Beyond relativistic Lagrangian perturbation theory: an exact-solution controlled model for structure formation

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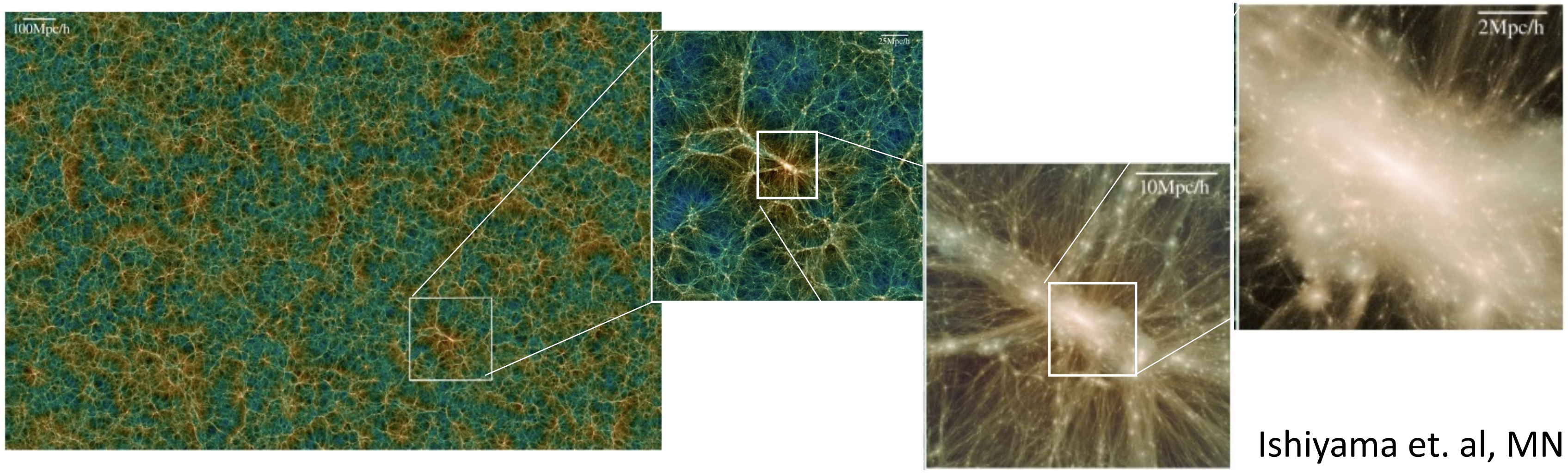
Introduction

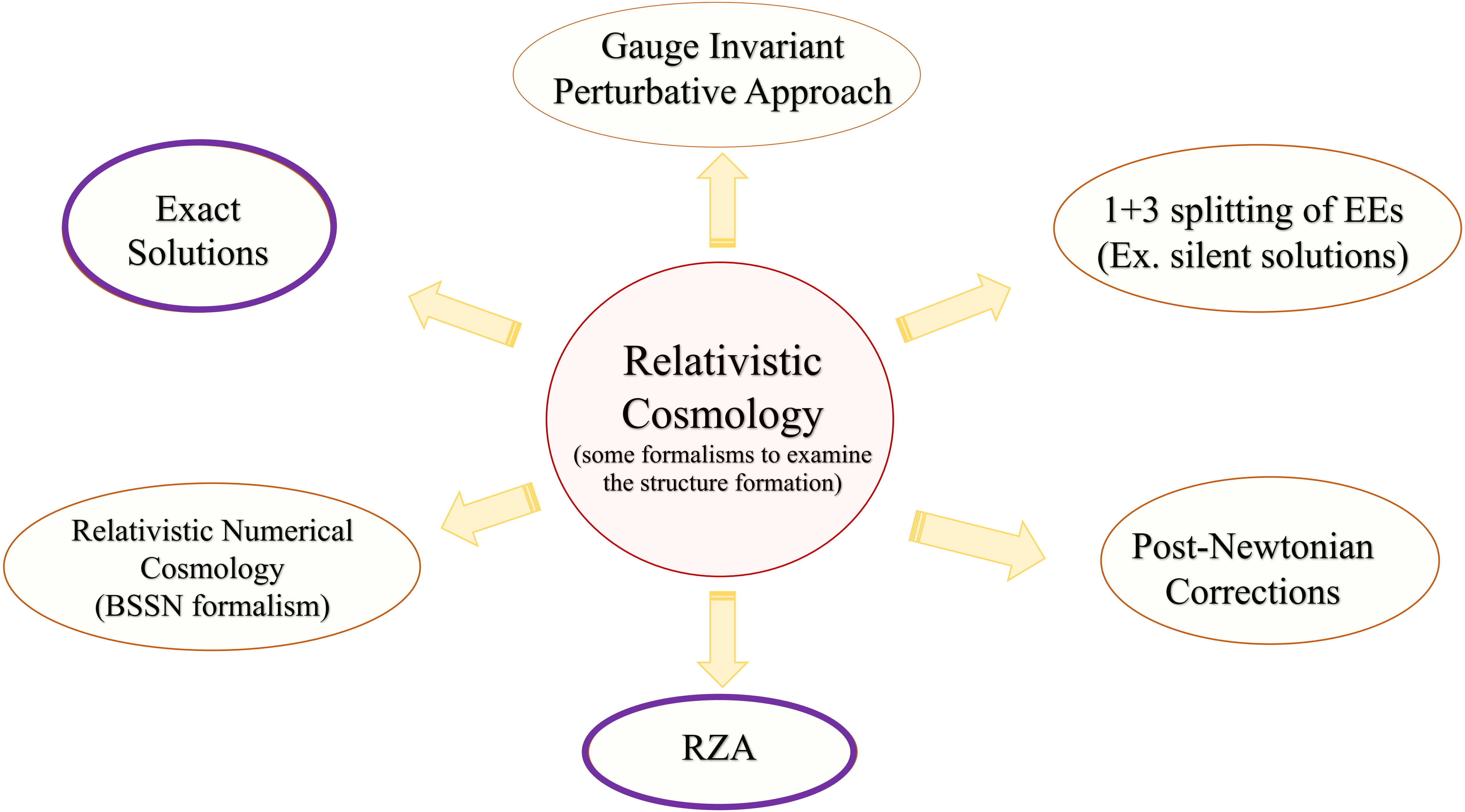
- The current era of "Precision Cosmology" has produced a large amount of high-quality observational data at all astrophysical and cosmic scales whose theoretical interpretation requires robust modeling of self-gravitating systems.

