Probing the Gluon Sivers Function with an Unpolarized Target: GTMD Distributions and the Odderons

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Probing the Gluon Sivers Function with an Unpolarized Target: GTMD Distributions and the Odderons

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Nucleon target coupling and its nucleon's spin dependence in a nutshell

Simplifying assumptions:

• $m_N = 0$, no momentum transfer $\Delta = P' - P = 0$, at high energy
$P = p,\ p^2 = 0$ light-cone vector

• the second light-cone vector $n^\mu, n^2 = 0$ is determined by a hard process
$n \cdot p = const$

General coupling to a nucleon:

$$\bar{u}(p,S)\Gamma u(p,S) \text{ with spin vector} S^\mu \text{ such that } \ S \cdot p = 0 \ \ S^2 = -1$$

with spinor Dirac matrix satisfy
$u(p,S)\bar{u}(p,S) = \frac{1}{2}\hat{p}(1+\gamma^5\hat{S})$ or $\frac{1}{2}\hat{p}(1+h\gamma^5)$ with $h=$ nucleon's helicity

Γ are in general 16 Fierz matrices
$\Gamma = 1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, i.e. 16=1+1+4+4+6

Three physically important couplings which lead to:

- unpolarised distributions

$$\bar{u}(p,S)\hat{n}u(p,S) \sim n \cdot p$$

no dependence on the spin vector S

- helicity distributions

$$\bar{u}(p,S)\hat{n}\gamma^5 u(p,S) \sim h\ n \cdot p$$

dependence on the helicity h, when spin vector S projected on momentum p

- transversity distributions

$r_\perp$ arbitrary transverse vector

$$\bar{u}(p,S)\sigma^{n\,r_\perp}u(p,S) \sim \ \epsilon^{n\,p\,r_\perp\,S} = \epsilon^{n\,p\,r_\perp\,S_\perp} = (r_\perp \times S_\perp)_z$$

dependence on $S_\perp$ components only, linear polarisation

diagonal in helicity $\bar{u}(p,S=h)\sigma^{n\,r_\perp}u(p,S=h) = 0$

Sivers distribution/function

$\equiv$ transversity coupling with $r_\perp$ being transverse partonic momentum $k_\perp$

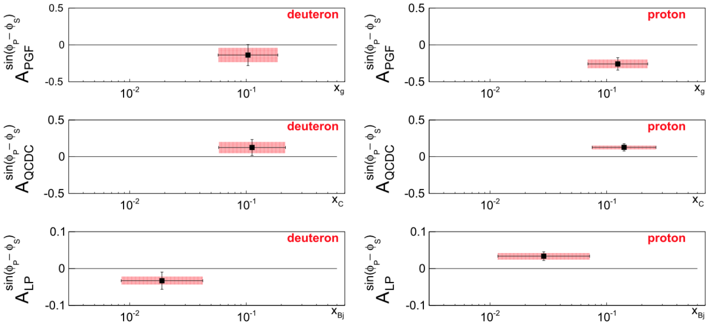
$$\bar{u}(p,S)\sigma^{n\,k_\perp}u(p,S) \sim \epsilon^{n\,p\,k_\perp\,S} = \epsilon^{n\,p\,k_\perp\,S_\perp} = (k_\perp \times S_\perp)_z$$

Physics:
correlation between transverse partonic momenta
AND
transverse target spin $S_\perp$

Experiment: polarized nucleon target is needed !!!! (?)

First measurement of the Sivers asymmetry for gluons using SIDIS data

The COMPASS Collaboration Physics Letters B 772 (2017) 854



Contrary to widespread views we asked:
is it possible to probe Sivers function in a process with UNPOLARISED target??

Our answer: is affirmative !

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Which process to choose ?

