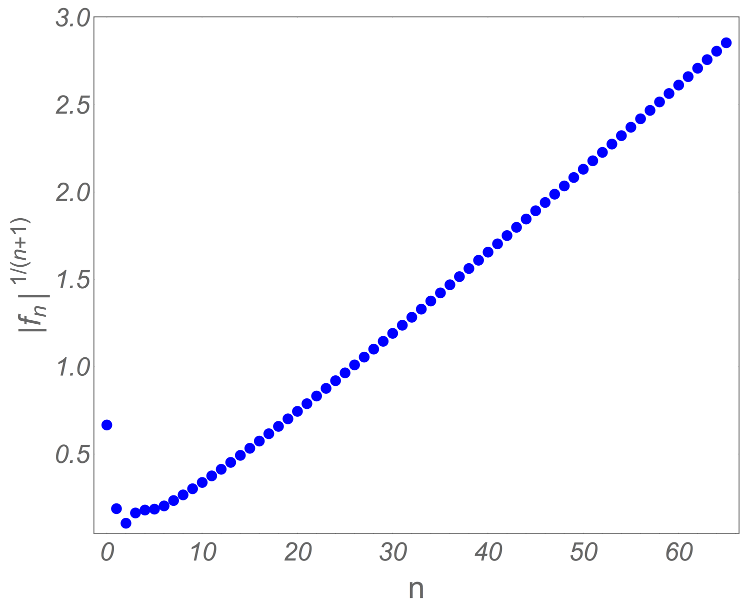
In particular, by imposing that the hydrodynamic gradient expansion dictated by Eq. (54) agrees with the $\mathcal{N} = 4$ SYM theory up to the second order in derivatives we get the following values of the 3 coefficients

$$C_\eta = \frac{1}{4\pi}, \quad C_{\tau\Pi} = \frac{2 - \log(2)}{2\pi} \quad \text{oraz} \quad C_{\lambda_1} = \frac{1}{2\pi}. \tag{56}$$

Although most of the calculations in the article [10] used the above values, it needs to be stressed that these coefficients are parameters of the MIS theory[18]. As a result, the conclusions of [10] apply to a whole class of theories.

seen in Fig. 12. Furthermore, the imaginary unit comes from the definition of the inverse Borel transform given by Eq. (52), whereas the factor of 3/2 follows directly from the adiabatic approximation on the hydrodynamic background which is discussed at length in the original articles.

[18] Up to additional conditions related to causality, see [39].



Rysunek 13: The plot, analogous to Fig. 11 demonstrates the asymptotic character of hydrodynamic gradient expansion for the MIS theory. The results are adopted from the original article [10].

Analogously to the $\mathcal{N}=4$ SYM theory and Eq. (38), the evolution equation (54) has a formal solution in the form of an infinite gradient expansion

$$f_{hydro} = \frac{2}{3}+\frac{4}{9}C_\eta w^{-1}+\frac{8}{27}C_\eta\left(C_{\tau\Pi}-C_{\lambda_1}\right)w^{-2}-\frac{16}{81}C_\eta(-2C_{\lambda_1}^2+4C_{\lambda_1}C_{\tau\Pi}+3C_\eta C_{\tau\Pi}-2C_{\tau\Pi})w^{-3}+\ldots \tag{57}$$

Imposing the condition (56) we get the agreement between Eq. (57) and the results for the $\mathcal{N}=4$ SYM theory up to terms $\mathcal{O}(w^{-2})$. The third order terms and higher are, of course, different.

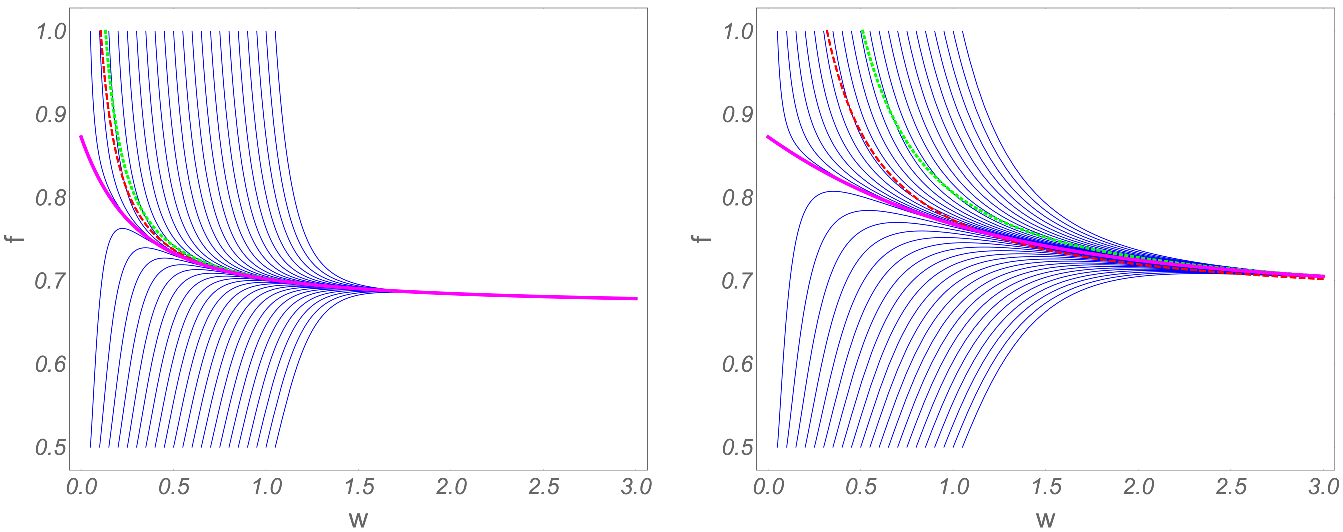
Similarly to the $\mathcal{N}=4$ SYM theory, small perturbations on top of the formal solution given by Eq. (57) exhibit exponential decay

$$\delta f(w) \sim \exp\left(-\frac{3}{2C_{\tau\Pi}}w\right) w^{\frac{C_\eta-2C_{\lambda_1}}{C_{\tau\Pi}}}\left(1+O(\frac{1}{w})\right) \tag{58}$$

with the decay time given essentially by the relaxation time τ_Π, which justifies its name. From the perspective of the holographic results, the exponentially decaying mode needs to be understood as an analog of non-hydrodynamic quasinormal modes. As it was discussed in the previous section, the main difference with holography is the absence of the oscillatory behaviour in Eq. (58) and the presence of only one such mode for each value of momentum.

In order to streamline the presentation, mostly the parameters from Eq. (56) will be used in numerical calculations. Hydrodynamic gradient expansion given by Eq. (57), as was the case for the $\mathcal{N}=4$ SYM theory, also diverges, see Fig. 13. The singularity structure of the Borel transform relates the leading singularities (a cut) with the frequency given by Eq. (58), in a complete analogy with Eq. (53). This result corroborates earlier findings from the article [5] that the hydrodynamic series expansion diverges since in the underlying collective state of matter there are excitations which are not described by the truncated derivative expansion.

The analysis presented in the article [10] demonstrates that the equations of motion for the MIS theory in the boost-invariant setup possess an attractor, see Fig. 14. For large values



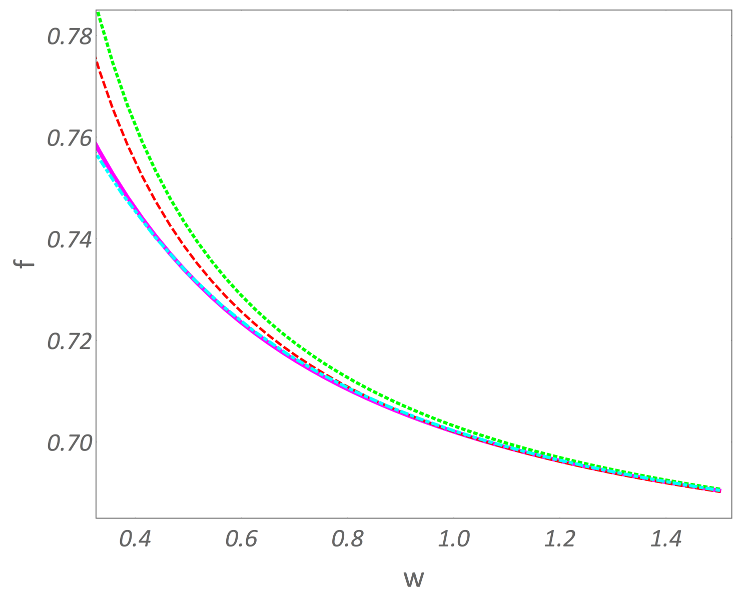
Rysunek 14: Blue curves denote numerical solutions of Eq. (54) for different initial conditions; the magenta curve is the numerically determined attractor approached by all solutions. Red and green dotted lines are, correspondingly, hydrodynamic constitutive relations up to first and second order in derivatives. Left: Parameters in Eq. (54) are fixed according to Eq. (56). Right: C_η and $C_{\tau\Pi}$ are both 3 times bigger than their previously considered values (correlating them is related to the issue of causality in the MIS theory). In the latter case, different non-equilibrium states approach the attractor at later times, in agreement with the physical intuition behind Eq. (58). It also needs to be stressed that in this case the range of w for which there is a difference between the attractor and a finite order hydrodynamics gets increased. The plots come from the original article [10].

of w (i.e. small gradients / late proper times), the attractor agrees with the predictions of the truncated hydrodynamic gradient expansion, but it also extends all the way to $w = 0$. It is then natural to associate the *resummed hydrodynamics* within the model given by Eqs. (8) with the attractor solution, as postulated in the article [10]. What is then crucial is that such a resummed hydrodynamics for the values of parameters of the order of Eq. (56) in the physically interesting range of w (i.e. for $w \geqslant 0.5$ or so) does not differ much from the truncated gradient expansion, see also Fig. 15. From this perspective, the article [10] can be understood as the very first example of (an indirect) resummation of the hydrodynamic gradient expansion justifying the use of truncated gradient expansion in the regime of large gradients.