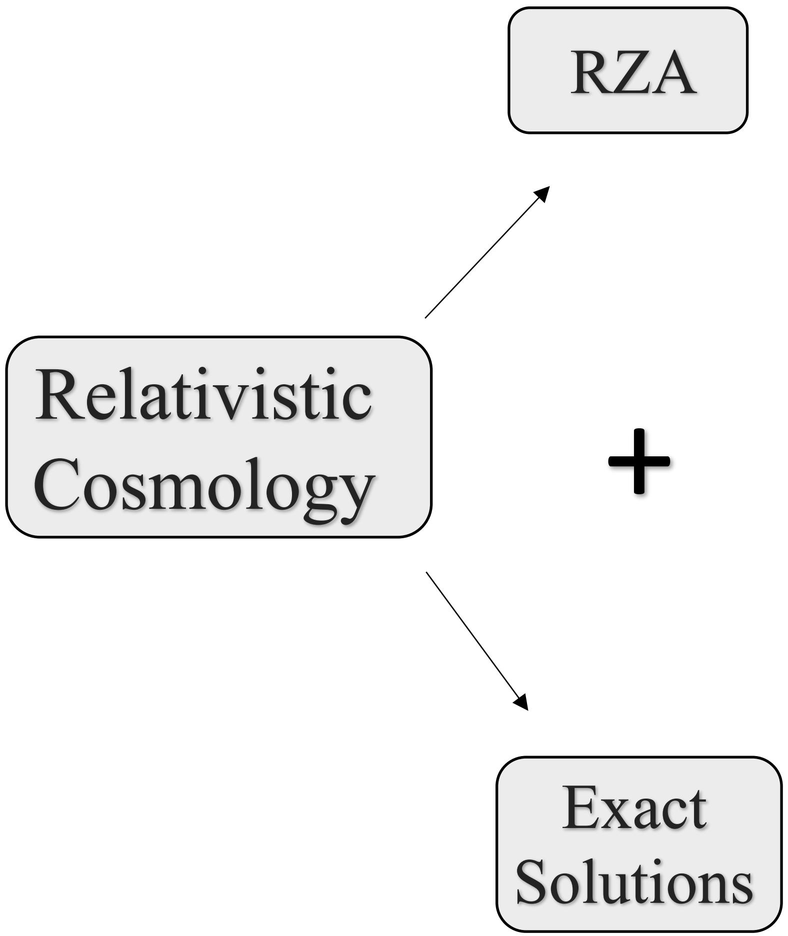
- The analysis of these observations by means of analytic or numerical solutions of Einstein's equations has been less favored due to their non-linear complexity.

Self-gravitational systems at galactic and galactic cluster scales →	Newtonian gravity
Large cosmic scale dynamics →	linear perturbations on a ΛCDM background



Ishiyama et. al, MNRAS, 2021


Gauge Invariant
Perturbative Approach
Exact
Solutions
1+3 splitting of EEs
(Ex. silent solutions)
Relativistic
Cosmology
(some formalisms to examine
the structure formation)
Relativistic Numerical
Cosmology
(BSSN formalism)
Post-Newtonian
Corrections
RZA



1. <u>*Beyond relativistic Lagrangian perturbation theory: an exact-solution controlled model for structure formation.*</u> **Ismael Delgado Gaspar,** Thomas Buchert, **Jan Jakub Ostrowski.** (2022) Accepted in ***Physical Review D***