Sivers target coupling is odd under charge conjugation:

$$\bar{u}(p,S)\sigma^{\hat{n}\,\hat{k}_\perp}u(p,S) = -\bar{v}(p,S)\sigma^{\hat{n}\,\hat{k}_\perp}v(p,S)$$

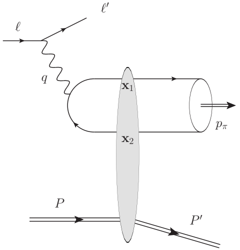
$$\bar{v}^T(p,S) = Cu(p,S)\ ,\ C\sigma^{n\,p}C^{-1} = -\sigma^{n\,p\,T}$$

$\Longrightarrow$ $t-$channel gluonic exchange is color singlet AND charge conjugation odd
$\equiv$ Odderon(s) exchange(s)

$\Longrightarrow$ simplest exclusive process: pion electroproduction on a nucleon

Exclusive pion π^0 electroproduction on a nucleon target at high energy

π^0- C even, γ^*- C odd: $(-1)\,1 = -1$, C odd exchange



$$\bar{P} = \frac{P' + P}{2}, \quad \Delta = P' - P,$$

relevant Lorentz invariants

$$Q^2 = -q^2, \quad x_B = \frac{-q^2}{2\,(P \cdot q)}, \quad t = \Delta^2, \quad y = \frac{(P \cdot q)}{(P \cdot \ell)}.$$

pion light-cone distribution amplitude

$$\langle \pi(p_\pi) | \bar{\psi}(x_1) \gamma^\lambda \gamma_5 \psi(x_2) | 0 \rangle = i f_\pi p_\pi^\lambda \int_0^1 dz e^{iz(p_\pi \cdot x_1) + i\bar{z}(p_\pi \cdot x_2)} \phi_\pi(z)$$

symmetry property: $\phi_\pi(z) = \phi_\pi(\bar{z} = 1 - z)$

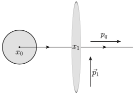
Feynman rules in a shock wave formalism:

Shock wave: in QED Liénard-Wiechert potentials when $v = c$

$$A^{\mu}_{Y_c}(x) = \delta(x^-) A^{+}_{Y_c}(x_\perp)\delta^{\mu +}$$

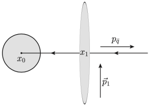
Wilson line

$$U_{\boldsymbol{x},Y_c} = \mathcal{P} e^{ig\int_{-\infty}^{+\infty} dx^- A^{+}_{Y_c}(x^-,\boldsymbol{x})} \equiv [-\infty, +\infty]_{\boldsymbol{x}}$$



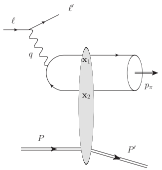
Coupling of shock wave to a quark

$$\bar{\psi}_{eff}(x_0) = \theta(-x_0^-)\int d^4x_1 \delta(x_1^-)\bar{\psi}(x_1)\gamma^- U_{\boldsymbol{x}_1} G(x_{10})$$



Coupling of shock wave to an anti-quark

$$\psi_{eff}(x_0) = -\theta(-x_0^-)\int d^4x_2 \delta(x_2^-) G(x_{02})\gamma^- U_{\boldsymbol{x}_2}\psi(x_2)$$



The scattering amplitude:

$$\mathcal{A} = \int d^4 x_0 \bar{u}_{\ell'} (-ie_\ell)\gamma^\mu u_\ell G_{\mu\nu}(q) e^{-i(q\cdot x_0)} \langle P'\pi|\bar{\psi}_{eff}(x_0)(-ie_f)\gamma^\nu \psi_{eff}(x_0)|P\rangle.$$

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Using Feynman rules and expression for π^0 distribution amplitude we get:

$$\mathcal{A} == \frac{e_f e_\ell}{4N_c} \bar{u}_{\ell'}\gamma^\mu u_\ell \int d^4x_0 d^4x_1 d^4x_2 \theta(-x_0^-)\delta(x_1^-)\delta(x_2^-) e^{-i(q\cdot x_0)}$$
$$\times\, if_\pi \int_0^1 dz e^{iz(p_\pi\cdot x_1)+i\bar{z}(p_\pi\cdot x_2)} \phi_\pi(z)\langle P'|\mathrm{Tr}(U_{\mathbf{x}_1} U^\dagger_{\mathbf{x}_2}) - N_c|P\rangle$$
$$\times\, 2p_\pi^- G_{\mu\nu}(q)\mathrm{Tr}\left[G(x_{10})\gamma^\nu G(x_{02})\gamma^-\gamma_5\right]$$

$$\langle P' | \mathrm{Tr}(U_{\boldsymbol{x}_1} U^\dagger_{\boldsymbol{x}_2}) - N_c | P \rangle$$

