Quantum droplets. New, interesting state of matter

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A small correction with huge consequences!

Energy density for Bose-Bose mixture:

$$\epsilon\left[\psi_1,\psi_2\right] = \frac{\hbar^2}{2m_1}|\vec{\nabla}\psi_1|^2 + \frac{\hbar^2}{2m_2}|\vec{\nabla}\psi_2|^2 + V_{trap}(\vec{r})(|\psi_1|^2+|\psi_2|^2) + \frac{1}{2}g_1|\psi_1|^4 + \frac{1}{2}g_2|\psi_2|^4 + g_{12}|\psi_1|^2|\psi_2|^2 + \epsilon_{LHY}[\psi_1,\psi_2]$$

The addition of the repulsive Lee-Huang-Young term stabilizes our mixture, since it competes with the attractive inter-species interaction.[1]

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$$\epsilon_{LHY} = \frac{8}{15\pi^2\hbar^3} m_1^{3/2}(g_1 n_1)^{5/2} f\left(\frac{m_2}{m_1}, \frac{g_{12}^2}{g_1 g_2}, \frac{g_2 n_2}{g_1 n_1}\right)$$

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for $m_1 = m_2 = m$ function f is analytical

$f(1,x,y) = \frac{1}{4\sqrt{2}}\sum_{\pm}\left(1 + y \pm \sqrt{(1-y)^2 + 4xy}\right)^{5/2}$

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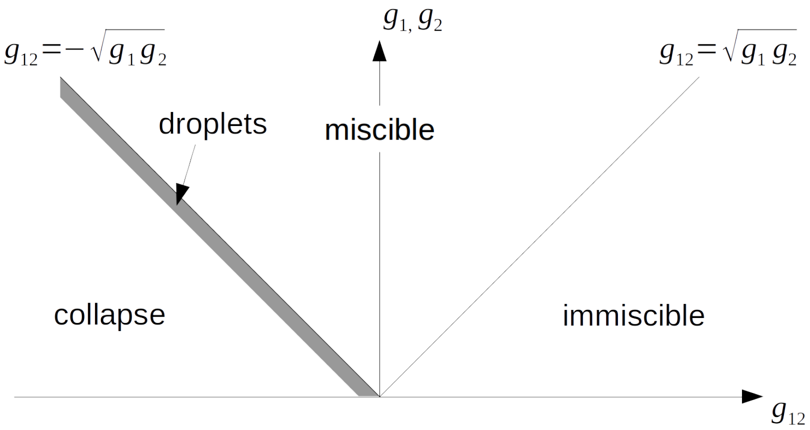
How to get a droplet?

We need to find a balance between intraspecies repulsion ($g_i > 0$) and interaction g_{12} between different atoms!



Courtesy of F. Gampel

Convenient dimensionless units:

- length $\xi = \sqrt{\frac{3}{2}\frac{\sqrt{g_1}/m_1+\sqrt{g_2}/m_2}{|\delta g|\sqrt{g_1}n_{10}}}$, $\delta g = g_{12} + \sqrt{g_1 g_2}$
- density $n_{10} = \frac{25\pi}{1024}\frac{\delta g^2}{a_1^3 g_1 g_2 f^2(m_2/m_1, 1, \sqrt{g_2/g_1})}$
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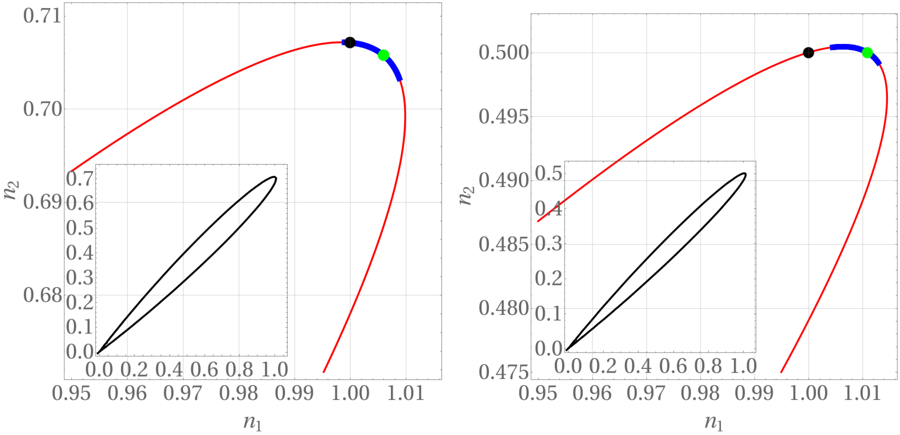
Gross-Pitaevskii equations