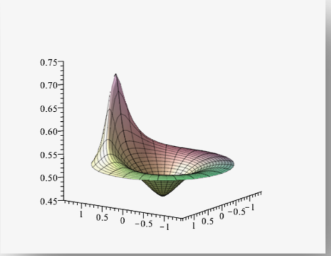
2. *On the maximum volume of collapsing structures.* ***Jan Jakub Ostrowski*** *and* ***Ismael Delgado Gaspar.*** (2022) **JCAP**

3. *On general-relativistic Lagrangian perturbation theory and its non-perturbative generalization.* Thomas Buchert, ***Ismael Delgado Gaspar, Jan Jakub Ostrowski***. (2022) **Universe**

Exact solutions: Sze.-Szafron

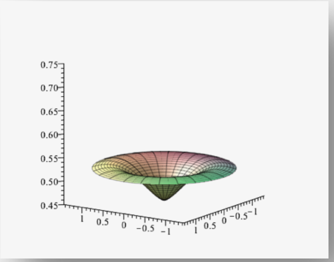
- Dust + Cosmological constant.
- No symmetries (but quasi-symmetries).
- Dynamics: Friedmann-like eq. and another eq. formally equivalent to the one leading to the growing and decaying modes in CPT.

Sze-Sza: dipolar dust distribution



It contains

LTB: SS dust distribution



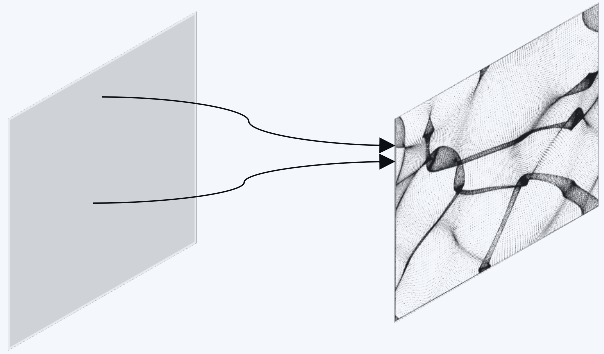
FLRW

Relativistic Zel'dovich Apprximation

Newtonian case:
Lagrangian deformation

$$dx^a = f^a{}_{|i} dX^i$$

$$\mathbf{x} = a(t)\left[\mathbf{X} + b(t)\mathbf{s}(\mathbf{X})\right] := \mathbf{f}(\mathbf{X}, t) \longrightarrow$$ Trajectory



Relativistic case case:

$$dx^a = f^a{}_{|i} dX^i \longrightarrow \left[\begin{array}{l} \boldsymbol{\eta}^a = \eta^a{}_i dX^i \\ g_{ij} = G_{ab}\eta^a{}_i \eta^b{}_j \end{array}\right.$$

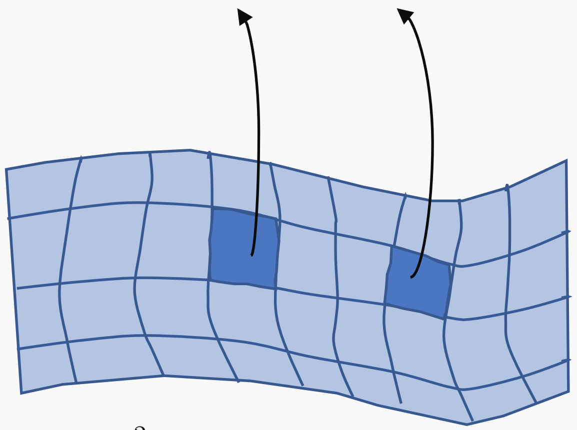
$$\boldsymbol{\eta}^a = \eta^a{}_i \mathbf{d}X^i = a(t)\left(\delta^a{}_i + P^a{}_i\right)\mathbf{d}X^i$$

We insert this ansatz into the Einstein equations and linearize the sol. in the deformation field.

Density : $\varrho = \varrho_i J^{-1}\ , \quad \text{with} \quad J = \sqrt{g}/\sqrt{G}$

GRZA

- GRZA does not assume the existence of a global FLRW model, but (inspired by the Szekeres solutions) considers an inhomogenous Friedmann-like reference model.



$$\left(\frac{\dot{A}}{A}\right)^2 = -\frac{\hat{k}}{A^2} + \frac{8\pi}{3}\frac{\hat{\varrho}_b}{A^3} + \frac{\Lambda}{3}$$

$$\boldsymbol{\eta}^a = \eta^a{}_i \mathbf{d}X^i = A\left(\delta^a_i + \hat{P}^a_i\right)\mathbf{d}X^i$$

$$A = A(t, \boldsymbol{r}) \ , \quad \hat{P}^a_i = \hat{P}^a_i(t, \boldsymbol{r})$$

