The trajectory approach of quantum mechanics

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Motivations: (quantum) cosmology

Homogeneous & Isotropic metric (FLRW):

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]$$

$\mathcal{K} \to$ Spatial curvature

Hubble rate $H \equiv \frac{\dot{a}}{a} = \frac{\mathrm{d}a}{a\,\mathrm{d}t}$

Matter component: perfect fluid: $T_{\mu\nu} = pg_{\mu\nu} + (\rho + p)u_\mu u_\nu$

$$p = w\rho \quad \begin{cases} w = 0 & \text{dust} \\ w = \frac{1}{3} & \text{radiation} \end{cases}$$

+ cosmological constant = Einstein equation:

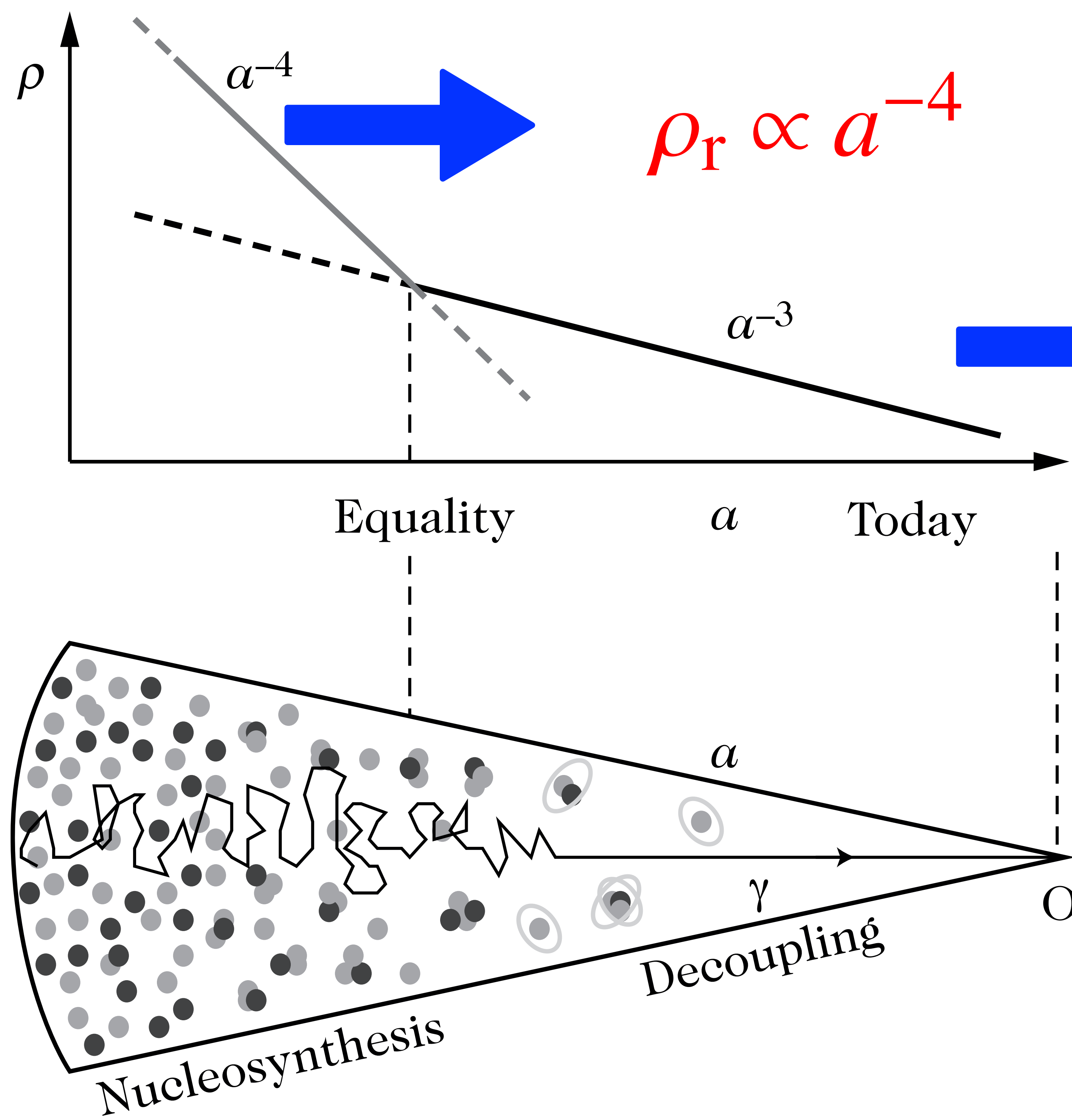
$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{1}{3}\left(8\pi G_{\mathrm{N}}\rho + \Lambda\right)$$

$$\frac{\ddot{a}}{a} = \frac{1}{3}\left[\Lambda - 4\pi G_{\mathrm{N}}\left(\rho + 3p\right)\right]$$

Particular solution: matter & radiation

Integrate conservation equation:

$$\rho[a(t)] = \rho_{\rm ini} \exp\left\{-3\int [1+w(a)]\,\mathrm{d}\ln a\right\} \underset{w\to\mathrm{cst}}{=} \rho_{\rm ini}\left(\frac{a}{a_{\rm ini}}\right)^{-3(1+w)}$$