On a related note, the article [10] resolved also the problem of ambiguities of in the Borel resummation of hydrodynamic gradient expansion. Based on the analogy with the perturbative expansions in quantum theories in which the weak-coupling expansion corresponds to the expansion in $1/w$ and non-perturbative objects such as instantons and renormalons correspond to non-hydrodynamic quasinormal modes, the article [10] postulates that the most general solution of Eq. (54) takes the trans-series form [119]

$$f(w) = \sum_{m=0}^{\infty} c^m \Omega(w)^m \sum_{n=0}^{\infty} a_{m,n} w^{-n}\,, \tag{59}$$

where $\Omega \equiv \exp\left(-\frac{3}{2C_{\tau\Pi}} w\right) w^{\frac{C_\eta - 2C_{\lambda_1}}{C_{\tau\Pi}}}$, coefficients $a_{m,n}$ are numbers set by C_η, C_{τ_Π} and C_{λ_1}, and the constant c is a parameter. The contribution with $m = 1$ comes from the non-hydrodynamic quasinormal mode in the MIS theory and terms higher order in m come from the nonlinear character of the equations of motion. For a set value of m, the expansion in powers of $1/w$



Rysunek 15: The attractor solution for the MIS theory with parameters given by Eq. (56) (magenta curve) together with the gradient expansion truncated at the first (red curve) and second (green curve) order. Blue dotted curve is the result of the trans-series resummation (up to $m = 2$ terms, see Eq. (59)). The plot suggests that in the physical range of parameters ($w \geqslant 0.5$) the direct resummation of the hydrodynamic gradient expansion does not lead to a significant alteration of the prediction based already on the gradient expansion truncated at shear viscosity level. The plot is adopted from the original article [10].

is divergent in a similar fashion to the hydrodynamic series corresponding to $m = 0$. The parameter c is in general complex and consist of the real parameter r responsible for the physical choice of the initial condition (initial state) in the first order differential equation (54) and a complex contribution associated with the choice of the integration contour in the Borel summation (inverse Borel transform). The fact that the choice of one complex parameter removes the ambiguity in the choice of the integration contour for any value of w is an example of the resurgence phenomenon, which encapsulates highly nontrivial relations between different contributions to the trans-series (i.e. terms with different values of the m index in Eq. (59)).

To summarize this part of the series, the results delivered in [5, 10] demonstrate that:

- The hydrodynamic gradient expansion, at least in certain relativistic theories, is an asymptotic series. This discovery can be understood as an explanation of the possibility of the hydrodynamization phenomenon discussed in the articles [3, 4, 6, 7] and in section 4.3.2 [**Q3**, **H5**].

- This divergence is intrinsically related to the presence of other excitations in collective states of matter, which cannot be described in terms of the gradient expansion. This led to an analogy with perturbative expansions in quantum mechanical systems and non-perturbative effects there within which the gradient expansion corresponds to the perturbative series, whereas quasinormal modes are non-perturbative excitations such as instantons or renormalons.

- Untruncated gradient expansion is not well-defined by itself due to ambiguities in the

Borel summation and requires adding non-perturbative terms cancelling this unphysical feature up to the freedom of specifying the initial state.

- Resummed hydrodynamics, at least within the MIS theory, can be associated with the attractor solution persisting all the way to $w = 0$. There is no significant difference between the attractor and the truncated gradient expansion in the first or second order within the physically relevant range of w, i.e. for $w \geqslant 0.5$ [**Q4**]. This result might be an indication that higher order transport coefficients do not contribute significantly to experimental estimates of the shear viscosity in heavy-ion collision experiments. The opposite situation was examined in [41] and the results delivered in the series suggest that this scenario might not be realized in physically interesting models.

4.3.5 Collision of two ultra-relativistic objects

The last part of the series is based on the original articles [6, 7] and concerns collisions of 2 ultra-relativistic objects in strongly-coupled quantum field theories. It can be considered a step towards modelling central ultra-relativistic heavy-ion collisions at RHIC and LHC under the assumption that the hydrodynamization process is dominated by the strong-coupling physics. Both publications focus on the analysis of the spatiotemporal profiles of $\langle T^{\mu\nu} \rangle$ after the collision until lab-frame times of the order of the hydrodynamization time at spacetime rapidity equal to 0.

The starting point for the analysis in [6, 7] is a superposition of 2 incoming packets of energy and momentum represented by $\langle T^{\mu\nu} \rangle$ given by Eq. (35). Compared to earlier results in [102], the key innovative aspect of [6,7] was studying different initial conditions which led to a discovery of a very rich pre-equilibrium dynamics in this setup. In [6] the initial conditions were set by the parameter e given by Eq. (36), which in central collisions involving the same nuclei would depend in the following way[19] on the Lorentz factor γ

$$e \sim \gamma^{-1/2}. \tag{60}$$

Within this association, setups characterized by a small value of the parameter e correspond to highly energetic collision, whereas large values of the parameter e will be associated with collisions at lower energies. In [6], 6 different values of this parameter were considered: $e = 2$, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{16}$ and $\frac{1}{8}\, e_{CY}$, where e_{CY} is the reference value used in [102] equal to approximately 0.64. Furthermore, in [7] a superposition of several Gaussian projectiles were considered in order to model a granular structure of atomic nuclei as seen along the longitudinal direction. This was meant as a step towards modelling asymmetric collisions of atomic nuclei of different sizes, at the same time neglecting the transversal dynamics. The latter, unphysical condition, was of a purely technical nature and it needs to be stressed that already after 1.5 years from publishing the original works, first results on collisions of objects with a transversal structure appeared in [120–122].

Fig. 16 depicts diagonal components of $\langle T^{\mu\nu} \rangle$,

$$(\mathcal{E}, \mathcal{S}, \mathcal{P}_L, \mathcal{P}_T) = \frac{2\pi^2}{N_c^2} \left(-\langle T^t{}_t \rangle, \langle T^z{}_t \rangle, \langle T^z{}_z \rangle, \langle T^{x_\perp}{}_{x_\perp} \rangle \right), \tag{61}$$

for 2 extreme values of the parameter e considered in [6]: $e = 2\, e_{CY}$ (collisions at "low energies") and $e = \frac{1}{8}\, e_{CY}$ (collisions at "high energies"). In order to understand the physics of the

[19] It is important to keep in mind that this is a mere analogy, since in our case the projectiles necessarily move with the speed of light.

processes depicted in Fig. 16, one needs to analyze the way hydrodynamic description becomes applicable in each of the cases. This is displayed in Fig. 17, which clearly demonstrates that at "low energies" even the collision process itself can be satisfactorily described by the means of viscous hydrodynamics. As a result, the immediate outcome of such a collision is a hydrodynamized matter, whose later expansion along the collision axis is governed by viscous hydrodynamics. This statement covered both the processes responsible for the maximal energy density significantly biggest that the sum of the maxima of incoming projectiles, as well as two receding peaks in the energy density afterwards. From this perspective, the "low energy" collisions resemble the Landau model [123] with a caveat that in this case the viscosity plays a significant role due to the phenomenon of hydrodynamization.

On the hand, in the case of the "high energy" collisions one discovers a large region originating from the light-cone boundary in which the viscous hydrodynamics is not applicable. This statement encapsulates both the remnants moving at a speed very close to the speed of light, as well as regions in which the local energy density is negative. The latter effect is well-known in quantum field theories and it reflects non-equilibrium nature of the state under consideration. Furthermore, there is also a region in which $\langle T^{\mu\nu} \rangle$ does not possess any timelike eigenvector. This is very surprising, since it implies that there is no local reference frame in which the matter is locally co-moving[20]. Moving on to the applicability of hydrodynamics, the setup of "high energy" collision provides another example of the fast hydrodynamization phenomenon discussed in section 4.3.2 in the context of the Bjorken flow results from [3, 4]. In the present case, at central rapidities hydrodynamics becomes applicable already after $t_{hyd} = 0.2/T_{hyd}$ and the pressure anisotropy is so big that the longitudinal pressure $\mathcal{P}_L$ is approximately equal to 0 at the hydrodynamic threshold, see Fig. 18.

Another topic covered in [6] is the question of the way the local energy density is distributed as a function of proper time τ and spacetime rapidity y coordinates. As it turns out, at fix proper time it is not flat (i.e. boost-invariant), but it widens when going from "low" to "high energies", see Fig. 19. This behaviour has to be contrasted with the weak-coupling results in which the rapidity profile at the leading order is boost-invariant [51]. Indeed, this quite narrow rapidity distribution seems to be a problem when the results of [6] are extrapolated to predict the spectrum of soft particles at RHIC and LHC [125]. An additional feature distinguishing between the "high energy" collisions at strong coupling [6] and the CGC results is the behaviour of the energy density at early times and central rapidity. The results depicted on Fig. 20 are consistent with the interpretation in terms of Eq. (31) in which the local energy density starts at 0 and grows quadratically with time: the parameter$n$ in Eq. (31) is equal to 2. On the other hand, in the CGC framework the local energy density is non-vanishing already at $\tau = 0^+$, see [51].

The point of departure for the last paper from the series [7] was the observation that despite considering more and more "energetic" collisions, for e smaller than a certain threshold value one nevertheless uncovers very similar profiles of the energy density in the mid-rapidity region. This led to the idea that in the centre of mass frame for the collisions at large-enough γ the longitudinal structure of the projectiles ceases to matter from the point of view of plasma at mid-rapidities. The article [7] clearly demonstrates that this phenomenon, called "coherence", indeed does occur provided the longitudinal structure of the projectiles in the centre of mass frame is smaller than approximately $0.26/T_{hyd}$, where T_{hyd} is the hydrodynamization temperature at mid-rapidity. This statement applies, in particular, to a collision seen on Fig. 21 (left),

[20] Trying to understand this observation from a weakly-coupled standpoint was a subject of the article [124] by other authors.

which is a crude[21] model of a p-A collision. The results from [7] indicate that for large-enough values of γ the distribution of the energy is a symmetric function of rapidity in the centre of mass frame. The lab-frame position of its maximum scales then in a simple fashion with atomic numbers of the projectiles.