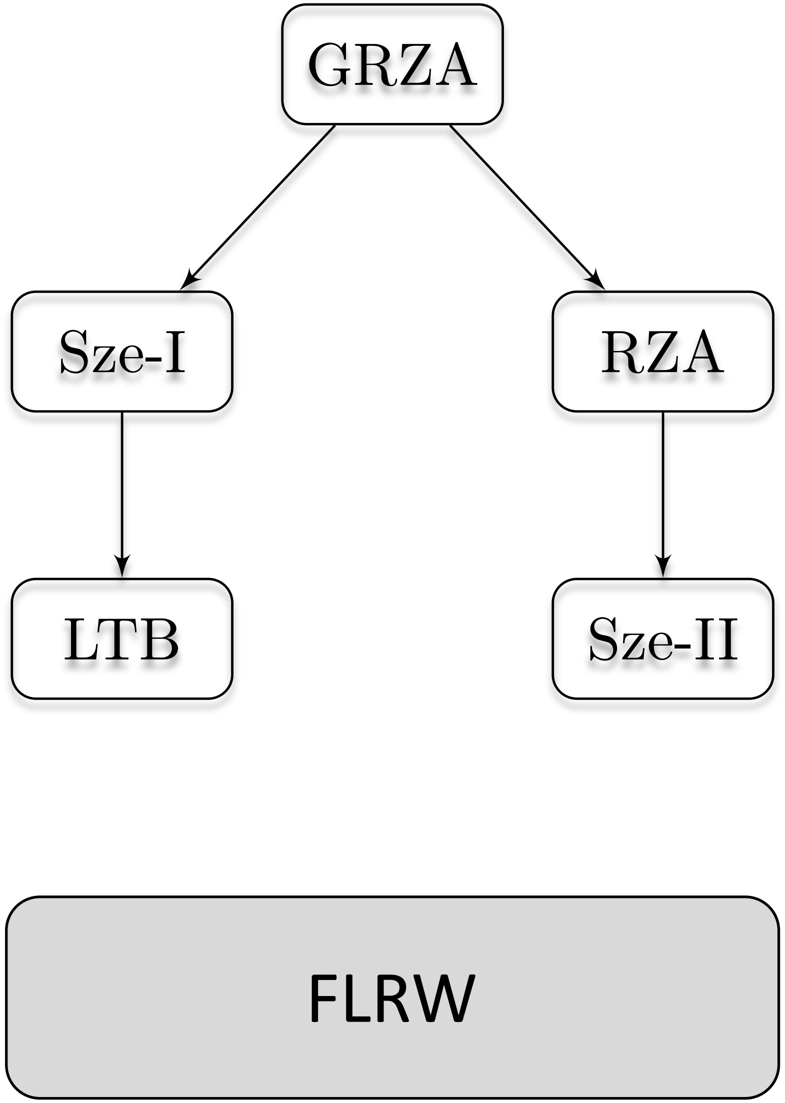
Approach:

(i) We insert the coframes into the Einstein equations

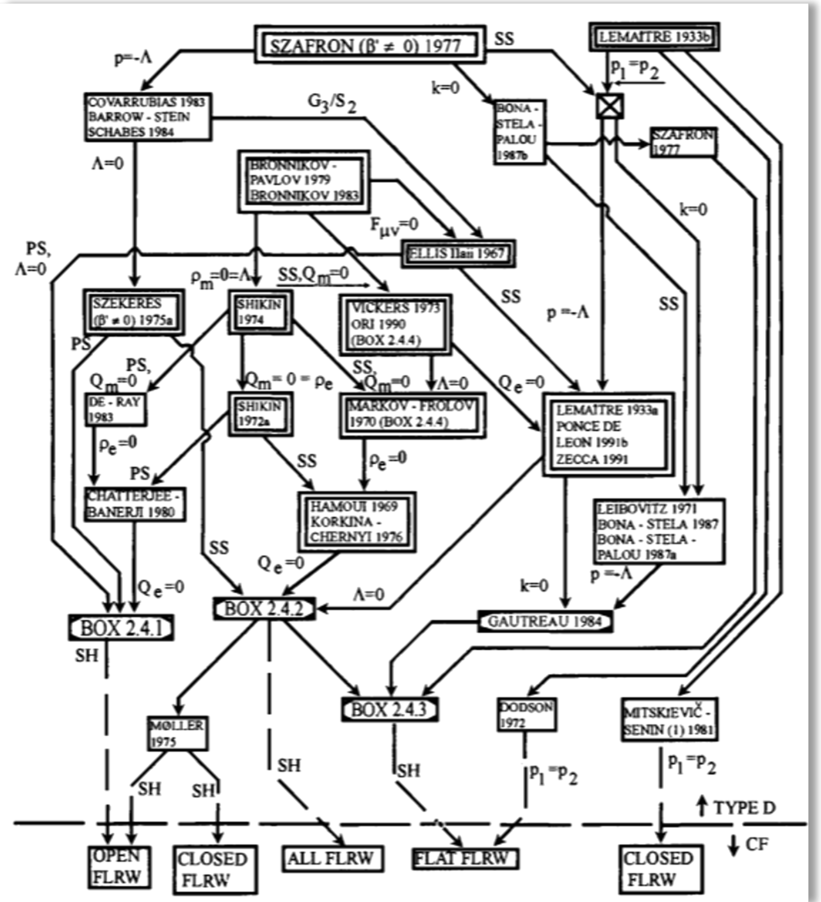
$$G_{\mu\nu}(g_{\mu\nu}) + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \qquad \begin{cases} g_{ij} = \mathcal{G}_{ab}\eta^a{}_i\eta^b{}_j \\ \mathcal{G}_{ab} = g_{ij}(t_{ini}, \mathbf{r}) \end{cases}$$