$$-\frac{1}{2}\nabla^2\phi_1 + g(n_1 - \sqrt{a}n_2)\phi_1 + \delta g n_2 \phi_1 + D(n_1 + a n_2)^{\frac{3}{2}}\phi_1 = \mu_1\phi_1$$

$$-\frac{1}{2}\nabla^2\phi_2 + g\sqrt{a}(\sqrt{a}n_2 - n_1)\phi_2 + \delta g n_1 \phi_2 + aD(n_1 + a n_2)^{\frac{3}{2}}\phi_2 = \mu_2\phi_2$$

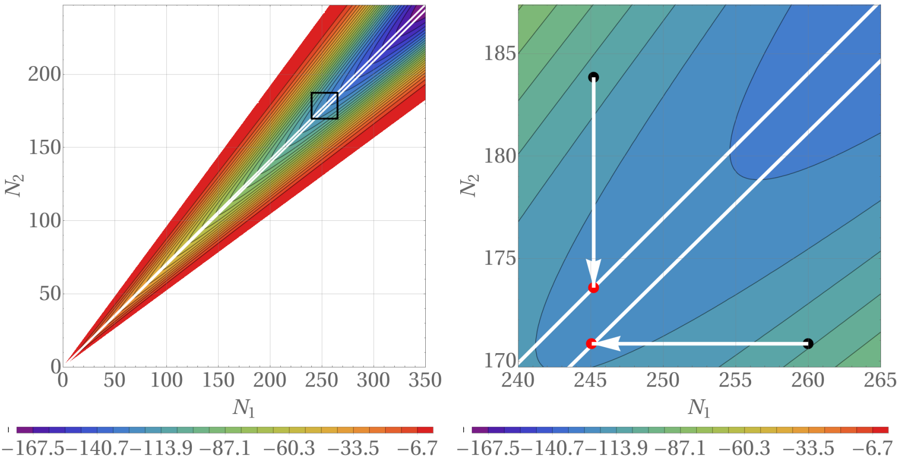
where $g = \frac{3a_{11}(\sqrt{a}+1)}{2|\delta a|}$, $\delta g = -\frac{3}{2}(\sqrt{a}+1)$, $D = \frac{5}{2}\frac{1}{\sqrt{a}(1+\sqrt{a})^{3/2}}$, and $a = a_{22}/a_{11}$

Stability of the droplets: $a = 2$ vs. $a = 4$



Paweł Zin, MP, Mariusz Gajda, *Phys. Rev. A*, **103**, 013312 (2021)

To evaporate or not to evaporate?



$a = 2$, "real" number of atoms $N_r \approx N \cdot 6300$

Droplet-droplet interaction potential

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If $R \gg r_0$

$$\Psi_i(\vec{r}) = \sum_{\alpha=L,R} \psi_i(\vec{r} - \vec{r}_\alpha) e^{-i\phi_i^\alpha}$$

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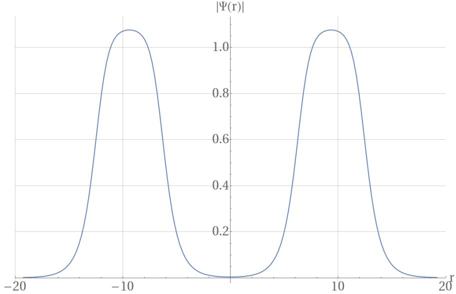