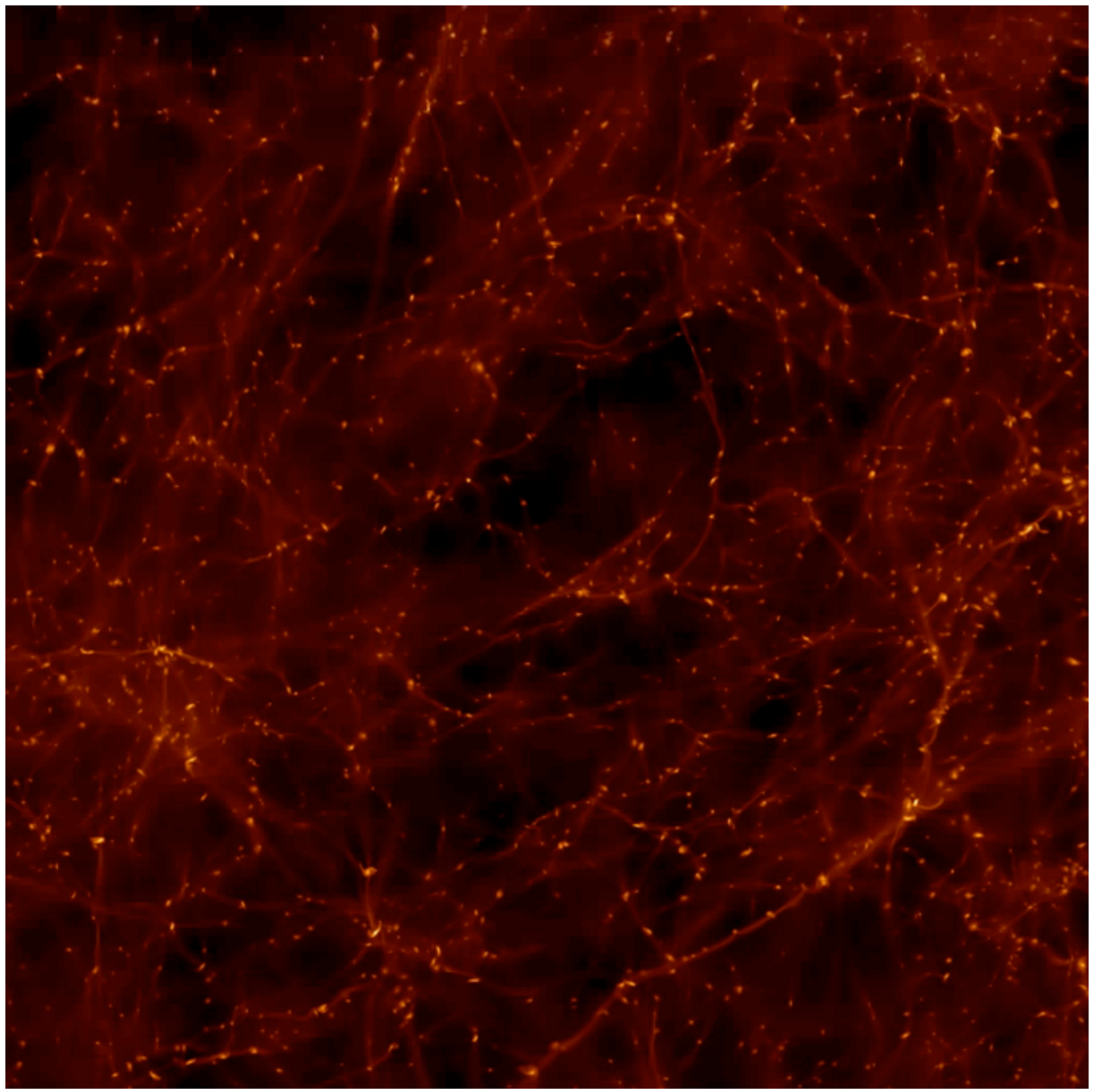


$$\rho_{\rm r} \propto a^{-4}$$

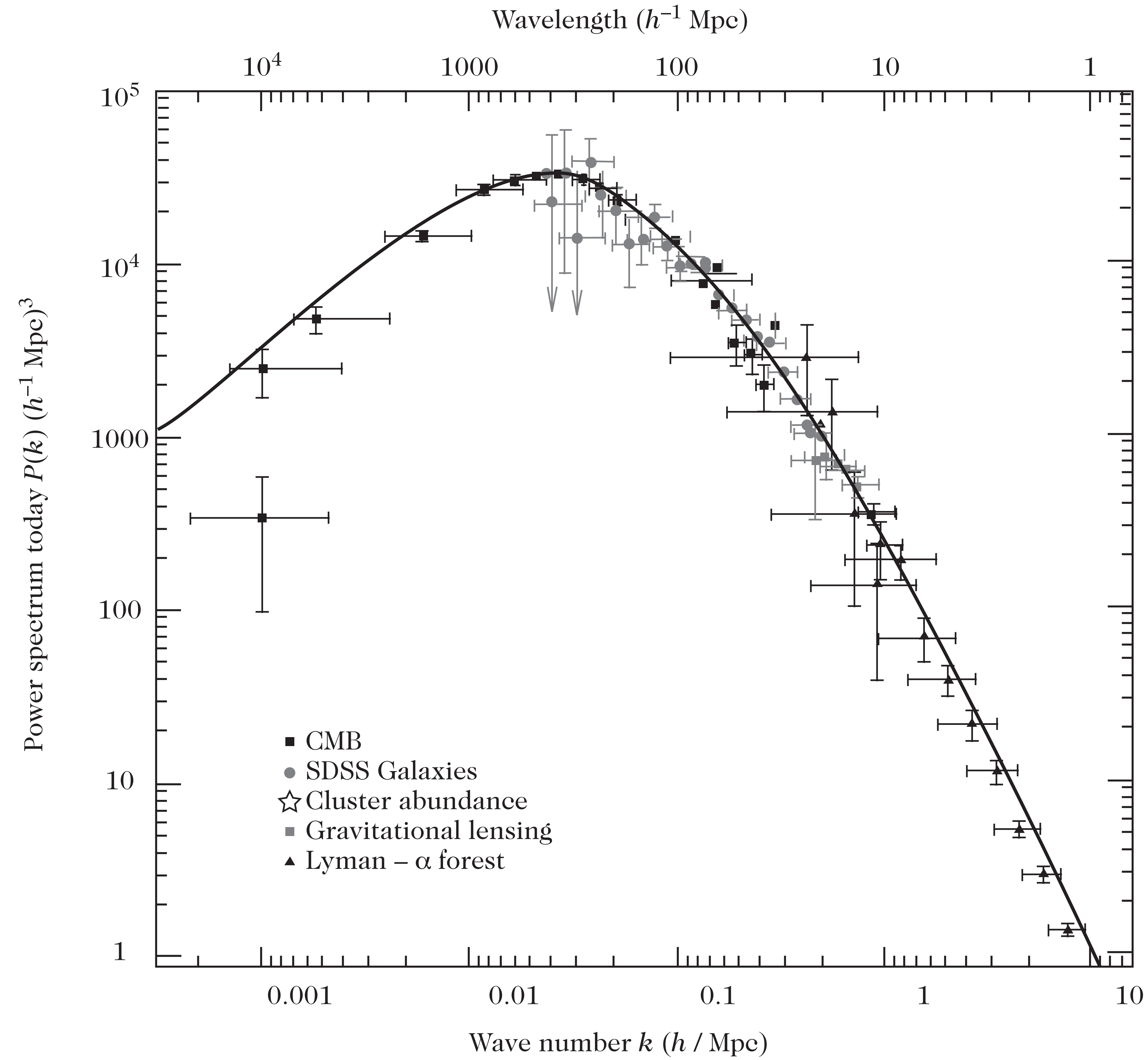
$$\rho_{\rm m} \propto a^{-3}$$

Phenomenologically valid description for almost 14 Gyrs!!!