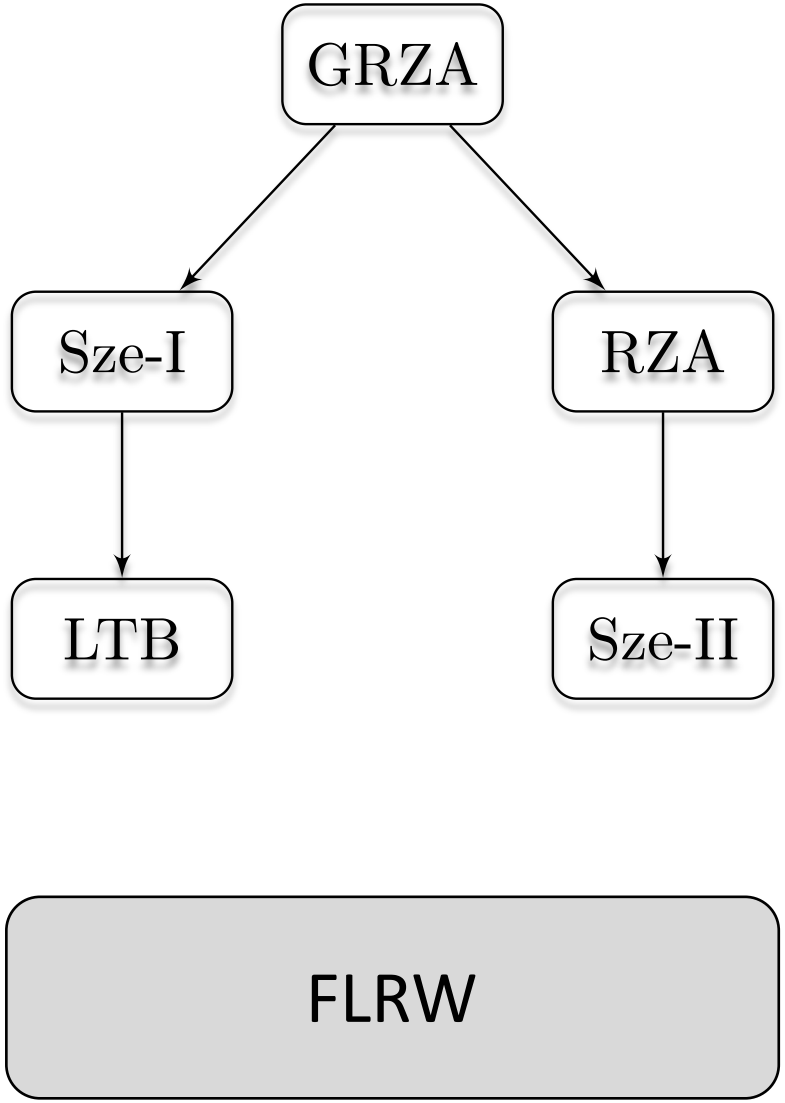
and linearize in the deformation.

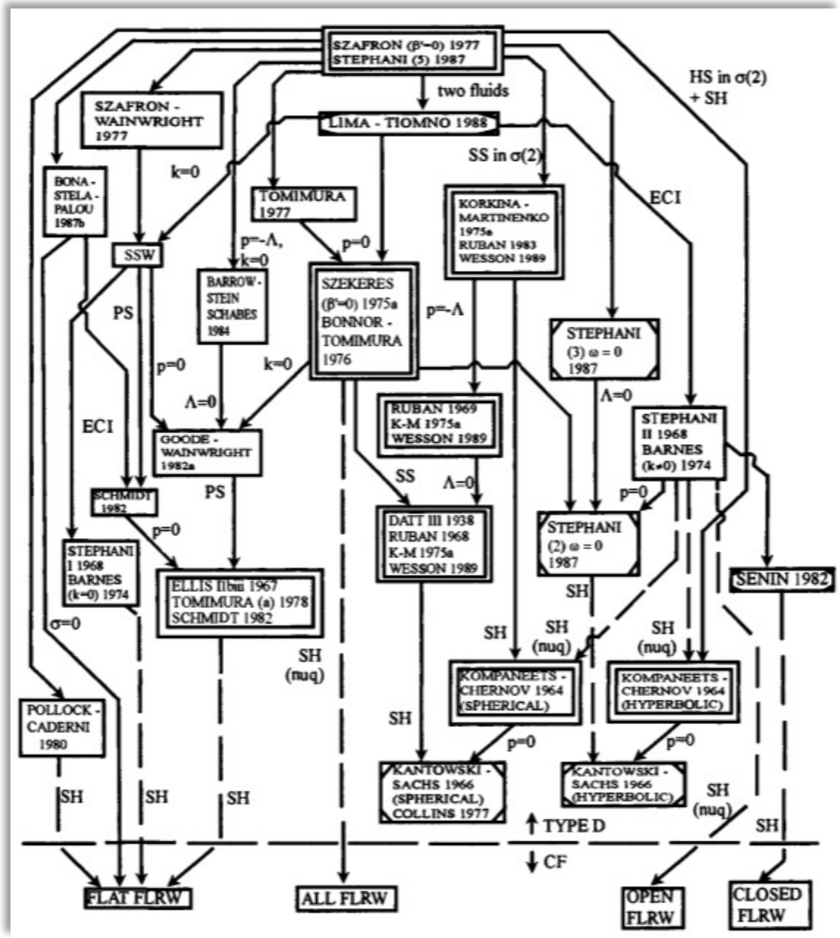
(i) Then, any quantity is evaluateted as a nonlinear functional of the deformation field.