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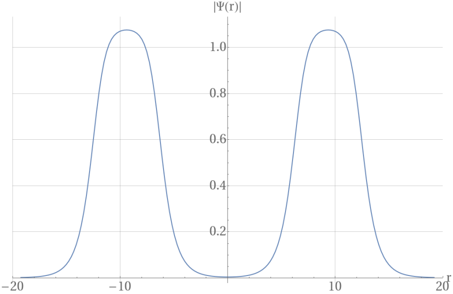

Energy functional

$$F[\Psi_1, \Psi_2] = \sqrt{\mathcal{B}_i} \sum_{i=1,2} \left(F_L + F_R + U_i(R, \Delta\phi_i)\right)$$

$$U(R, \Delta\phi_1, \Delta\phi_2) = -\sum_i A_i^2 \frac{4\pi}{R} e^{-\lambda_i R} \cos\left(\Delta\phi_i\right)$$

where $\lambda_i = \sqrt{-2\mu_i}$

$$\mathcal{H} = \sum_{\alpha=L,R} \left(\frac{(\vec{p}_\alpha)^2}{2M^\alpha} + \frac{(p_H^\alpha)^2}{2M_H^\alpha} + \frac{(p_S^\alpha)^2}{2M_S^\alpha} + U + \sum_{i=1,2} \mu_i^\alpha \delta N_i^\alpha \right)$$

[2] P.Zin, MP, M. Gajda, NJP 23, 033022 (2021)

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Kinetic momenta of hard and soft modes:

$p_{S,H}^\alpha = (\sqrt{\mu_{1,1}^\alpha}\delta N_1^\alpha \pm \sqrt{\mu_{2,2}^\alpha}\delta N_2^\alpha)/\sqrt{2}$, where $\mu_{i,j}^\alpha = \frac{\partial \mu_i^\alpha}{\partial N_j^\alpha}$

Coefficients $M^\alpha = N_1^\alpha + N_2^\alpha$ and $M_{S,H}^\alpha = \left(1 \pm \mu_{1,2}^\alpha / \sqrt{\mu_{1,1}^\alpha \mu_{2,2}^\alpha}\right)^{-1}$ are "masses" of translational mode and two phase-modes.

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$$\dot{P} = U_1'(R)\cos(\Delta\phi_1) + U_2'(R)\cos(\Delta\phi_2),$$
$$\dot{R} = P/\mathcal{M},$$
$$\delta\dot{N}_i^R = -\delta\dot{N}_i^L = U_i(R)\sin(\Delta\phi_i),$$
$$\Delta\dot{\phi}_i = \sum_j \left(\mu_{i,j}^R \delta N_j^R - \mu_{i,j}^L \delta N_j^L\right) + \Delta\mu_i,$$

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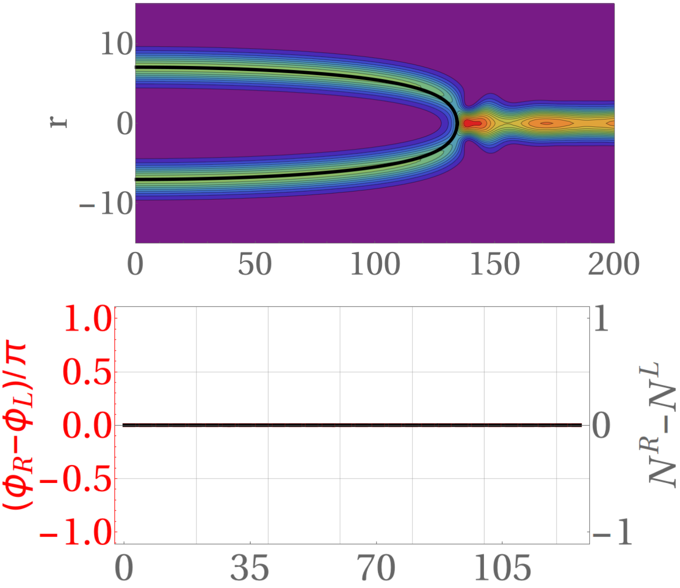
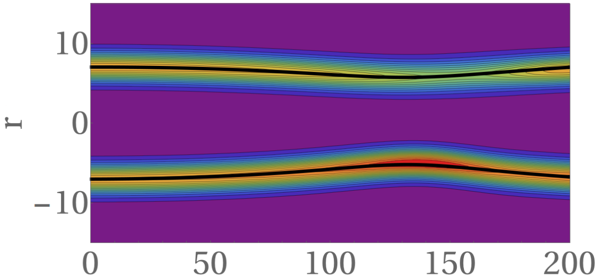
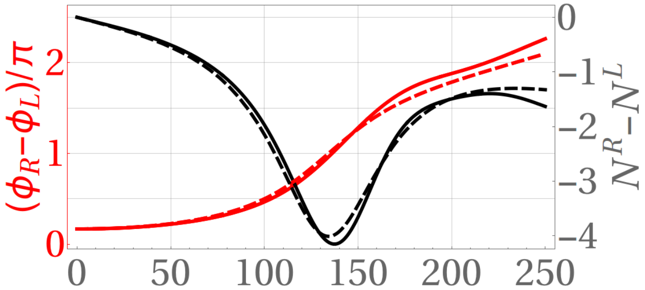
Small droplets: $\Delta\phi_1 = \Delta\phi_2 = 0$