The results delivered in [6, 7] can be briefly summarized as follows:

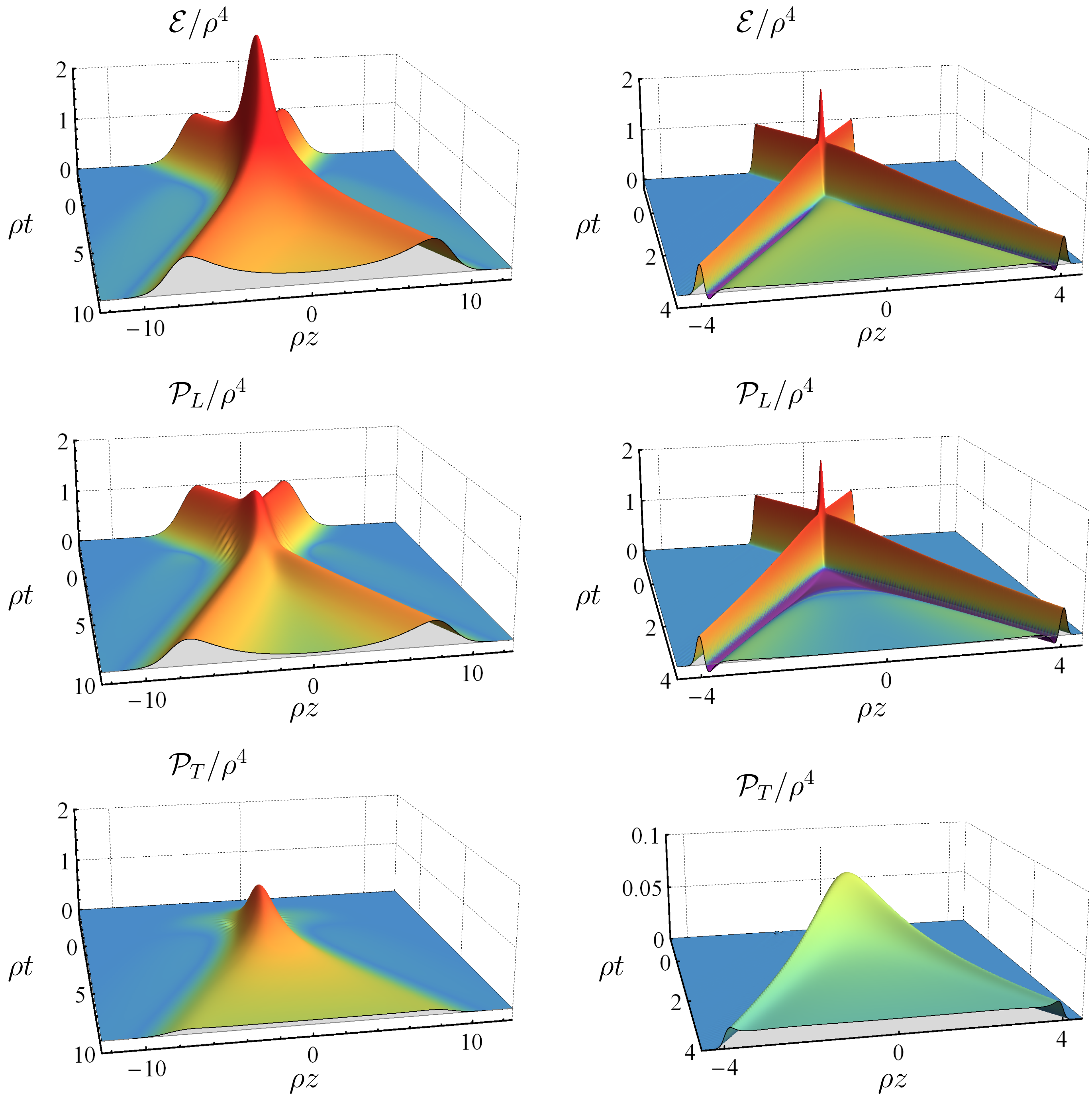
- Holographic collisions provide another instance of the phenomenon of fast hydrodynamization [**Q1**, **Q2**, **Q5**, **Q5'**].
- The form of $\langle T^{\mu\nu}\rangle$ for time-scales of the order of hydrodynamization times can be strongly-dependent on the initial condition interpreted in [6] as specified by the Lorentz factor γ. Collisions at "low energies" are characterized by an immediate hydrodynamization, whereas at "high energies" the hydrodynamization occurs after a rich far-from-equilibrium evolution.
- In the "high energy" regime, one finds regions without a local rest-frame as well as regions where the lab-frame energy density is negative. Furthermore, the resulting plasma is not boost-invariant in the sense of Bjorken [50] [**H8**] and the time-dependence of the energy density at mid-rapidity and early times is different than in the CGC framework at the leading order in the QCD coupling constant [51].
- At "high energies", collisions display the phenomenon of coherence, i.e. the longitudinal structure of projectiles does not leave an imprint on the hydrodynamized matter in the centre of mass frame [**H7**]. This is one of the first results in numerical holography that was directly motivated by the physics of p-A collisions.

4.4 Summary

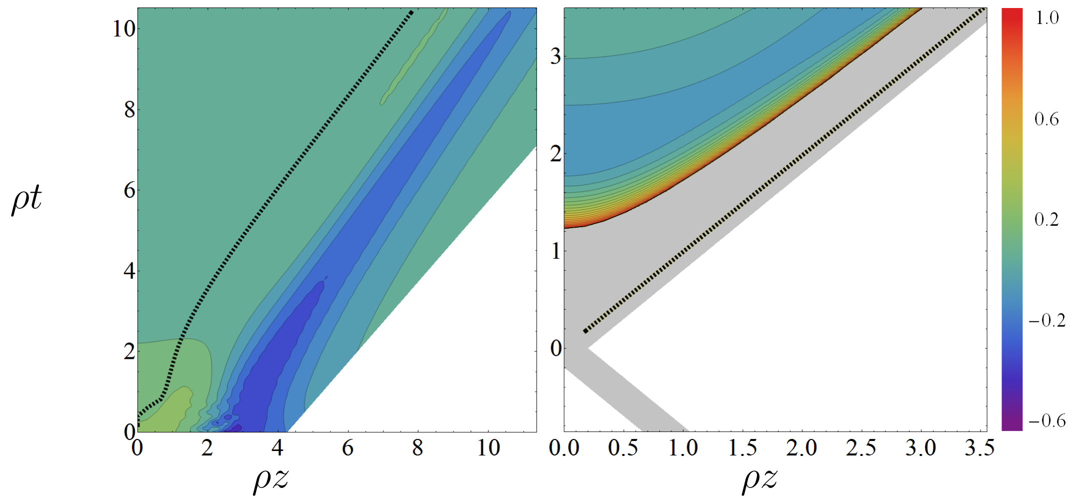
The most important results delivered by the series [1–10] can be summarized as follows:

- In strongly-coupled quantum field theories it is the temperature at the hydrodynamic threshold which sets the timescale over which viscous hydrodynamics becomes the appropriate description of $\langle T^{\mu\nu}\rangle$ in a given non-equilibrium process.
- The applicability of hydrodynamics does not require approximate isotropy. In order to make this finding more apparent to the community, a significant part of the literature now refers to the transition to the hydrodynamic regime as to the hydrodynamization process ($\neq$ thermalization).
- Existing formulations of relativistic viscous hydrodynamics can be generalized to account for the interactions of hydrodynamic and non-hydrodynamic degrees of freedom in strongly-coupled systems.
- Hydrodynamic gradient expansion is divergent. However, its resummation does not seem to lead to significant deviations from the predictions of viscous hydrodynamics truncated at the shear viscosity level.
- Ultra-relativistic collisions in strongly-coupled quantum field theories have different phenomenology than the weakly-coupled QCD models.

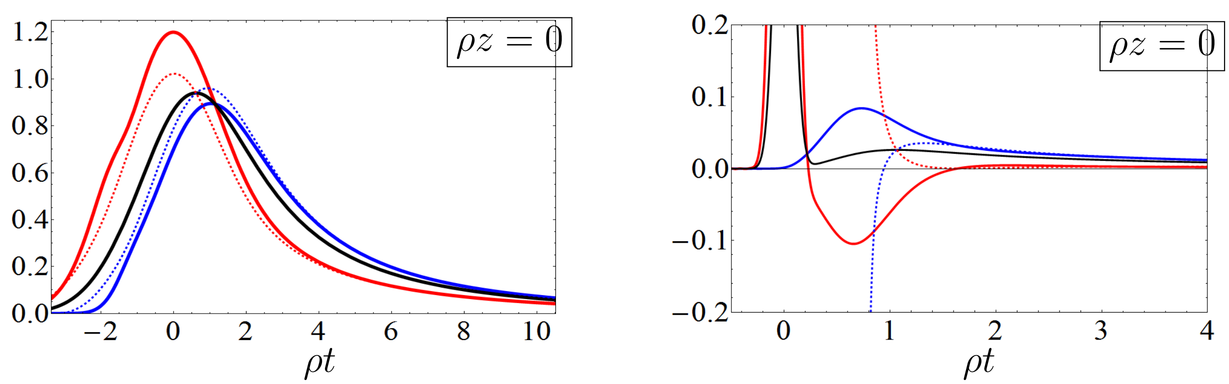
[21] Because it neglects the transversal dynamics.



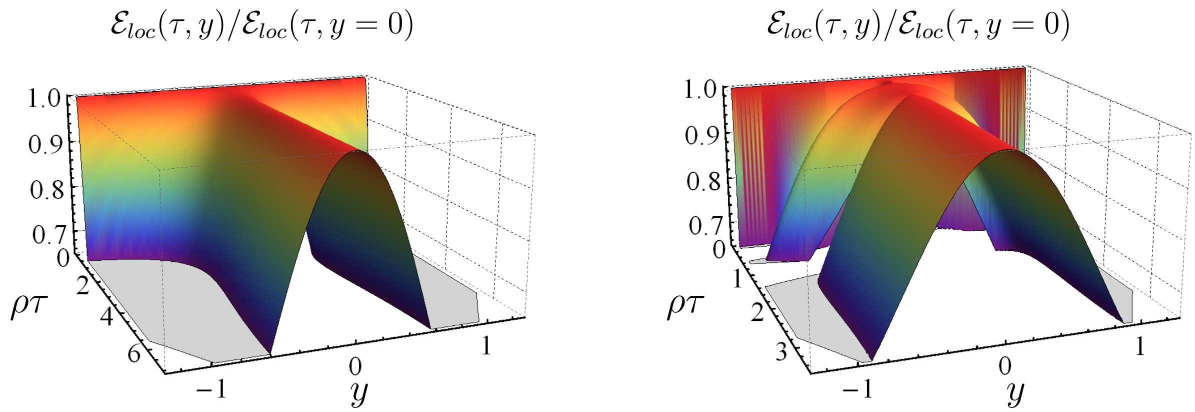
Rysunek 16: The energy density and the longitudinal pressure for the collisions at "low" (left) and "high energies" (right). All the quantities are normalized so that the maximal energy density before the collision is equal to 1. Furthermore, the grey plane corresponds to the vertical axis value equal to 0. The origin of the horizontal axis was chosen so that the maxima of the projectiles would meet there in the absence of any interactions between them. The plots are adopted from the original article [6].