GRZA
Sze-I
RZA
LTB
Sze-II
FLRW

A. Krasinski, Inhomogeneous Cosmological Models. Cambridge University Press (1997).
Fig. 2.1 and 2.4





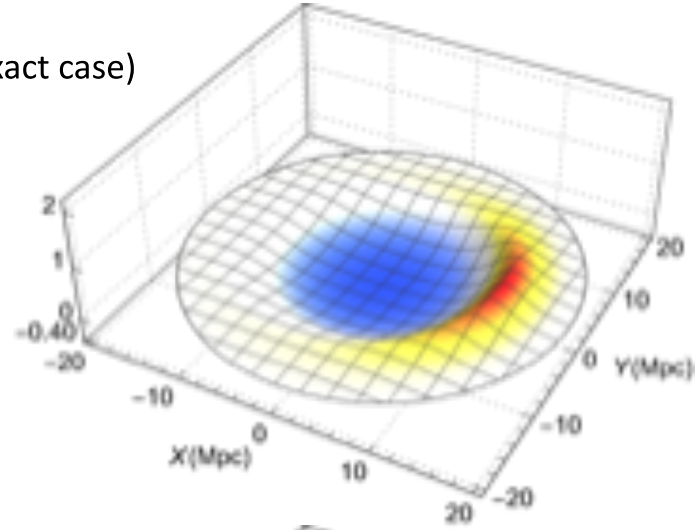


Numerical Example: we explore the numerical solutions of a family of locally 1-d models, containing Szekeres as a particular case.

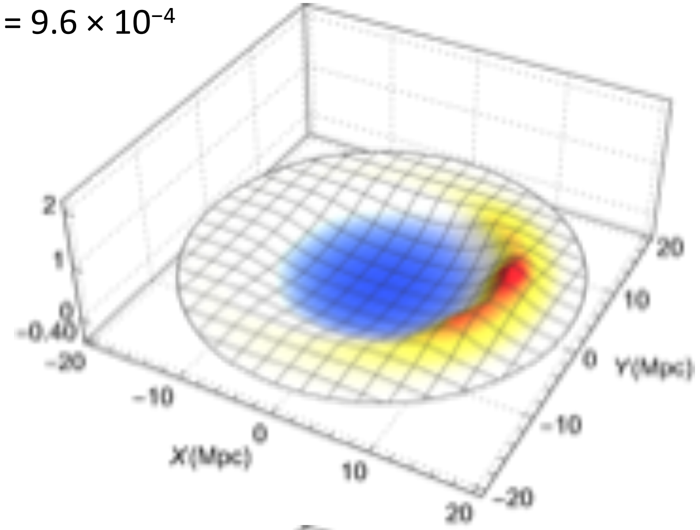
$$G_{ij} = \mathrm{Diag}\left[\left(\mathcal{A} - \hat{\mathcal{F}}_{ini}\right)^2 \left(\frac{\mathcal{W}}{\chi}\right)^2, \left(\frac{e^{\nu}}{\chi}\right)^2, \left(\frac{e^{\nu}}{\chi}\right)^2\right]$$

$$P^a_i = \frac{\hat{\mathcal{F}} - \hat{\mathcal{F}}_{\rm ini}}{\mathcal{A} - \hat{\mathcal{F}}_{\rm ini}} \delta^a_3 \, \delta^3_i \qquad \hat{\mathcal{F}} = \mathcal{F} + \delta\mathcal{F} \ , \quad \delta\mathcal{F} = \alpha\left(\delta\beta_+ f_+ + \delta\beta_- f_-\right)$$