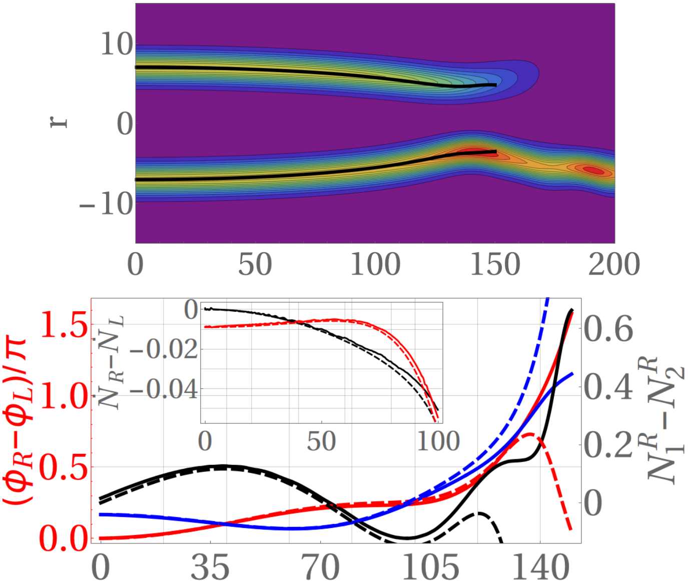


Small droplets: $\Delta\phi_1 = \Delta\phi_2 = \pi/6$

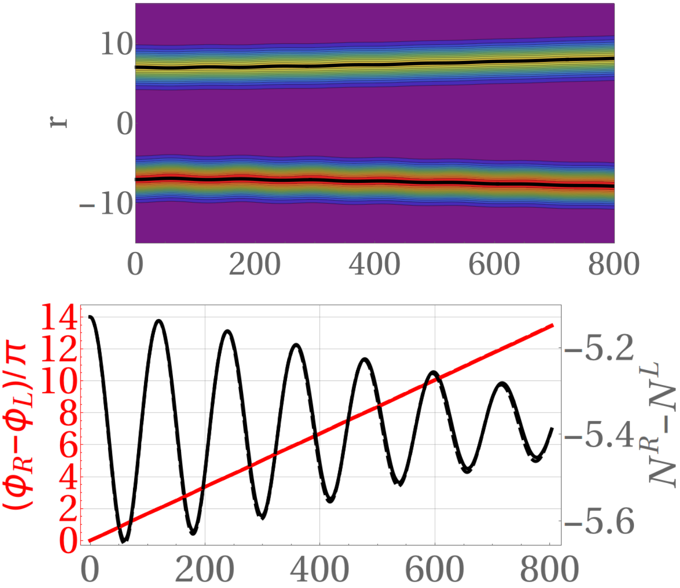




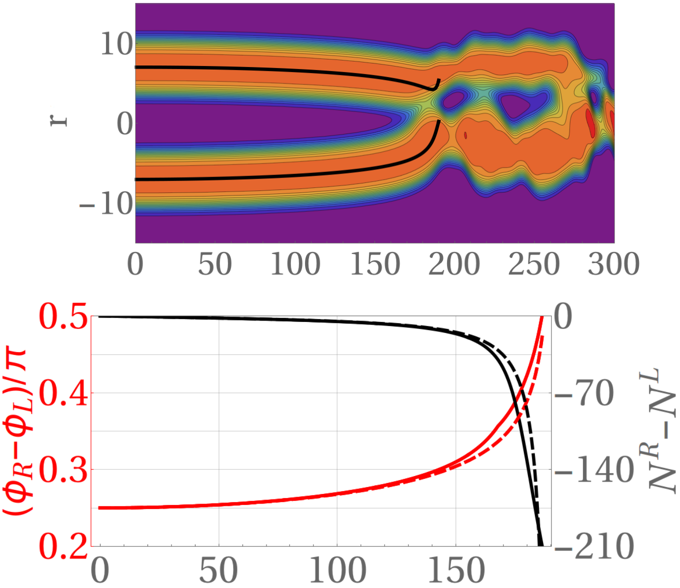
Small droplets: $\Delta\phi_1 = 0$, $\Delta\phi_2 = \pi/6$



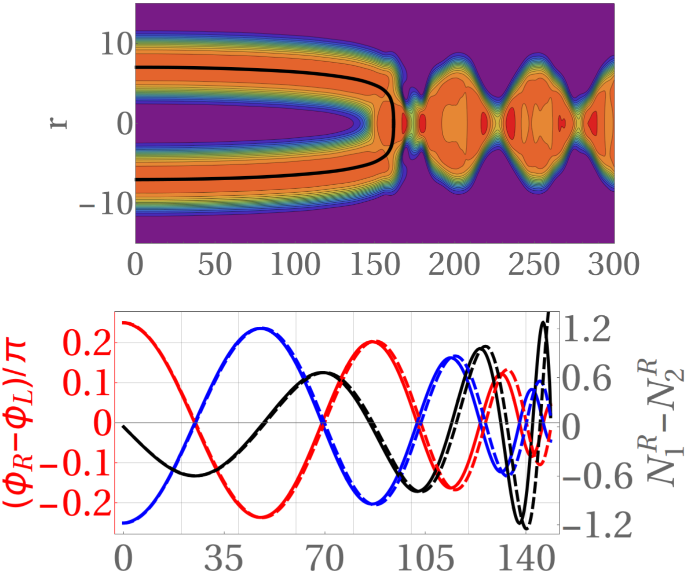
Small droplets: $\Delta\phi_1 = \Delta\phi_2 = 0$, $N_1^L = 25.4$, $N_1^R = 20.3$