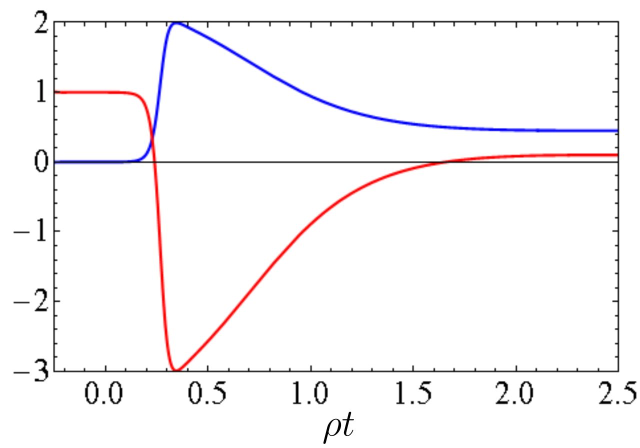
Rysunek 17: One of the measures of the applicability of viscous hydrodynamics defined as the ratio of the difference between the longitudinal pressures between the ab initio results and the viscous hydrodynamics prediction normalized to what would be the equilibrium pressure, $\Delta\mathcal{P}_L^{\rm loc}/(\frac{1}{3}\mathcal{E}_{\rm loc})$, for the collisions at "low" (left) and "high energies" (right). Dotted curves denoted the maxima of the energy flux. White reflects the vacuum in the region outside the causal contact with the projectiles, whereas grey shows the regions where the deviations from the applicability of viscous hydrodynamics are bigger than 100%. In particular, in the collisions at "high energies" there are regions where one cannot find a local rest frame, i.e.. $\langle T^{\mu\nu}\rangle$ does not possess a timelike eigenvector which after normalization defines the fluid velocity u^μ in the Landau frame. Better understanding of this observation was a subject of the paper [124] by other authors. The plots are adopted from the original article [6].



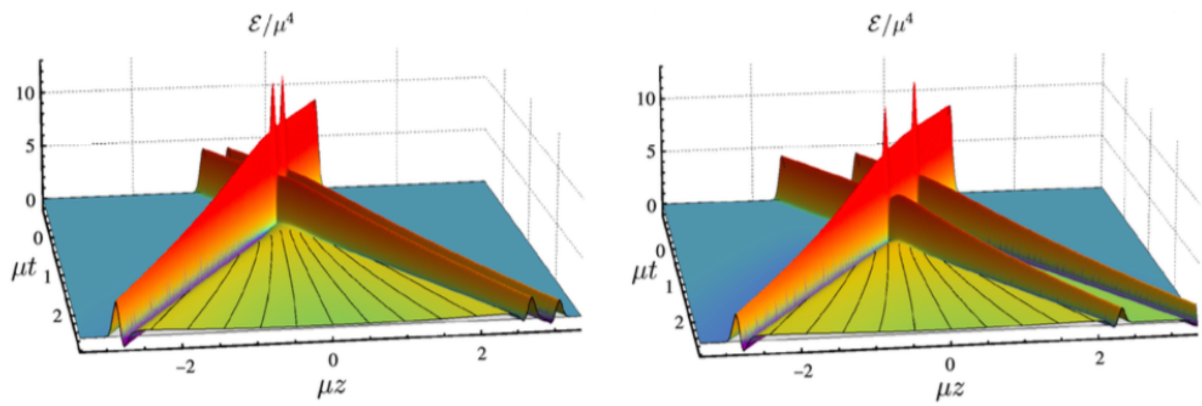
Rysunek 18: The equilibrium pressure corresponding to a given value of the energy density ($\mathcal{E}/3\rho^4$, black), the longitudinal pressure ($\mathcal{P}_L/\rho^4$, red) and the transversal pressure ($\mathcal{P}_T/\rho^4$, blue) as functions of the lab-frame time for the holographic collisions at "low" (left) and "high energies" (right). The quantities are measures in the centre of the created matter, i.e. at $\rho\, z = 0$. Dotted curves describe the corresponding viscous hydrodynamics predictions. The plots are adopted from the original article [6].



Rysunek 19: The energy density in the local rest frame as a function of spacetime rapidity y and proper time τ for the collisions at “low” (left) and “high energies” (right). The plots show a clear tendency for the distribution to widen as the “collision energy” grows, but exhibit no sign of its flattening. Furthermore, the discontinuity of the plot for the “high energy” collision follows from the absence of a local rest frame in the far-from-equilibrium regime. The plots are adopted from the original article [6].



Rysunek 20: Ratios of $\mathcal{P}_L/\mathcal{E}$ (red curve) i $\mathcal{P}_T/\mathcal{E}$ (blue curve) at $z = 0$ in the case of a “high energy collisions”. After a short time needed for the projectiles to pass through each other, which one can see on the plot through the ratios being equal to the corresponding values following from Eq. (35), pressures temporarily reach the values of $-3 \times \mathcal{E}$ and $2 \times \mathcal{E}$. This observation is consistent with the behaviour of Eq. (31) for $n = 2$. The plots are adopted from the original article [6].



Rysunek 21: An example of an asymmetric collision as seen in the centre of mass frame and corresponding to small (left) and large (right) values of the Lorentz factor γ. Within the analogy pursued in the text, the projectile with 2 maxima in its energy density corresponds to a "large nuclei" (each maximum is a "nucleon"), whereas the projectile with only one maximum corresponds to a proton. When the longitudinal structure of each of the projectiles in the centre of mass frame lies within $0.26/T_{hyd}$ (left), it does not leave an observational imprint on the structure of the energy density at central rapidities. This phenomenon is what was called in [7] as the "coherence", since one does not observe then any direct effects of individual collisions between the participating "nucleons". The plots are adopted from the original article [7].

All the delivered results are based on ab initio calculations, i.e. without making any ad hoc simplifying assumptions about the microscopic dynamics, in a class of strongly-coupled quantum field theories. Next sections discuss methodological aspects of the series, as well as the influence of articles [1–10] on the field.

4.5 Novel aspects of the methodology

Vast majority of the developments in the series (apart from the articles [8, 10]) is based on applying numerical techniques to the Einstein gravity in the presence of negative cosmological constant. Furthermore, the article [9] considered also additional matter fields and associated nontrivial configuration of sources in the dual $\mathcal{N} = 2^*$ gauge theory.

One of the most novel aspects of the series, together with the earlier developments in [100–102], lies in using the numerical relativity techniques in the context of the AdS spacetimes before it became the mainstream activity in holography. Solving Einstein's equations on computers till recently was the research domain of relativists predominately developed in the context of asymptotically flat spacetimes. The main feature of the AdS spacetimes is the presence of timelike boundary at infinity, which requires imposing conditions on the behaviour of fields propagating in the bulk. In the context of holography, the appropriate boundary conditions are interpreted as configurations of sources in dual quantum field theories. The necessity of imposing these novel boundary conditions and the lack of obvious physics motivation for that type of calculations seem to be the key factors why the significant development of numerical relativity applied to AdS spacetime occurred only in the second decade of the 21$^{\text{st}}$ Century.

All the calculations presented in the series were performed on personal computers (in most cases without parallelization) and, apart from [5–7], generating the relevant numerical data took at most an hour, in most of the cases much less. This fact follows from considering setups of high symmetry, which nevertheless provided rich physics insight. As the field developed, the symmetry assumptions got gradually relaxed and new developments appeared which required much more elaborated numerical methods and necessarily used supercomputers, see, e.g., [126].

When it comes to the details of numerical implementation, all the works within the series used Mathematica. Furthermore, numerical data for [3, 4] were obtained using computer codes written in C++. Discretization in time direction was done using high-order Runge-Kutta methods, whereas for spatial directions the pseudospectral methods were used [127]. The latter approach allowed to retain a very good precision capacities with a rather small number of grid points (typically less than 100). Furthermore, numerical calculations in [5] required extended precision (Mathematica tracked intermediate results up to several hundreds of decimal places) due to error propagation in 240 steps of iteration and summing very many terms with alternating signs.

When it comes to numerical cross-checks in modelling real-time processes:

- Normalized gravitational constraint equations were monitored (articles [1–4, 6, 7]);
- Obtained $\langle T^{\mu\nu} \rangle$ was compared with the known early time expansion (articles [3, 4]);
- The results were compared with viscous hydrodynamics predictions (articles [3, 4, 6, 7]);
- Quasinormal modes late time tail was obtained from the numerical data for $\langle T^{\mu\nu} \rangle$ (articles [1, 2]);
- $\langle T^{\mu\nu} \rangle$ was compared with the competing group numerical data [99, 102] (articles [6, 7]).

When it comes to elliptic problems

- Numerically-generated transport coefficients were checked against known analytic results at low orders (article [5]);
- we compared obtained quasinormal mode frequences with known results at low and large temperatures (article [9]).

All those tests were successful which provides very stringent verification of the correctness of the calculations on which the series is based. Last, but not least, it might be useful to mention that in the case of article [5] the most nontrivial test is reproducing the frequency of the lowest quasinormal mode with the accuracy of 5 decimal places from the high order behavior of the hydrodynamic series.