Numerical simulation for large scale structure formation

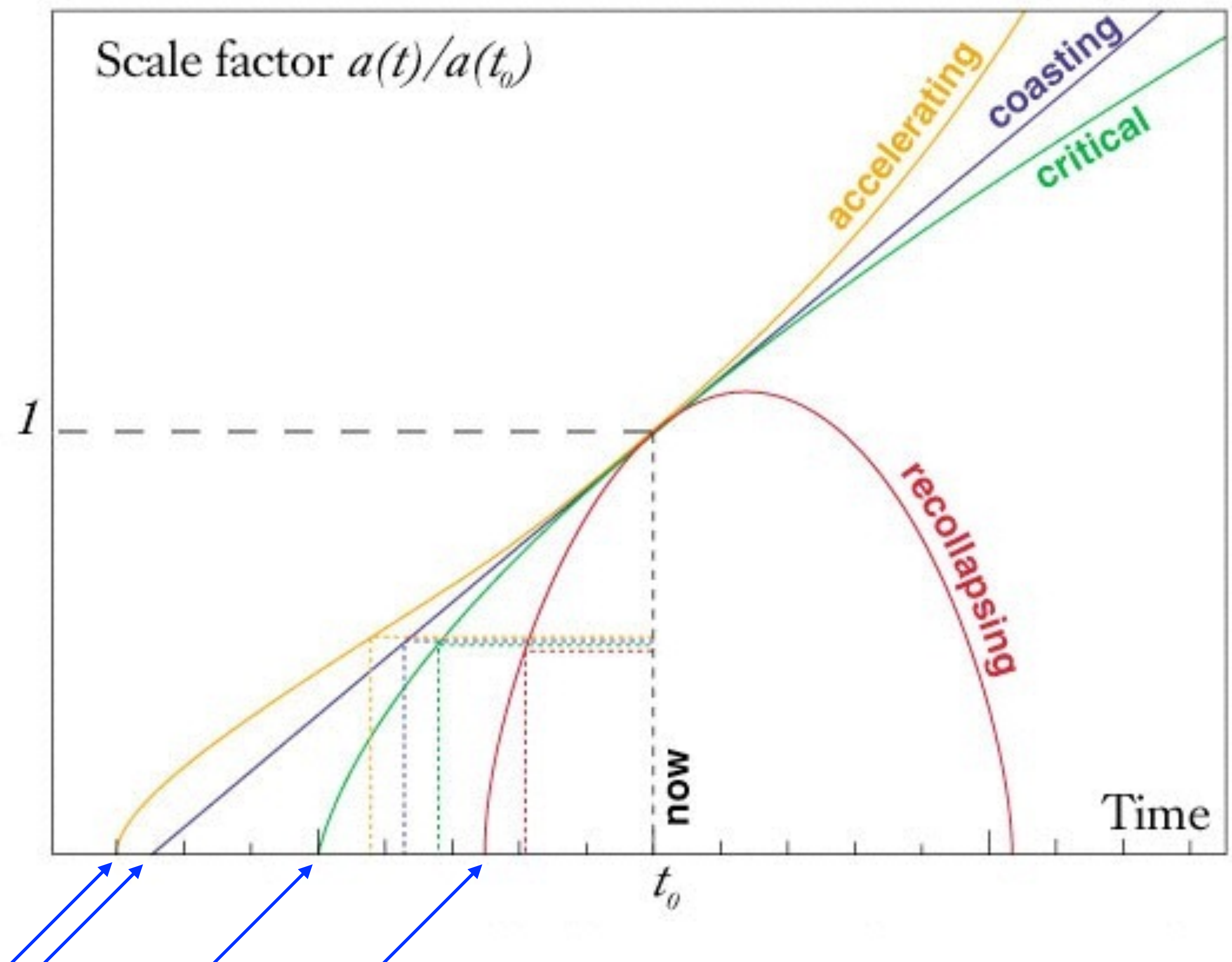


Numerical simulations: C. Pichon @ IAP