Big droplets: $\Delta\phi_1 = \Delta\phi_2 = \pi/4$

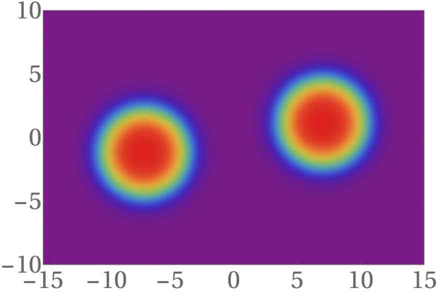


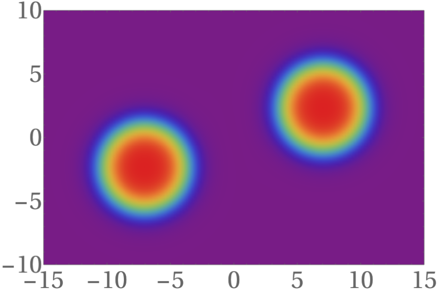
Big droplets: $\Delta\phi_1 = \pi/4$, $\Delta\phi_2 = -\pi/4$



Big droplets: $\Delta\phi_1 = \Delta\phi_2 = \pi$, $V = 1.5$

For central collisions and $\Delta\phi_i = \pi$ reflection from the barrier
What if the collision is not central?