When it comes to novel technical aspects of the series, the following features are worth listing:

- Pioneering the analysis of many non-equilibrium states due to solving gravitational constraints equations on an initial time slice [1–4, 6, 7]. This is certainly the most innovative aspect as compared to the earlier developments by other authors [100–102] which founded thenumerical holography;
- Implementing the so-called ADM formalism for the AdS gravity in a novel way [3, 4], which allowed to follow non-equilibrium evolution of initial states analyzed earlier in [21];
- Pioneering calculations in hydrodynamics at large orders possible due to very precise and fast numerics [5];

- Related to the former point, pioneering the use of the method of trans-series in the context of relativistic hydrodynamics [10];
- Initialization[22] of the studies of equilibration processes in strongly-coupled quantum field theories without the conformal symmetry, which required using numerics in the presence of non-trivial boundary conditions corresponding to conformal symmetry breaking sources in the $\mathcal{N} = 2^*$ gauge theory.

4.6 Influence on the field and other researchers

The scientific program developed by articles [1–10] has been a part of an extremely active interdisciplinary research front. Between 2010 and 2015, and especially after 2013 when almost all the "numerical" works from the series were already available online, generating numerical solutions of Einstein's equations in AdS spacetimes and analyzing their properties became one of the main research activities in holography. As a result, articles [1, 3, 6] are regarded as standard references in this newly emerged research field which is reflected, in particular, by their citation record. When it comes to the numerical holography, these works provided important clues about the apparent universality of the hydrodynamization time when measured in the units of local effective temperature. An example of article by other authors which builds on these ideas is [109]. In this context, one of the last articles published within the series, [9], is especially worth mentioning since it initiated investigations of equilibration time scales in strongly-coupled quantum field theories without the conformal symmetry. Currently, it is one of the most active research trends in numerical holography.

In parallel to the developments in the series, an enormous progress has been made in studies of equilibration processes in the weakly-coupled QCD in the so-called statistical classical field theory (see, in particular, [54] and [53]), as well as in the effective kinetic description of QCD (see, e.g., [128]). The main aim of these developments is understanding the hydrodynamization process in the weakly-coupled QCD starting from the CGC-type initial conditions corresponding to heavy-ion collisions at asymptotically high energies. Also developments on this front refer to the presented here publication series (especially to [3, 6]) as to standard references on the problem of hydrodynamization in strongly-coupled quantum field theories. When it comes to the initial state physics in heavy-ion collisions at RHIC and LHC, the key articles developed within the series are [6, 7]. The most interesting results delivered in them from the current standpoint are: unexpected far-from-equilibrium phenomena of the negative local energy density and the absence of local rest frame in the regions close to the collision light-cone, different from the one predicted by the CGC form of $\langle T^{\mu\nu} \rangle$ right after the collision and relation with the Landau model. In the latter context, the article [6] is cited, e.g., by other authors in [129]. Also, the papers [6, 7] led to many further developments by the co-authors and other researchers, including [112, 120–122, 124–126, 130, 131].

Another research direction influence by the series are the studies of relativistic hydrodynamics and its generalizations which aim at describing some of the non-hydrodynamic effects in the heavy-ion collisions at RHIC and LHC. In this context, the key developments from the series are [3, 4], which demonstrated very clearly the phenomenon of hydrodynamization. When it comes to the phenomenological role of the other work from the series, [5], it changed the way of thinking about the hydrodynamic gradient expansion, see, e.g., [132].

[22] Article [9] appeared on the arXiv in the same listing as the competing works [113, 114] and day before other [111].

The articles [5, 10], on the other hand, opened a new interdisciplinary research front: applying trans-series techniques to hydrodynamic gradient expansions at high orders. The trans-series method is currently being widely applied to the analysis of quantum-mechanical perturbative series and the article [10] fits very well into this active research front at the same time finding a new application area for these methods, see, e.g., article [119]. Another indication of this influence are recent works by the leading specialists from the discipline, [133] and [134], which build directly on [10]. On top of that, it should be noted that efforts towards understanding hydrodynamics at the third and higher order of the gradient expansion, such as [135], are also partially motivated by the research presented in the series.

An equally important measure of the influence of the series on other research is its coverage by review articles. The articles from the series are discussed in particular in the following review works:

- The book [80] published by Cambridge University Press, which discusses at length [1–4];
- The article [78] discusses [3–5];
- The article [38] pays a lot of attention to the developments in [3, 4];
- The book "Quark-Gluon Plasma 5" published by World Scientific[23] discusses the following articles: [1–7].

Apart from these review works, the articles from the series are cited in several other summaries and reviews, which can be found on the arXiv in categories [hep-th], [hep-ph], [nucl-th] and [gr-qc]. This reflects the interdisciplinary character of the series and the scope of its influence. Last but not least, a significant fraction of the developments from the series will be discussed at length in the review article "New theories of relativistic hydrodynamics in the LHC era" written for Reports on Progress in Physics (Impact Factor 17.062). The article is currently under the development and is expected to be ready in the first half of 2017. The co-authors in this project are Michał Spaliński and Wojciech Florkowski.

5 Overview of other academic activities

5.1 Other scientific publications

After the Ph.D. (October 2010 onwards)

11. A. Buchel, M. P. Heller and J. Noronha, "Beyond adiabatic approximation in Big Bang Cosmology: hydrodynamics, resurgence and entropy production in the Universe," submitted to Phys. Rev. Lett., arXiv:1603.05344 [hep-th].
12. J. de Boer, M. P. Heller, R. C. Myers and Y. Neiman, "Holographic de Sitter Geometry from Entanglement in Conformal Field Theory," Phys. Rev. Lett. 116, 061602 (2016), arXiv:1509.00113 [hep-th].
13. J. de Boer, M. P. Heller and N. Pinzani-Fokeeva, "Effective actions for relativistic fluids from holography," JHEP 1508, 086 (2015), arXiv:1504.07616 [hep-th].
14. J. de Boer, M. P. Heller and N. Pinzani-Fokeeva, "Testing the membrane paradigm with holography," Phys. Rev. D 91, 026006 (2015), arXiv:1405.4243 [hep-th].

[23] The appropriate chapter is available online as [136].

15. V. Balasubramanian, B. D. Chowdhury, B. Czech, J. de Boer and M. P. Heller, "Bulk curves from boundary data in holography," Phys. Rev. D 89, 086004 (2014), arXiv:1310.4204 [hep-th].

16. J. de Boer, B. D. Chowdhury, M. P. Heller and J. Jankowski, "Towards a holographic realization of the Quarkyonic phase," Phys. Rev. D 87, 066009 (2013), arXiv:1209.5915 [hep-th].

17. I. Booth, M. P. Heller, G. Plewa, M. Spalinski, "On the apparent horizon in fluid-gravity duality," Phys. Rev. D 83 (2011) 106005, arXiv:1102.2885 [hep-th].

18. I. Booth, M. P. Heller, M. Spalinski, "Black brane entropy and hydrodynamics," Phys. Rev. D 83 (2011) 061901, arXiv:1010.6301 [hep-th].

Before the Ph.D. (before October 2010)

19. I. Booth, M. P. Heller, M. Spalinski, "Black brane entropy and hydrodynamics: the boost-invariant case," Phys. Rev. D 80 (2009) 126013, arXiv:0910.0748 [hep-th].

20. A. Buchel, M. P. Heller, R. C. Myers, "sQGP as hCFT," Phys. Lett. B 680, 521 (2009), arXiv:0908.2802 [hep-th].

21. G. Beuf, M. P. Heller, R. A. Janik, R. Peschanski, "Boost-invariant early time dynamics from AdS/CFT," JHEP 0910, 043 (2009) , arXiv:0906.4423 [hep-th].

22. M. P. Heller, P. Surowka, R. Loganayagam, M. Spalinski and S. E. Vazquez, "Consistent Holographic Description of Boost-Invariant Plasma," Phys. Rev. Lett. 102, 041601 (2009), arXiv:0805.3774 [hep-th].

23. M. P. Heller, R. A. Janik and T. Lukowski, "A new derivation of Luscher F-term and fluctuations around the giant magnon," JHEP 0806, 036 (2008), arXiv:0801.4463 [hep-th].

24. P. Benincasa, A. Buchel, M. P. Heller and R. A. Janik, "On the supergravity description of boost invariant conformal plasma at strong coupling," Phys. Rev. D 77, 046006 (2008), arXiv:0712.2025 [hep-th].

25. Michal P. Heller and Romuald A. Janik, "Viscous hydrodynamics relaxation time from AdS/CFT," Phys. Rev. D 76 (2007) 025027, arXiv:hep-th/0703243.

Proceedings

26. M. P. Heller and P. Surowka, "AdS/CFT correspondence, viscous hydrodynamics and time-dependent D7-brane embedding," Acta Phys. Polon. B 38, 3809 (2007).

27. M. P. Heller, "Second Order Viscous Hydrodynamics and AdS/CFT Correspondence," eConf C 0706044, 08 (2007).

28. M. P. Heller, R. A. Janik and R. Peschanski, "Hydrodynamic Flow of the Quark-Gluon Plasma and Gauge/Gravity Correspondence," Acta Phys. Polon. B 39, 3183 (2008), arXiv:0811.3113 [hep-th].

In total, 24 published scientific articles and 1 preprint. Among them, 10 papers published in Physical Review Letters which is the most prestigious magazine in the field of theoretical high energy physics. During 5.5 years after the Ph.D. the publication list grew by 18 records, half of which were Physical Review Letters. Including 3 proceedings, the articles were cited 1037 times according to the INSPIRE HEP database. This number includes 2 articles cited more than 100 times, [3, 25], and 7 others cited more than 50 times, [1, 4, 6, 15, 21, 23, 24]. After removing all the self-cites (including citations by the co-authors) from the analysis, 869 independent citations. Google Scholar has recorded 1117 citations to date. All the bibliometric data are provided as of 31 May 2016.

5.2 Scientific talks

5.2.1 Colloquia

1. "String theory, thermalization and ultra-relativistic heavy ion collisions", University of Western Ontario, London, 2015.

2. "Gauge fields out of equilibrium – a holographic approach", University of Helsinki, 2014.

3. "Gauge fields out of equilibrium – a holographic approach", University of Utrecht, 2014.