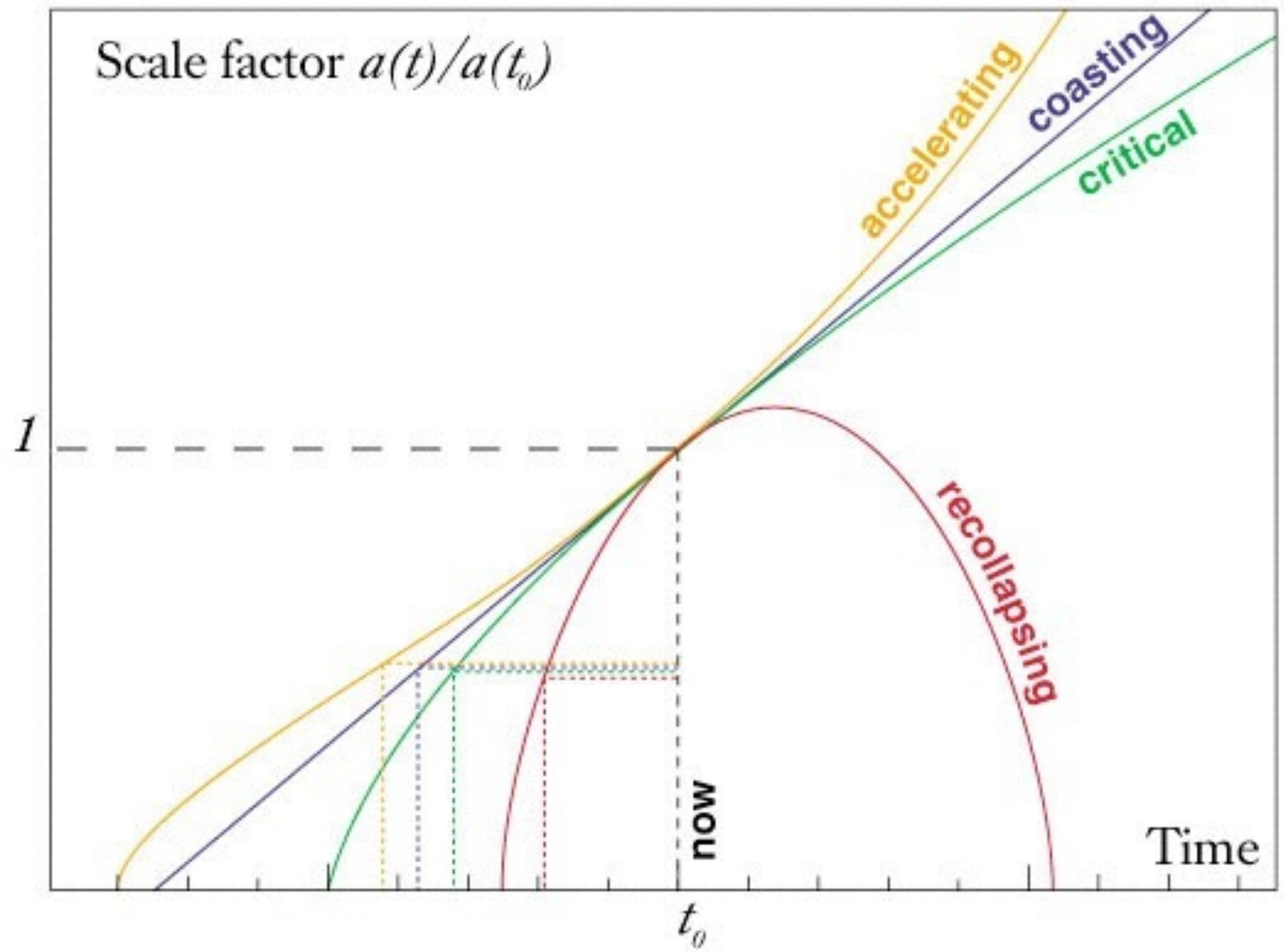


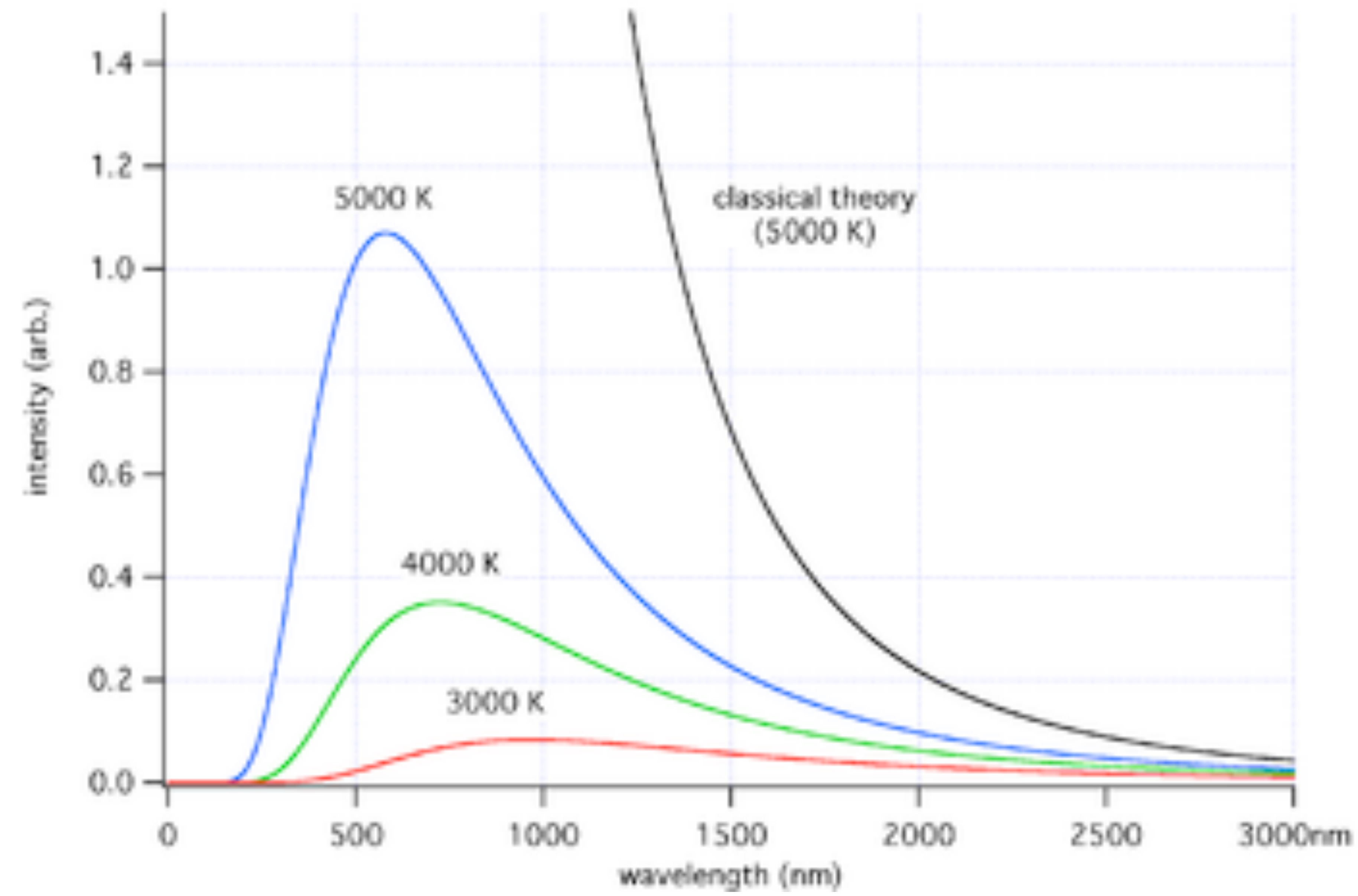
Comparison with observational data