Introducing:

center of a dipol: $\boldsymbol{b} = z\boldsymbol{x}_1 + \bar{z}\boldsymbol{x}_2$

size of a dipol: $\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2$

$$\langle P'|\mathrm{Tr}(U_{\boldsymbol{x}_1}U^\dagger_{\boldsymbol{x}_2}) - N_c|P\rangle$$

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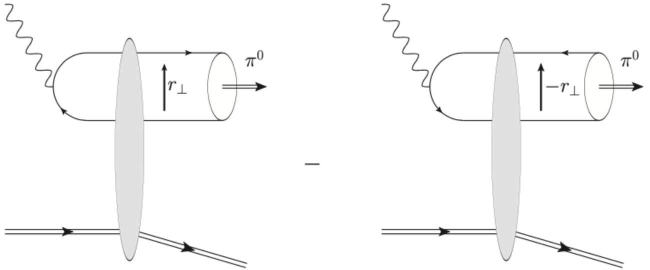
Calculation of a simple Dirac trace, performing gaussian integration and integration over the light-cone time x_0^- leads to

$$\mathcal{A} = -\frac{e_f e_\ell f_\pi}{N_c}(2\pi)\delta(q^- - p_\pi^-)\bar{u}_{\ell'}\gamma^\mu u_\ell \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{-i(\boldsymbol{p}_\pi - \boldsymbol{q})\cdot\boldsymbol{b}}$$
$$\times \int d^2\boldsymbol{r} \int_0^1 dz \phi_\pi(z) r_{\perp\alpha}\sqrt{\frac{z\bar{z}Q^2}{\boldsymbol{r}^2}} K_1(\sqrt{z\bar{z}Q^2\boldsymbol{r}^2})$$
$$\times \langle P'|\mathrm{Tr}(U_{\boldsymbol{b}+\bar{z}\boldsymbol{r}}U^\dagger_{\boldsymbol{b}-z\boldsymbol{r}}) - N_c|P\rangle G_{\mu\nu}(q)\epsilon^{\alpha\beta+-}\left(q^- g^\nu_{\perp\beta} - n^\nu q_{\perp\beta}\right).$$

Odderon exchange: C even meson production



$$\frac{1}{2}\left[\mathrm{Tr}\left(U_{b+\frac{r}{2}}U^{\dagger}_{b-\frac{r}{2}}\right) - \mathrm{Tr}\left(U_{b-\frac{r}{2}}U^{\dagger}_{b+\frac{r}{2}}\right)\right]$$

From Wilson lines to TMD distributions

- Derivative of a shockwave operator allows to extract a physical gluon

$$(\partial^i U^\dagger_{x_\perp})U_{x_\perp} = -ig\int dx^+ [+\infty, x^+]_{x_\perp}\, F^{-i}(x^+, x_\perp)\, [x^+, +\infty]_{x_\perp}$$

$$\partial^i A^-(x^+, x_\perp) \Rightarrow F^{-i}(x^+, x_\perp)$$

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- Definition of TMD

$$\frac{4M}{\bar{P}^+} \int \frac{d^4 v}{(2\pi)^3} \delta(v^+) e^{ix\bar{P}^+ v^- - i(\boldsymbol{k}\cdot\boldsymbol{v})}$$
$$\times \langle P' S' | \mathrm{Tr}\left[F^{i+}\left(-\frac{v}{2}\right) \mathcal{U}^{[+]}_{(v/2),-(v/2)} F^{i+}\left(\frac{v}{2}\right) \mathcal{U}^{[-]}_{-(v/2),(v/2)} \right] | PS \rangle$$
$$= \bar{u}_{P'} \left[F^g_{1,1} + i \frac{\sigma^{i+}}{\bar{P}^+} (\boldsymbol{k}^i F^g_{1,2} + \boldsymbol{\Delta}^i F^g_{1,3}) + i \frac{\sigma^{ij} \boldsymbol{k}^i \boldsymbol{\Delta}^j}{M^2} F^g_{1,4} \right] u_P,$$