4. "Gauge fields out of equilibrium – a holographic approach", National Seminar Theoretical High Energy Physics organized by the Dutch Research School of Theoretical Physics, NIKHEF, Amsterdam, 2013.

5. "Gauge fields out of equilibrium – a holographic approach", Konwersatorium im. Leopolda Infelda, University of Warsaw, 2013.

5.2.2 Selected seminars

1. "Entanglement, Holography and Causal Diamonds", Stanford University, Palo Alto, 2016.

2. "Entanglement, Holography and Causal Diamonds", UC Berkeley, 2016.

3. "Hydrodynamics at large gradients", McGill University, Montreal, 2015.

4. "Relativistic hydrodynamics and beyond", Ohio State University, Columbus, 2015.

5. "Hydrodynamics beyond the gradient expansion: resurgence and resummation", Kent State University, Kent, 2015.

6. "Hydrodynamics beyond the gradient expansion: resurgence and resummation", MIT, Cambridge, 2015.

7. "Hydrodynamics beyond the gradient expansion: resurgence and resummation", Brookhaven National Laboratory, Upton, 2015.

8. "Hydrodynamics beyond the gradient expansion: resurgence and resummation", 2 seminars: YITP and Nuclear Theory, State University of New York, Stony Brook, 2015.

9. "Non-dissipative hydrodynamics from linearized gravity in AdS", Durham University, 2013.

10. “New lessons about hydrodynamics from gravity”, Perimeter Institute for Theoretical Physics, Waterloo, 2013.
11. “A hole-ographic spacetime”, Yukawa Institute for Theoretical Physics, Kyoto, 2013.
12. “Holographic excursions beyond hydrodynamics”, CERN, Geneva, 2013.
13. “Holographic excursions beyond hydrodynamics”, Centre de Physique Theorique Saclay, Paris, 2013.
14. “Recent lessons about hydrodynamics from holography & Towards holographic heavy ion collisions”, HKUST Institute for Advanced Studies, Hong Kong, 2013.
15. “Two interesting lessons about hydrodynamics from holography & Condensed matter physics of holographic QCD: a quest for the quarkyonic phase”, University of Leiden, 2013.
16. “On the applicability of linearized Einstein’s equations in the studies of holographic isotropization”, VUB Brussels, 2012.
17. “Holographic thermalization – an update”, CP3-Origins, Odense, 2012.
18. “Strong coupling isotropization of non-Abelian plasmas simplified”, Niels Bohr Institute, Copenhagen, 2012.
19. “Cold nuclear matter in $N_f = 1$ holographic QCD”, University of Warsaw, 2011.
20. “The characteristics of thermalization of boost-invariant plasma from holography”, University of Crete, Heraklion, 2011.
21. “The characteristics of thermalization of boost-invariant plasma from holography”, University of Barcelona, 2011.
22. “The characteristics of thermalization of boost-invariant plasma from holography”, Niels Bohr Institute, Copenhagen, 2011
23. “Gauge-gravity duality and near-equilibrium entropy production”, University of Warsaw, 2011
24. “Black brane entropy and hydrodynamics”, University of Utrecht, 2010.
25. “Applied string theory”, University of Wroclaw, 2010.
26. “Transport properties of holographic conformal field theories: beyond $\eta/s = 1/(4\pi)$”, Jagiellonian University, 2010.
27. “AdS/CFT, real-time dynamics at strong coupling and black holes”, University of Amsterdam, 2010.
28. “AdS/CFT, real-time dynamics at strong coupling and black holes”, Imperial College, London, 2010.
29. “Black brane entropy and hydrodynamics: the boost-invariant case”, University of Oxford, 2010.

30. "Black brane entropy and hydrodynamics: the boost-invariant case", Princeton University, 2009.

31. "Boost-invariant early time dynamics from AdS/CFT correspondence", Brown University, Providence, 2009.

32. "Boost-invariant flow from string theory", Weizmann Institute of Science, Rehovot, 2009

33. "Boost-invariant hydrodynamics and AdS/CFT correspondence", State University of New York, Stony Brook, 2008.

34. "Hydrodynamika w AdS/CFT", Instytut Problemów Jądrowych im. A. Sołtana, Warsaw, 2008.

35. "From RHIC to non-equilibrium AdS/CFT", University of British Columbia, Vancouver, 2008.

5.2.3 Selected conference talks

1. "Hydrodynamics at large orders," Numerical methods for asymptotically AdS spaces, Technion, Haifa, 2016.

2. "Equilibration processes in strongly-coupled gauge theories: timescales and models of heavy-ion collisions," Mini workshop on collective effects in p+p, p+A and A+A, IFJ, Cracow, 2016.

3. "Entanglement Holography," AdS/CFT and Quantum Gravity, University of Montreal, 2015.

4. "Entanglement Holography", Quantum Information and Quantum Gravity II, Perimeter Institute for Theoretical Physics, Waterloo, 2015.

5. "Relativistic Hydrodynamics at Large Gradients", Equilibration Mechanisms in Weakly and Strongly Coupled Quantum Field Theory, Institute for Nuclear Theory, Seattle, 2015.

6. "Dynamical fields in de Sitter from CFT entanglement", Benasque Gravity Meeting, 2015.

7. "Generalized Müller-Israel-Stewart theories of relativistic hydrodynamics and non-equilibrium degrees of freedom in strongly coupled quark-gluon plasma", Conference on Non-equilibrium Phenomena in Condensed Matter and String Theory, ICTP, Trieste, 2014.

8. "The Approach to Equilibrium in Strongly Interacting Matter", RBRC Workshop on The Approach to Equilibrium in Strongly Interacting Matter, Brookhaven National Lab, 2014.

9. "Numerical solutions of AdS gravity: new lessons about dual equilibration processes at strong coupling", New frontiers in dynamical gravity, University of Cambridge, 2014.

10. "Holographic approach to non-equilibrium gauge theories: from new lessons about hydrodynamics to toy models of heavy ion collisions", Strongly interacting field theories, Friedrich Schiller University Jena, 2013.

11. "Towards a holographic realization of the quarkyonic phase", Holography & QCD - recent progress and challenges, Kavli IPMU, Kashiwa, 2013.

12. "Thermalization at strong coupling", invited review talk at Initial State 2013, Santiago de Compostella.

13. "Holographic hydrodynamization", Gauge/Gravity Duality Conference, Munich, 2013.

14. "From full stopping to transparency in a holographic model of heavy ion collisions", invited talk at the Non-perturbative QFT and String Theory parallel session, conference EPS-HEP 2013, Stockholm.

15. "Hydrodynamic gradient expansion in gauge theory plasmas", Benasque Gravity Meeting, 2013.

16. "New insights about hydrodynamics from holography", Workshop & Conference on Geometrical Aspects of Quantum States, ICTP, Trieste, 2013.

17. "What's new in applications of AdS/CFT?", stringtheory.pl/2013, Cracow, 2013.

18. "Holographic thermalization for expanding plasmas", invited review talk at the Holograv 2013 meeting, Helsinki.

19. "AdS/CFT out of equilibrium – review", invited review talk at Iberian Strings, Lisbon, 2013.

20. "Holographic view on thermalization of strongly coupled matter", Physics@FOM, Veldhoven, 2013.

21. "Holography for thermalization: transition to hydrodynamics and its features", planary talk at Strong and Electroweak Matter, Swansea, 2012.

22. "Strong coupling isotropization simplified", workshop Novel Numerical Methods for Strongly Coupled Quantum Field Theory and Quantum Gravity, KITP UCSB, Santa Barbara, 2012.

23. "The characteristics of thermalization of boost-invariant plasma from holography", invited talk at the Non-perturbative QFT and String Theory parallel session, conference EPS-HEP 2011, Grenoble.

24. "Holografia AdS/CFT – wprowadzenie", stringtheory.pl/2011, Warsaw, 2011

25. "Holographic approach to far-from-equilibrium dynamics of non-Abelian media", First String Meeting: A String Theoretic Approach to Cosmology and Quantum Matter, Groningen, 2010.

26. "Quasilocal notions of horizons in the fluid/gravity duality", ESI Programme on AdS Holography and the Quark-Gluon Plasma, Erwin Schrödinger Institute, Vienna, 2010.

27. "Gauge/gravity duality and dynamical properties of strongly coupled plasmas", Krakowsko-Warszawskie Warsztaty LHC, Cracow, 2010.

28. "Boost-invariant dynamics – near and far from equilibrium physics and AdS/CFT", Fluid/Gravity Correspondence, Munich, 2009.

29. “Boost-invariant flow from string theory – near and far from equilibrium physics and AdS/CFT”, invited talk at one of parallel sessions of the String Phenomenology 2009 conference, Warsaw;

30. “Boost-invariant hydrodynamics from AdS/CFT”, Young Enrage Meeting, London, 2009

31. “Hydrodynamics from AdS/CFT”, workshop Black Holes: A Landscape of Theoretical Physics Problems, CERN, 2008.

32. “Second order viscous hydrodynamics from AdS/CFT”, workshop From Strings to Things, Institute for Nuclear Theory, Seattle, 2008.

33. “Second order viscous hydrodynamics from AdS/CFT”, conference Rencontres de Moriond: QCD and High Energy Interactions, La Thuile, 2008.

34. “AdS/CFT and Second Order Viscous Hydrodynamics,” seminar during XLVII Cracow School of Theoretical Physics, Zakopane, 2007

35. “Viscous hydrodynamics relaxation time from AdS/CFT,” 9th Workshop on Non-Perturbative QCD, Paris, 2007.

5.3 Participation in research grants

1. Co-PI in the NCN Opus grant *Zastosowanie metod holograficznych do badania silnie sprzężonej plazmy teorii Yanga-Millsa* (2013-2015), PI: Michał Spaliński (NCNR).

2. PI in the NWO Veni grant *The gauge-gravity duality and its applications to the physics of strong interactions* supporting a 3-year position at University of Amsterdam and providing generous research funds, 218k€ (2012-2014).

3. Researcher in the FOM grant *A String Theoretic Approach to Cosmology and Quantum Matter* supporting a post-doctoral position at University of Amsterdam, PI: Jan de Boer (University of Amsterdam) (2010-2012).