$\Delta = 10$ vs. $\Delta = 20$

Conclusions

Work in progress. . .

- the dynamics of inter-acting droplets and their ultimate fate depend crucially on the relative phases of their wavefunction,
- Two droplets, which constitute identical macroscopic objects, can be made to merge, repel or evaporate only by manipulating their phases

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- the dynamics of inter-acting droplets and their ultimate fate depend crucially on the relative phases of their wavefunction,
- Two droplets, which constitute identical macroscopic objects, can be made to merge, repel or evaporate only by manipulating their phases they are really quantum objects
- Our Josephson-junction equations do not have any free parameters, everything is from the solution of stationary GP equations,

Thank you for your attention. . .

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How to get a droplet? Some theoretical background

Within mean-field approximation

$$E[\psi, \psi^\star] = \frac{\hbar^2}{2m} \int |\nabla \psi(\vec{r})|^2 \, d^3r + \frac{1}{2} \int V_{int}(\vec{r} - \vec{r}')|\psi(\vec{r})|^2 |\psi(\vec{r}')|^2 \, d^3r \, d^3r' + \\ + \int V_{trap}(\vec{r})|\psi(\vec{r})|^2 \, d^3r$$

For mixture of two Bose gases with contact interaction $V_{int}^{(i)}(\vec{r} - \vec{r}') = g_i \cdot \delta(\vec{r} - \vec{r}')$:

$$E[\psi_1, \psi_2] = \frac{\hbar^2}{2m_1} \int |\vec{\nabla} \psi_1|^2 \, d^3r + \frac{\hbar^2}{2m_2} \int |\vec{\nabla} \psi_2|^2 \, d^3r + \\ + \frac{1}{2} \int \left(g_1 |\psi_1|^4 + g_2 |\psi_2|^4 \right) d^3r + g_{12} \int |\psi_1|^2 |\psi_2|^2 \, d^3r + \\ + \int V_{trap}(\vec{r}) \left(|\psi_1|^2 + |\psi_2|^2 \right) d^3r$$

where

$$g_i = \frac{4\pi\hbar^2 a_i}{m_i}, \quad g_{12} = \frac{2\pi\hbar^2 a_{12}}{m_r}$$

Stability of the droplets

The droplet is stable if:

$$dE = \frac{\partial E}{\partial N_1} dN_1 + \frac{\partial E}{\partial N_2} dN_2 = \mu_1 dN_1 + \mu_2 dN_2 > 0$$

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In a typical experimental situation the number of particles can only decrease $dN_1 \leqslant 0$, so $\mu_1 < 0$ and $\mu_2 < 0$.
Moreover, for uniform droplet (without a surface) we get an additional constraint: a droplet will stabilize its volume if internal pressure vanishes:

$$p = -\frac{\partial E}{\partial V} = \mu_1 n_1 + \mu_2 n_2 - \epsilon(n_1, n_2) = 0$$

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- ...but also on the phase difference between the droplets
- and may be attractive or repulsive...or much more interesting if $\Delta\phi_1 \neq \Delta\phi_2$phase matters!
- it may be generalized to the situation in which there is a small population imbalance between the droplets:

$$U(R, \Delta\phi_1, \Delta\phi_2) = -\sum_i A_i^{(L)} A_i^{(R)} \frac{4\pi}{R} e^{-(\lambda_i^{(L)}+\lambda_i^{(R)})R/2} \cos\left(\Delta\phi_i\right)$$

Josephson junction-like equation

- The exponential density tails overlap forming a weak link

[3] J. Dziarmaga, Phys. Rev. A 70, 063616 (2004)
[4] P. Zin, MP, and M. Gajda, New J. Phys. 23, 033022 (2021)

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- The phase difference between the left and right parts of the two wavefunctions will trigger a coherent flow of atoms between the droplets
- These are the Josephson-junction-like oscillations of particle number and relative phase
- Instead of solving the full set of GP equations, the oscillations may be described adequately by only considering the zero-energy or Goldstone modes[3,4]

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