Exclusive low x amplitude = GTMD amplitude

T. Altinoluk and R. Boussarie, JHEP 10 (2019) 208

$$\left\langle P', S' \left| \mathrm{Tr}\left(U_{\boldsymbol{x}_1} U_{\boldsymbol{x}_2}^\dagger\right) - N_c \right| P, S \right\rangle$$

$$= \frac{\alpha_s \bar{P}^-}{M} e^{-i\boldsymbol{\Delta}\cdot\left(\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2}\right)} \delta\left(\Delta^-\right) \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}}$$

$$\times \left[e^{-i(\boldsymbol{k}\cdot\boldsymbol{r})} - \frac{1}{2}\left(e^{i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)} + e^{-i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)}\right) + \frac{(\boldsymbol{k}\cdot\boldsymbol{r})}{(\boldsymbol{\Delta}\cdot\boldsymbol{r})}\left(e^{i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)} - e^{-i\left(\boldsymbol{\Delta}\cdot\frac{\boldsymbol{r}}{2}\right)}\right)\right]$$

$$\times \bar{u}_{P',S'} \left[F_{1,1}^g + i\frac{\sigma^{i-}}{\bar{P}^-}\left(\boldsymbol{k}^i F_{1,2}^g + \boldsymbol{\Delta}^i F_{1,3}^g\right) + i\frac{\sigma^{ij}\boldsymbol{k}^i\boldsymbol{\Delta}^j}{M^2} F_{1,4}^g \right] u_{P,S}$$

Every exclusive low x process probes a Wigner distribution!

Fourier transform of the $(r_\perp \leftrightarrow -r_\perp)$-antisymmetric dipole (Odderon)

$$\int d^2\boldsymbol{r} e^{-i(\boldsymbol{k}\cdot\boldsymbol{r})}\langle P', S'|\mathcal{O}(\boldsymbol{r})|P, S\rangle$$
$$= \frac{g_s^2}{2} N_c (2\pi)^2 \delta(P'^+ - P^+) \frac{1}{\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}}$$
$$\times\, i \frac{\boldsymbol{k}^j}{M} \bar{u}_{P',S'} \left[\frac{\boldsymbol{\Delta}^j}{M} \gamma^+ g_{1,1} + i\sigma^{i+} \left(\delta^{ij} g_{1,2} + \frac{\boldsymbol{\Delta}^i \boldsymbol{\Delta}^j}{M^2} (g_{1,3} - \frac{1}{2} g_{1,1}) \right) \right] u_{P,S}.$$

With explicit spinors, we see 3 types of coupling to the target:

- The Vector Odderon $i(\boldsymbol{k}\cdot\boldsymbol{\Delta})g_{1,1}$
- The Spin Odderon $(\boldsymbol{k}\times\boldsymbol{S})^z g_{1,2}$
- The Spin-vector Odderon $(\boldsymbol{k}\cdot\boldsymbol{\Delta})(\boldsymbol{\Delta}\times\boldsymbol{S})^z g_{1,3}$

Probing the Sivers function

Odderon/GTMD equivalence implies, that the cross section for exclusive π^0 electroproduction at small x and small t with unpolarized lepton and proton beams is a direct probe for the gluon Sivers function

$$\frac{d\sigma}{d\xi dQ^2 d|t|} \simeq (2\pi)^3 \frac{\alpha_{\rm em}^2 \alpha_s^2 f_\pi^2}{8\xi N_c M^2 Q^2} (1 - y + \frac{y^2}{2}) \times \left[\int_0^1 dz \frac{\phi_\pi(z)}{z\bar{z}Q^2} \int d\boldsymbol{k}^2 \frac{\boldsymbol{k}^2}{\boldsymbol{k}^2 + z\bar{z}Q^2} x f_{1T}^{\perp}(x, \boldsymbol{k}^2) \right]^2 .$$

In other words: we can understand the gluonic content of the transversely polarized protons without polarizing the proton beam.

I thank deeply the award jury for appreciation of our publication !!

Thank you for your attention !!