4. Co-PI in the Ministry of Science and Higher Education grant *Badanie silnie sprzężonych kwantowych teorii pola w oparciu o zasadę holograficzną* (2010-2012), PI: Michał Spaliński (NCNR).

5. Researcher in the Ministry of Science and Higher Education grant *Teoria strun i korespondencja AdS/CFT* (2008-2011), PI: Romuald A. Janik (Jagiellonian University).

6. PI in the grant *Black holes in the AdS/CFT correspondence* from the Polish-British Young Scientists Program supporting a long-term stay at Durham University (host: Mukund Rangamani), 2008.

7. PI in the Ph.D. grant from Ministry of Science and Higher Education *Badanie wybranych zagadnień dynamiki nieperturbacyjnej teorii cechowania metodami korespondencji AdS/CFT* (2008-2010).

5.4 Selected service for the community

Reviewer for:

1. *National Science Centre (Poland)*
2. *Austrian Academy of Sciences*
3. *Durham University (Junior Research Fellowship Scheme)*
4. *Physical Review Letters*
5. *Physical Review D*
6. *Journal of High Energy Physics (JHEP)*
7. *European Physical Journal C*
8. *Classical and Quantum Gravity*
9. *International Journal of Modern Physics A*
10. *Acta Physica Polonica B.*

5.5 Selected leadership activities

1. Initiator and the main organizer of a series of conferences Quantum Gravity in Cracow: *Kwantowa Grawitacja w Krakowie*, 12-13 January 2008; *Quantum Gravity in Cracow*^{2}, 19-21 December 2008 and *Quantum Gravity in Cracow*^{3}, 24-25 April 2010.
2. One of the creators of the *stringtheory.pl* initiative.
3. Member of the organizing committee in the following annual meetings of Polish string theorists: conferences *stringtheory.pl/2011* (Warsaw, 15-17 April 2011) and *stringtheory.pl/2013* (Cracow, 05-07 April 2013) and international schools *stringtheory.pl/2012* (Wrocław, 13-15 April 2012) and *stringtheory.pl/2014* (Warsaw, 25-27 April 2014).
4. Member of the organizing committee of the *Amsterdam String Workshop 2012*, University of Amsterdam, the Netherlands, 02-13 July 2012.
5. Initiator and member of the organizing committee of the *Holographic thermalization* conference, Lorentz Center, the Netherlands, 08-12 October 2012.
6. Member of the organizing committee of a conference *Holography and QCD - Recent progress and challenges*, Kavli IPMU, University of Tokyo, Japan, 24-28 September 2013.
7. Member of the organizing committee of the *CERN-CKC TH Institute on Numerical Holography*, CERN, Switzerland, 08-18 December 2014.

5.6 Long-term visits

1. *University of Barcelona, Spain* (August and September 2014, 4.5 weeks), host: David Mateos.
2. *CERN, Switzerland* (August 2013, 2.5 weeks),host: Urs Wiedemann.
3. *Hong Kong Institute for Advanced Studies, PRC* (June 2013, 2 weeks), host: Gary Shiu.
4. *Niels Bohr Institute, Denmark* (August 2011 and August 2012, 2 weeks in total), host: Niels Obers.
5. *Oxford University, UK* (January 2010), host: Andrei Starinets.
6. *MIT, USA* (October - December 2009, 8 weeks), host: Hong Liu.
7. *Weizmann Institute of Science, Israel* (April - May 2009, 6 weeks), host: Ofer Aharony.
8. *Perimeter Institute for Theoretical Physics, Canada* (January-February 2008, July 2008, February 2009, 8 weeks in total), host: Alex Buchel.
9. *Durham University, UK* (October - November 2008 and January 2009, 8 weeks in total), host: Mukund Rangamani.
10. *Service de physique theorique du CEA/SACLAY, France* (November - December 2008, 4 weeks), host: Robi Peschanski.

6 Selected teaching and outreach activities

6.1 Graduate-level courses and lecture series

LVI Cracow School of Theoretical Physics, Zakopane

Holography, Thermalization and Heavy Ion Collisions (May 2016, 3 hours): Series of invited lectures on the topics of articles [1–10]. Lectures were attended by about 40 participants.

Summer School on String Theory and Holography, Instituto Superior Técnico in Lisbon

Holographic collisions (July 2014, 4 hours): Series of invited lectures on the articles [1–7]. Lectures were attended by about 80 participants including many Ph.D. students working on holography.

Helsinki University

Introduction to numerical holography (May 2014, 15 hours): Delivering an invited course (hosts: Keijo Kajantie and Alexi Vuorinen) on [1,2] and general methodology of numerical holography.

Universität Regensburg

Introduction to numerical holography (September 2013, 6 hours): Delivering an invited (host: Andreas Schäffer) crash course on how to write a numerical relativity code performing simulations in [1, 2].

Hong Kong University of Science and Technology

Crash course on applied AdS/CFT (June 2013, 9 hours): Basic introduction to applied holography.

Paris-Amsterdam-Brussels Ph.D. School

Crash course on applied AdS/CFT (December 2012, 6 hours): Part of a larger course on applied holography covering basic solutions of Einstein's equations with negative cosmological constant and their properties.

Practical course on applications of AdS/CFT correspondence (December 2011, 15 hours): Introduction to applied holography delivered for the first year graduate students from several leading string theory groups in Europe.

University of Warsaw

Crash course on applied AdS/CFT (December 2011, 9 hours): 3 hours of an overview lecture aimed for advanced students and researchers in theoretical physics introducing the concept of holography. The lecture was followed by 2 exercise sessions for students, each lasting 3 hours.

6.2 Undergraduate classes

University of Amsterdam

String theory (April - May 2012, 15 hours): Conducting half of the exercise classes to the string theory lecture by Kostas Skenderis and being a replacement teacher for a 2-hour lecture on D-branes and T-duality.

Jagiellonian University

Warsztaty metod fizyki teoretycznej (2007-2008): Innovative workshops in theoretical physics for undergraduates created in collaboration with Jan Kaczmarczyk. Materials are available online[24]. In 2011 similar activities were conducted at University of Warsaw[25].

6.3 Work with talented youth

Physics Preschool 2016 organized by Jagiellonian University in Zakopane

"Najlepsze szeregi są rozbieżne – fizyka z Wolfram Programming Lab": 1.5 hours of workshop for talented high school students presenting asymptotic expansions on the example of article [10].

[24] https://sites.google.com/site/kaczek/warsztaty
[25] http://skfiz.fuw.edu.pl/problem-solving

August Witkowski 5th High School and Jagiellonian University

Kółko fizyczne: (2006-2009) Initiating, together with Jan Kaczmarczyk, extracurricular activities preparing high school students from Cracow to compete in the Polish Physics Olympiad. In total, several hundreds of hours of classes between 2006 and 2009. Activities are continued to date by undergraduate and graduate students of the Department of Physics, Astronomy and Applied Computer Science of Jagiellonian University[26].

Polish Children's Fund

Research workshops at the University of Warsaw Physics Department (2009): Preparing and executing, together with Michał Spaliński, 90 hours of a workshop on the elements of string theory for 4 gifted high school students.

Spotkanie na Kresach, Reszel (2008): Workshops for gifted high school students on basic mathematical methods used in the theoretical physics.

Science Camp, Świder (2008): 4-hour long workshop on the elements of theoretical high energy physics for gifted high school students.

6.4 Outreach activities

Outreach initiatives

1. *blog Świat: jak to działa? (2008-2009)*: Contributions to an outreach blog on physics and related subjects supported by Onet.pl and Tygodnik Powszechny. These activities had led to an honourable mention in the science outreach competition Popularyzator Nauki 2008. The blog is still available online[27].

2. *Uniwersytet dla szkół (2007-2008)*: The initiator and the main organizer of a series of weekly lectures on physics, biology, biotechnology and computer science for high school students from Cracow. The initiative was supported by Jagiellonian University and the actual lectures were given at the August Witkowski 5th High School. In total, more than 20 lectures.

Outreach talks

1. "The power of fantasy", symposium Viva Fysica, University of Amsterdam, February 2013.

2. "O fizyce jako o sztuce opisu rzeczywistości", Fine Arts Academy in Cracow, January 2013.

3. "Dlaczego fizyka teoretyczna jest super?", Astronomical Observatory in Poland, January 2009.

[26] http://www.fais.uj.edu.pl/dla-szkol/warsztaty-z-fizyki/szkoly-ponadgimnazjalne
[27] http://swiat-jaktodziala.blog.onet.pl/

Outreach articles

1. Michał P. Heller, "Najdoskonalszy płyn w przyrodzie", Forum Akademickie nr 02/2009.
2. Michał P. Heller, "Fizyka de Lux", Tygodnik Powszechny Nr 48 (3151), 29 November 2009.
3. Michał P. Heller, "Wypatrywanie Ziem",Tygodnik Powszechny Nr 10 (3113), 8 March 2009.
4. Michał P. Heller, "Ile jest Wszechświatów?", Tygodnik Powszechny Nr 9 (3112), 1 March 2009.
5. Michał P. Heller, "Czas przewrotów", Tygodnik Powszechny Nr 38 (3089), 21 September 2008.

Articles on physics education

1. Michał P. Heller, Jan Kaczmarczyk, "Nowa forma zajęć z fizyki", Postępy Fizyki Tom 59, rocznik 2008.
2. Michał P. Heller, Jan Kaczmarczyk, "O kółku fizycznym", Fizyka w Szkole 04, 2007.