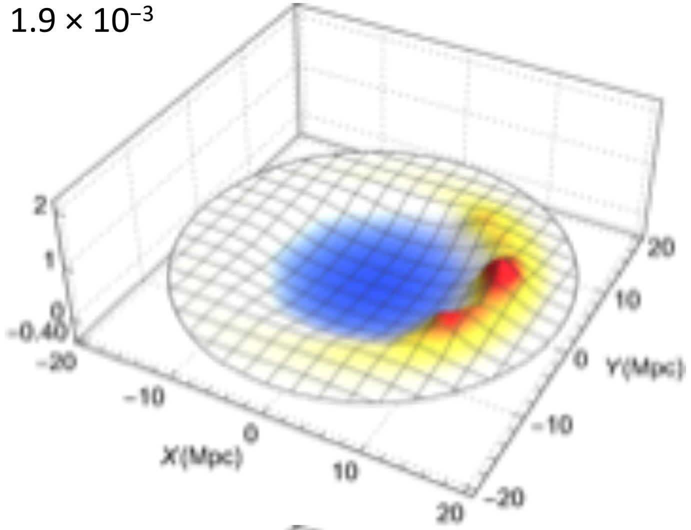
α = 0
(Sze. exact case)



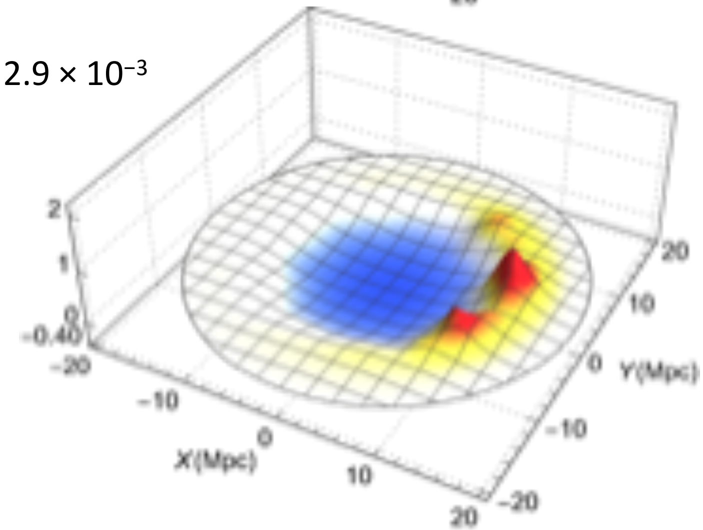
$\alpha = 9.6 \times 10^{-4}$



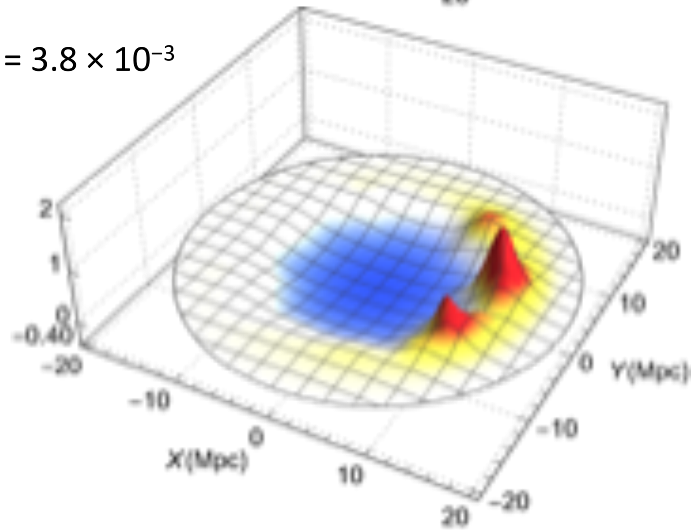
$\alpha = 1.9 \times 10^{-3}$



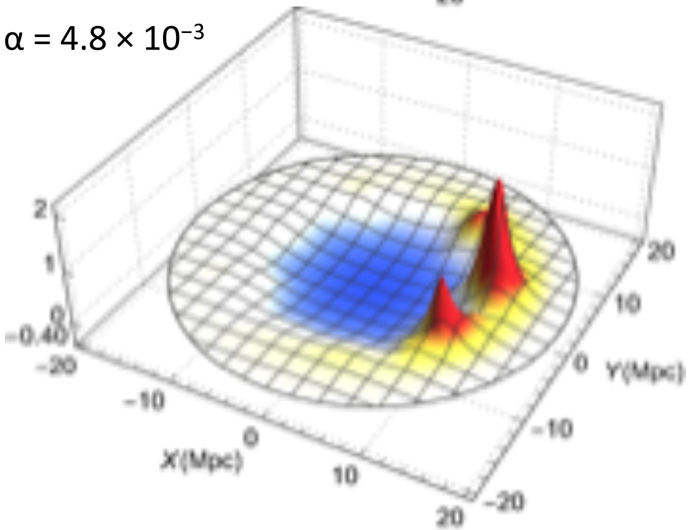
$\alpha = 2.9 \times 10^{-3}$



$\alpha = 3.8 \times 10^{-3}$



$\alpha = 4.8 \times 10^{-3}$

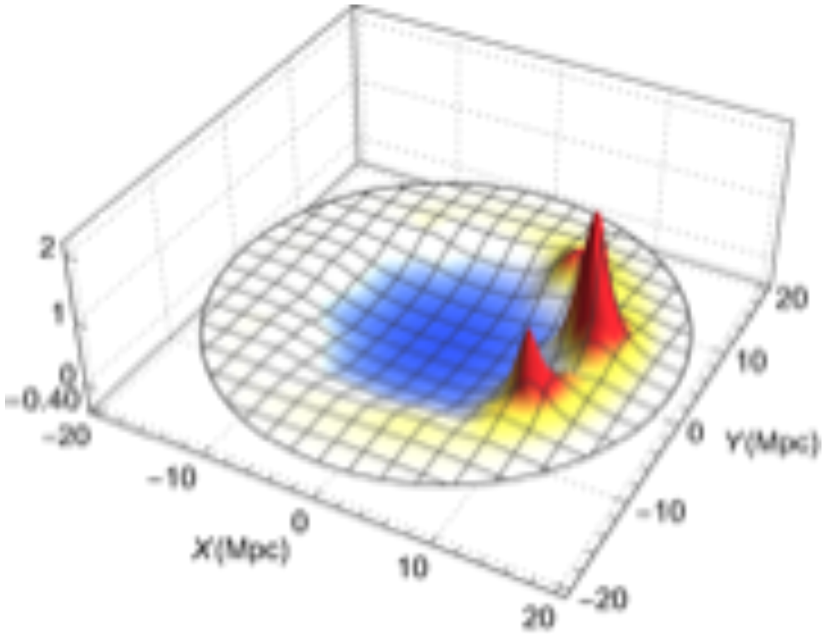


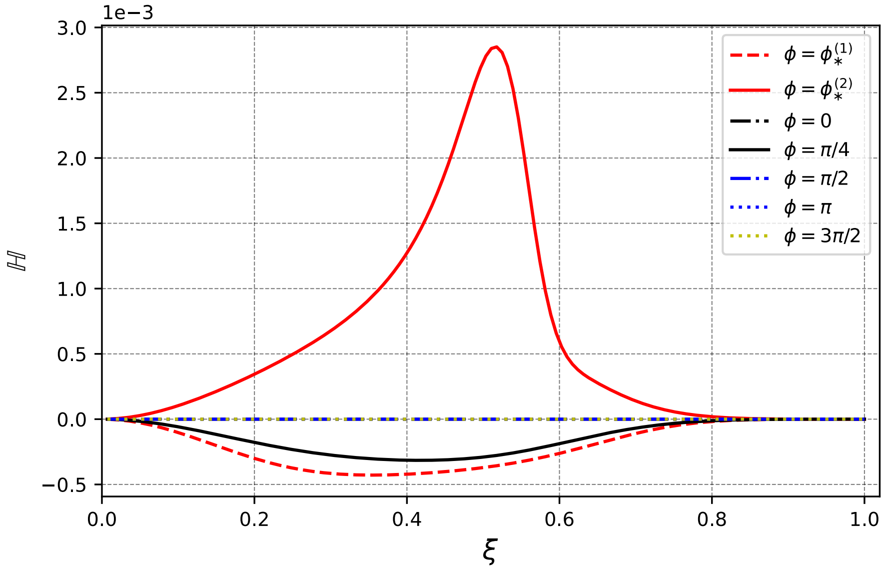
Numerical Example: we explore the numerical solutions of a family of locally 1-d models, containing Szekeres as a particular case.

$$G_{ij} = \mathrm{Diag}\left[\left(\mathcal{A}-\hat{\mathcal{F}}_{ini}\right)^2\left(\frac{\mathcal{W}}{\chi}\right)^2, \left(\frac{e^{\nu}}{\chi}\right)^2, \left(\frac{e^{\nu}}{\chi}\right)^2\right]$$

$$P^a_i = \frac{\hat{\mathcal{F}} - \hat{\mathcal{F}}_{\rm ini}}{\mathcal{A} - \hat{\mathcal{F}}_{\rm ini}} \delta^a_3\, \delta^3_i \qquad \hat{\mathcal{F}} = \mathcal{F} + \delta\mathcal{F}\ , \quad \delta\mathcal{F} = \alpha\left(\delta\beta_+ f_+ + \delta\beta_- f_-\right)$$

α = 4.8 × 10^{-3}





Concluding remarks

- We have developed a new nonlinear approach to model the large-scale structure formation in the Universe (GRZA). It merges elements of exact solutions and the Lagrangian perturbation theory: the dynamics is described in terms of a deformation field evolving on a background model. But, it generalizes the global FLRW background to an inhomogeneous Friedmann-like reference model.

- GRZA contains Szekeres (then LTB) models and RZA as particular limits. However, applications will ultimately reveal the quality of the approach and whether or not its use is justified.

- Potential applications include:

    I. Relativistic corrections to the current N-body numerical simulations.

    II. Impact of the inhomogeneities on the prediction of H_0 or the Universe' large-scale average evolution (i.e., to address the Hubble tension and the backreaction problem).

    III. Corrections of non-linear effects in quantities or observable processes like weak lensing.