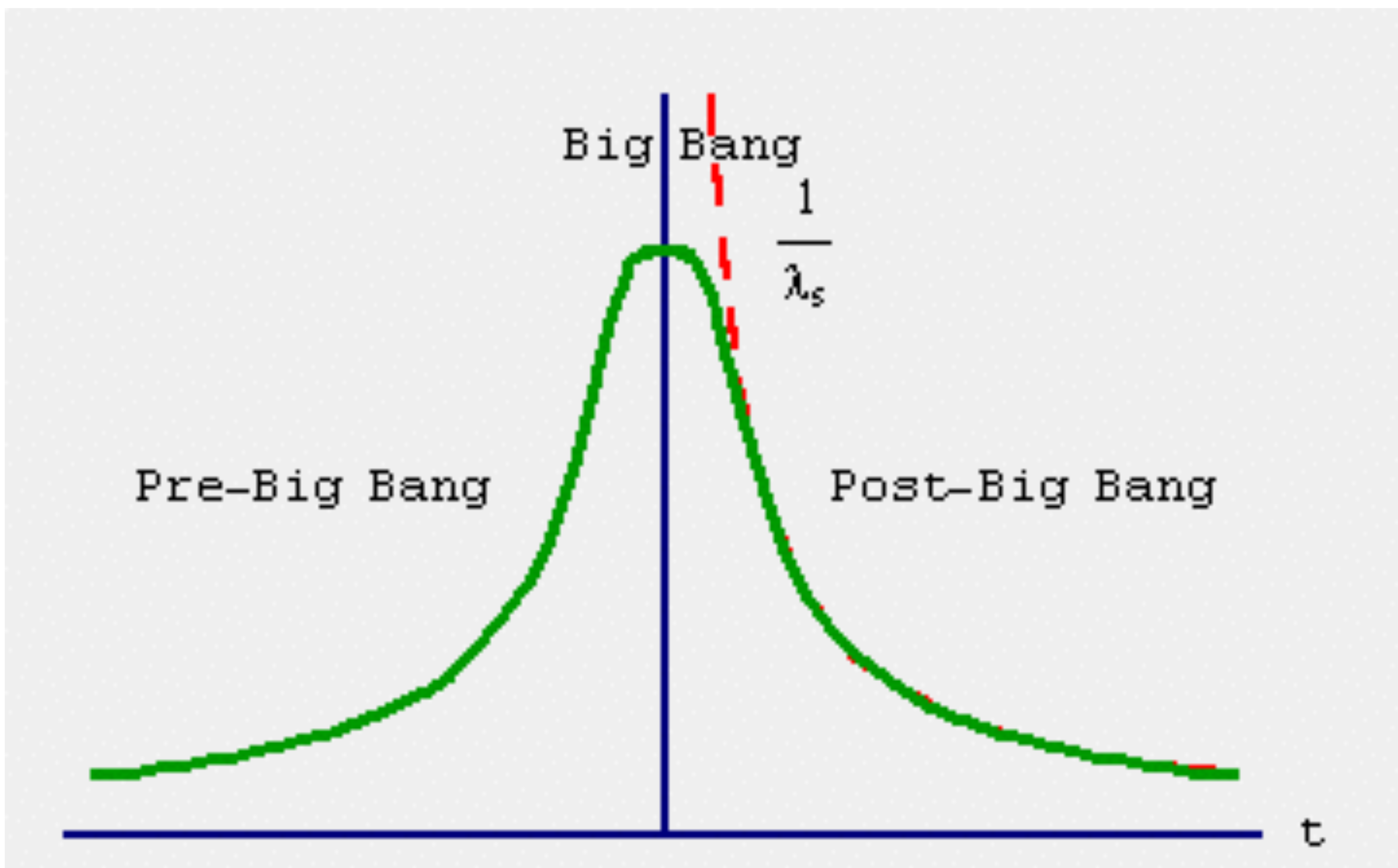
A central problem (though not often formulated thus…): the singularity