7 Students

1. *Natalia Pinzani-Fokeeva* (University of Amsterdam): co-advisor (advisor: Jan de Boer, second co-advisor: Marika Taylor) of the Ph.D. thesis "Holography in the interior of space" defended in September 2015. Natalia Pinzani-Fokeeva is currently a post-doc at Amos Yarom's group at Technion in Israel (2015-2018).
2. *Jakub Jankowski* (University of Wrocław): co-advisor (advisor: Michał Spaliński) of the Ph.D. thesis "A holographic perspective on strongly-interacting matter properties at finite temperatures and densities" defended in June 2013. The dissertation was based on 2 research articles including [16]. Jakub Jankowski is currently a post-doc at Romuald Janik's group at Jagiellonian University, where he holds an individual fellowship NCN Fuga *Holografia: zastosowania i podstawy* (2013-2016).
3. *Jewel Kumar Ghosh* (Perimeter Institute for Theoretical Physics): daily supervisor of the M.Sc. thesis "Entanglement and holography" defended in June 2016. Starting in the Fall 2016, Jewel Kumar Ghosh will pursue Ph.D. studies in string theory under the supervision of Elias Kiritsis at Paris Diderot University in France.
4. *Miquel Triana* (University of Amsterdam): daily supervisor of the M.Sc. thesis "Strongly coupled anisotropic plasmas in the gauge-gravity duality" defended in July 2013. The project resulted in the scientific publication [2]. Miquel Triana is currently a Ph.D. student of David Mateos supported by the ERC Starting Grant *Holography for the LHC era* at University of Barcelona (2013-2017).
5. *Stanislav Fort* (Perimeter Institute for Theoretical Physics): co-supervisor of the summer research project for advanced undergraduate sutidents on the stability of AdS spacetime lasting 8 weeks in 2015. Stanislav Fort is about to finish Mathematical Tripos at

University of Cambridge and in the Fall 2016 he joins Stanford University as a Ph.D. student.

During the period 2010-2014, while not being an official co-advisor, working with the following Ph.D. students:

3. *Przemysław Witaszczyk* (Jagiellonian University): Ph.D. student of Romuald Janik, co-author of [3–5].

4. *Wilke van der Schee* (University of Utrecht): former Ph.D. student of Gleb Arutyunov, co-author of [1, 2, 6, 7]. Since October 2014, a post-doc at MIT.

8 Career in physics

8.1 Pre-doctoral level

In 2003 I started my undergraduate education by following elite Studies in Mathematics and Natural Sciences at Jagiellonian University, where I specialized in theoretical physics. During that time, I was a laureate of the Undergraduate Scholarship for Outstanding Students from the Polish Ministry of Science and Higher Education (2004-2007). In 2007 I defended my M.Sc. thesis on theoretical high energy physics written under the supervision of Romuald A. Janik. My thesis presented the first calculation of hydrodynamic transport properties at second order in gradients in strongly-coupled quark-gluon plasma. This result was recognized with the 2$^{\text{nd}}$ prize in a nationwide competition for the best M.Sc. thesis in physics organized by the Polish Physical Society. The scientific article based on this calculation, [25], is one of my most important and most cited publications. In particular, this result led to a realization, by other authors [39], that widely-used MIS theory is incomplete and played an important part in the discovery of the fluid-gravity duality [98].

Between 2007 and 2010 I was a Ph.D. student at the Department of Physics, Astronomy and Applied Computer Science of Jagiellonian University, where I continued my collaboration with Romuald A. Janik. The earliest results delivered during that period, [22, 24], focused on the crucial question of the consistency of gravitational solutions dual to hydrodynamics and led to a very first publication in Physical Review Letters, [22]. This article pointed out that real curvature singularities (i.e. regions of spacetime with unbounded curvature not shielded by an event horizon) need to be distinguished from the singularities of the approximate expansion that is devised to describe spacetime geometry. This results is certainly one of my important contributions to the field. Another important result delivered during my graduate education was solving in [21] the early time dynamics of strongly-coupled Yang-Mills plasma, see also section 4.2.2, which was one of the very first results on far-from-equilibrium physics of strongly-coupled quantum field theories in the literature. Apart from these works, as a Ph.D. student I also published on the higher curvature gravity [20], black hole horizons [19] and integrability [23].

One of the most important experiences as a Ph.D. student was organizing the series of scientific meetings called Quantum Gravity in Cracow, which gathered Poland-based researchers working on quantizing gravity and related subjects. These meetings led to the consolidation of the Polish string theory community. This resulted in 2011 in forming the initiative *stringtheory.pl*, which remains very active with 3 conferences (*stringtheory.pl/2011* in Warsaw, *2013* in Cracow and *2015* in Wrocław) and 3 international schools organized to date (*stringtheory.pl/2012* in Wrocław, *2014* in Warsaw and *2016* in Zakopane). The latter meeting was also advertised as the 56th Cracow School of Theoretical Physics and the Holograv network workshop.

In the last 2 years of my Ph.D. studies I was a laureate of the START Fellowship from the Foundation for Polish Science for the best scientists < 30. In 2010 I started my employment at the National Centre for Nuclear Research, where I am currently holding a post of an assistant professor.

8.2 Post-doctoral level

My first post-doctoral position was at the University of Amsterdam, which hosts one of the biggest string theory group in the world (2010-2014). In 2012 I was awarded prestigious Veni Fellowship from the Netherlands Organisation for Scientific Research (NWO) aimed for the top 15% of researchers at the beginning of their careers. In 2013 I was nominated to the first edition of the Poles with Verve context and finished second in sciences. In 2014 I started my second post-doc at the Perimeter Institute for Theoretical Physics, which is the biggest research centre in this discipline worldwide.

The focal aspect of my research are black holes, both from the perspective of applications of holography to QCD as well as more fundamental questions in classical and quantum gravity. The first research thread underlies both the series as well as my article [16] in which I discussed with my co-authors the possibility of existence of the so-called quarkyonic phase [137] in the holographic QCD model of Sakai and Sugimoto [83, 84]. The quarkyonic phase is a hypothetic phase of the low-temperature QCD at intermediate chemical potential in which quark confinement coincides with the restoration of the chiral symmetry[28]. Originally, the quarkyonic phase was postulated was postulated in the context of the large-N_c QCD. The existence of this phase for the real world QCD, i.e. when $N_c = 3$, would have an enormous influence on our understanding of the phase diagram of strong interactions, might lead to observable consequences in heavy-ion collisions, as well as come with astrophysical implications. The main motivation for searching the quarkyonic phase in holographic QCD models was for us, on the one hand, the absence of the relevant ab initio results within the QCD itself and, on the other, the fact that the arguments for it were devised by using the large-N_c limit which is automatically implied by the model we considered. The results from [16] became a major part of Jakub Jankowski's Ph.D. thesis, which I co-advised (see also section 7).

Regarding the black hole physics, the articles [17, 18] building upon my earlier contribution in [19] considered the fundamental properties of black hole horizons in the context of their hydrodynamic interpretation. The paper [18] demonstrated using holography that the existence of a hydrodynamic entropy current is completely equivalent to a question of the applicability of the area theorem to a rather general family of hypersurfaces representing entropy in a dual gravitational picture. In the follow-up work [17] I considered with my collaborators the so-called apparent horizon (or more precisely the future outer trapping horizon [139]) for long-wavelength perturbations of black branes, which is an example of an aforementioned surfaced satisfying the appropriate area theorem. Apparent horizons, as opposed to event horizons, are causal, i.e. they evolve only when absorbing gravitational waves or matter [139]. From this perspective, they are conceptually attractive definitions of black objects boundaries both in numerical simulations of black hole mergers or distorted black hole spacetimes, as well as possible entropy carriers of these objects. My work [17] is one of the most advanced analytic examples of an apparent horizon calculation.

[28] This is only an approximate statement since the original scenario outlined in [137] requires modifications due to instabilities breaking both the translational invariance, as well as the restored chiral symmetry [138].

The results of another paper of mine, [14], also fit very well into the question of black objects boundaries as it considered the so-called membrane paradigm [140, 141]. The latter idea is a simplified approach to black hole dynamics, which, instead of the event horizon, considers a fiducial membrane located right above it (the so-called stretched horizon). The membrane has very simple physical properties such as constant conductivity. In [14] I demonstrated with my co-authors that this simplistic approach fails in a few situations and postulated that the membrane has to be thought of, in general, as synonymous to the event horizon. This research continued with [13], which demonstrates how to formulate a variational principle for the relativistic hydrodynamics of perfect fluids using holography.

When it comes to the microscopic interpretation of dynamical spacetime in the language of non-gravitational holographic degrees of freedom, my paper [15] put forward an equation relating within the AdS_3/CFT_2 correspondence lengths of a certain class of closed curves on a constant time slice with a linear combination of entanglement entropies of certain intervals in a dual conformal field theory. This work influenced many further developments including generalizations to a higher number of dimensions, [142, 143], as well as to time-dependent setups, [144]. The results presented in [15] provide in particular further indications that selecting volumes in diffeomorphism-invariant theories does not correspond to a factorization of the microscopic Hilbert space, which has a bearing on the Hawking's information paradox [145] and its latest incarnation in the form the firewalls discussion [146, 147].

The most interesting paper not touching upon the topic of non-equilibrium physics of quantum field theories that I delivered while at the Perimeter Institute is certainly [12]. In this article, motivated by the earlier results from [148], I discuss with co-authors a surprising relation between the geometry of de Sitter spacetimes and entanglement entropy in unitary conformal field theories. These results already led to several generalization, including [149–151], and are the main motivation for my present research endeavours.

9 Selected prizes and achievements

1. 2$^{\text{nd}}$ prize in sciences in the first edition of the *Poles with Verve* contest, which targets the most talented and creative Poles and promotes their achievements, 2013.

2. Award of the Director of the National Centre for Nuclear Research in Poland for the series of work on black hole horizons [17–19], 2012.

3. START Fellowship from the Foundation for Polish Science for the best scientists < 30, 2009 and 2010.

4. Adam Krzyżanowski Fund Scholarship for the best Ph.D. student, Jagiellonian University, Poland, 2009.

5. 1$^{\text{st}}$ place in *Skomplikowane i proste* contest for a popular article on own research titled "The most perfect fluid in the universe", Poland, 2009.

6. Florentyna Kogutowska Fund Scholarship, Jagiellonian University, Poland, 2008,

7. 2$^{\text{nd}}$ prize in a nationwide competition for the best M.Sc. thesis in physics, Polish Physical Society, 2007.

8. Undergraduate scholarship for outstanding students from the Polish Ministry of Science and Higher Education, 2004-2007.

9. Participation in the educational program for exceptionally gifted children organized by the Polish Children's Fund, 2002-2003.

10. Finalist (2002) and laureate (13th place in 2003) of the Polish Physics Olympiad.

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