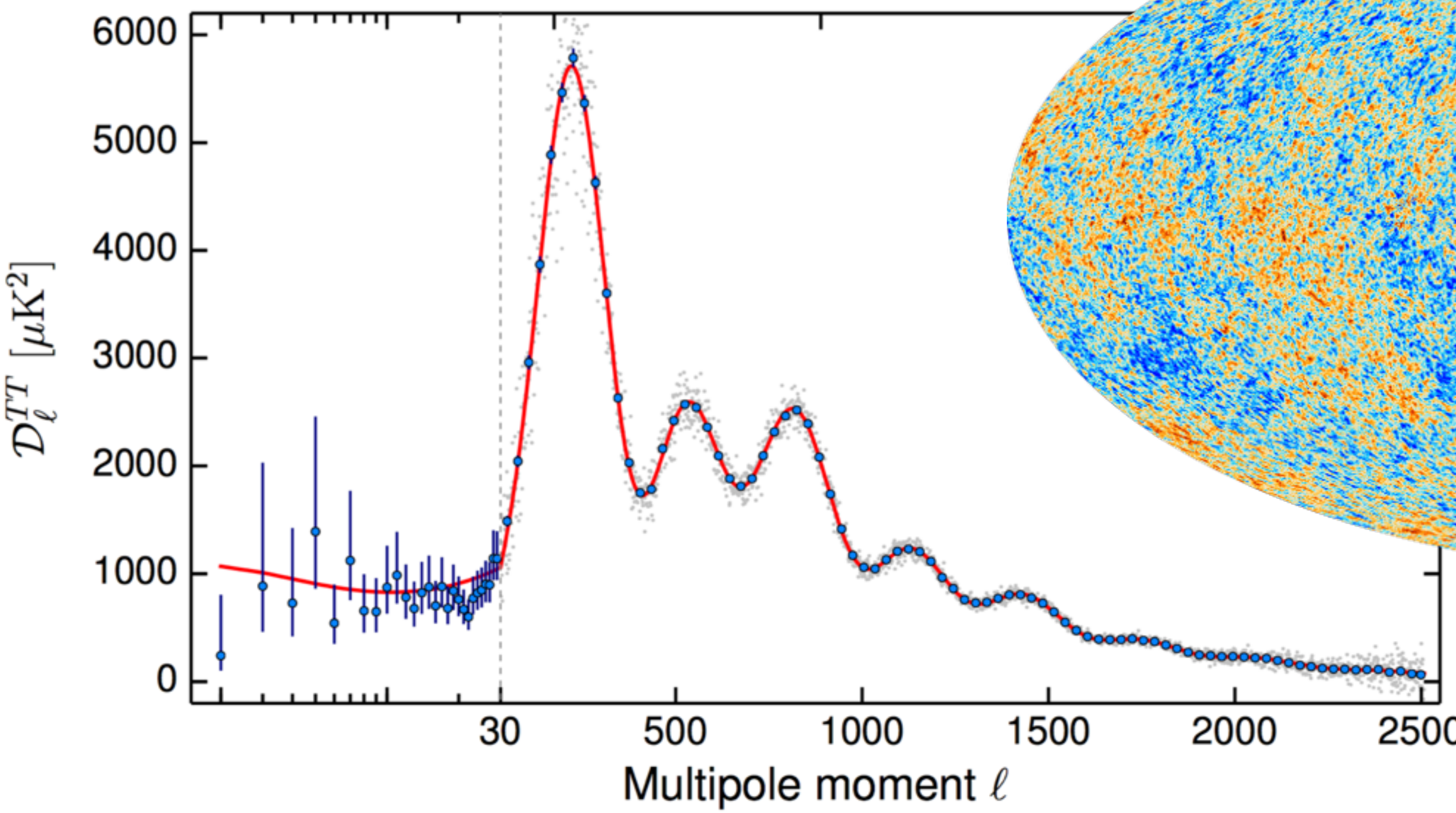


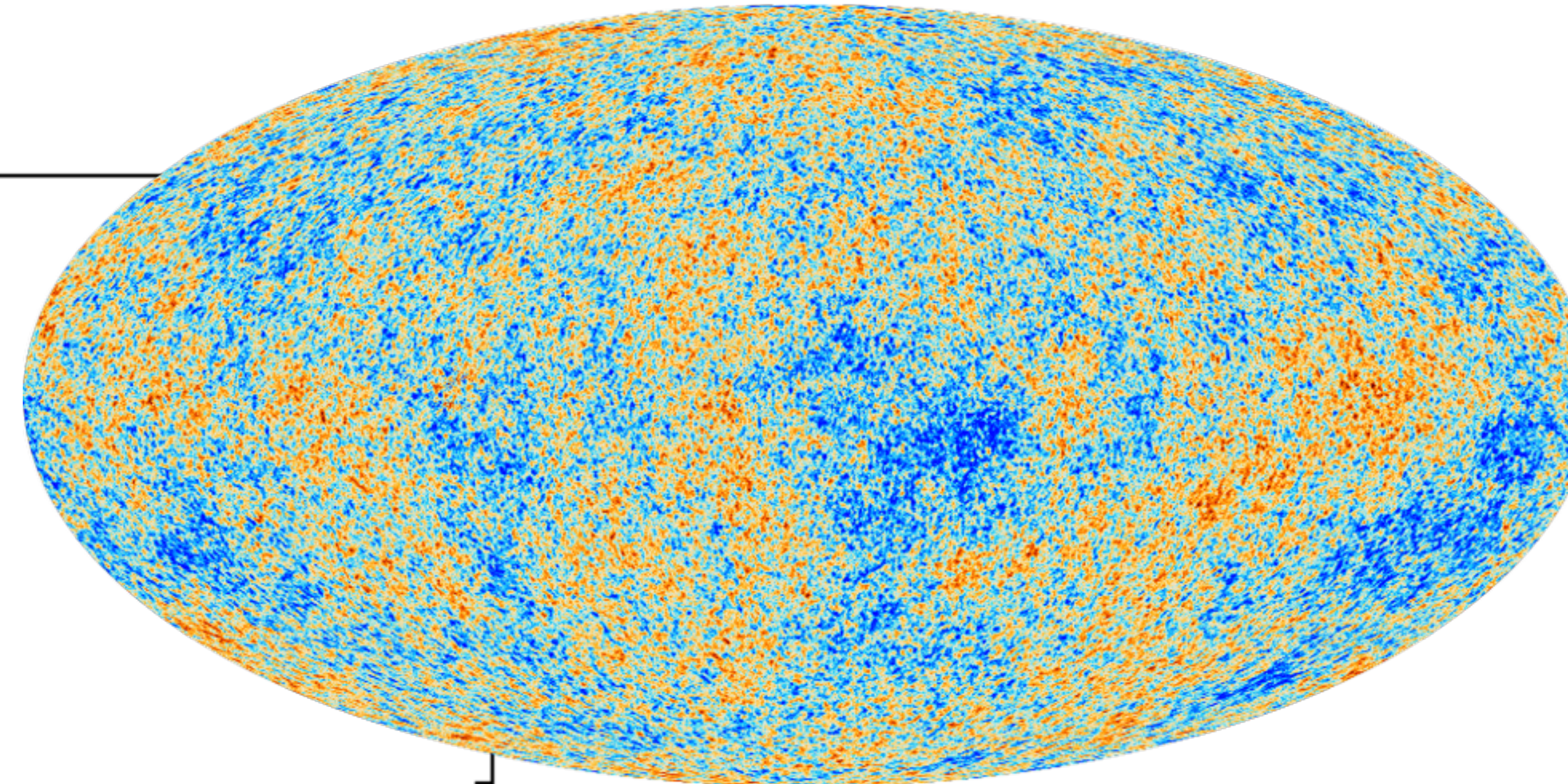
Singularity problem   Quantum effect?











$\Omega_{\mathcal{K}} = 0.000 \pm 0.005$

$n_{\rm s} = 0.9639 \pm 0.0047$ almost $\underbrace{\text{scale invariant}}_{\text{excluded}}$

quantum vacuum fluctuations of a single scalar d.o.f

$$\left.\begin{aligned} f_{\rm NL}^{\rm local} &= 0.71 \pm 5.1 \\ f_{\rm NL}^{\rm equil} &= -9.5 \pm 44 \\ f_{\rm NL}^{\rm ortho} &= -25 \pm 22 \end{aligned}\right\}$$ gaussian signal

$\longrightarrow$ compatible with ***INFLATION***

isocurvature $\lesssim 1\%$

$r < 0.11$

Quantum mechanics

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution
Wavepacket reduction = non linear / stochastic

Mutually incompatible

+ *External observer*

Predictions for a quantum theory

Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$

Usually in a lab:
repeat the experiment

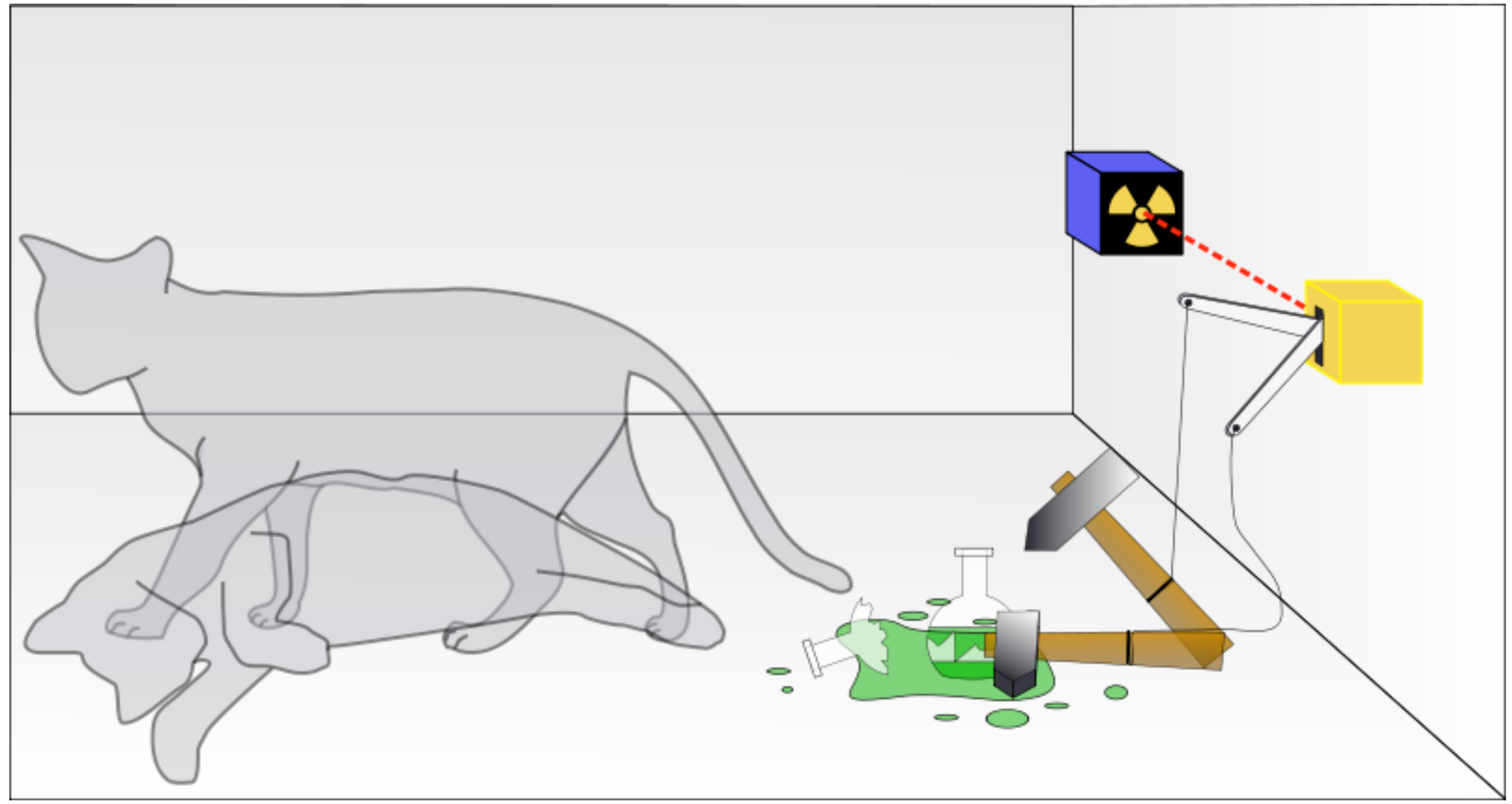
Ensemble average over experiments ⟷ Quantum average

Here one has a single experiment (a single universe)

Ergodicity

Spatial average over directions in the sky ⟷ Quantum average

The measurement problem in quantum mechanics

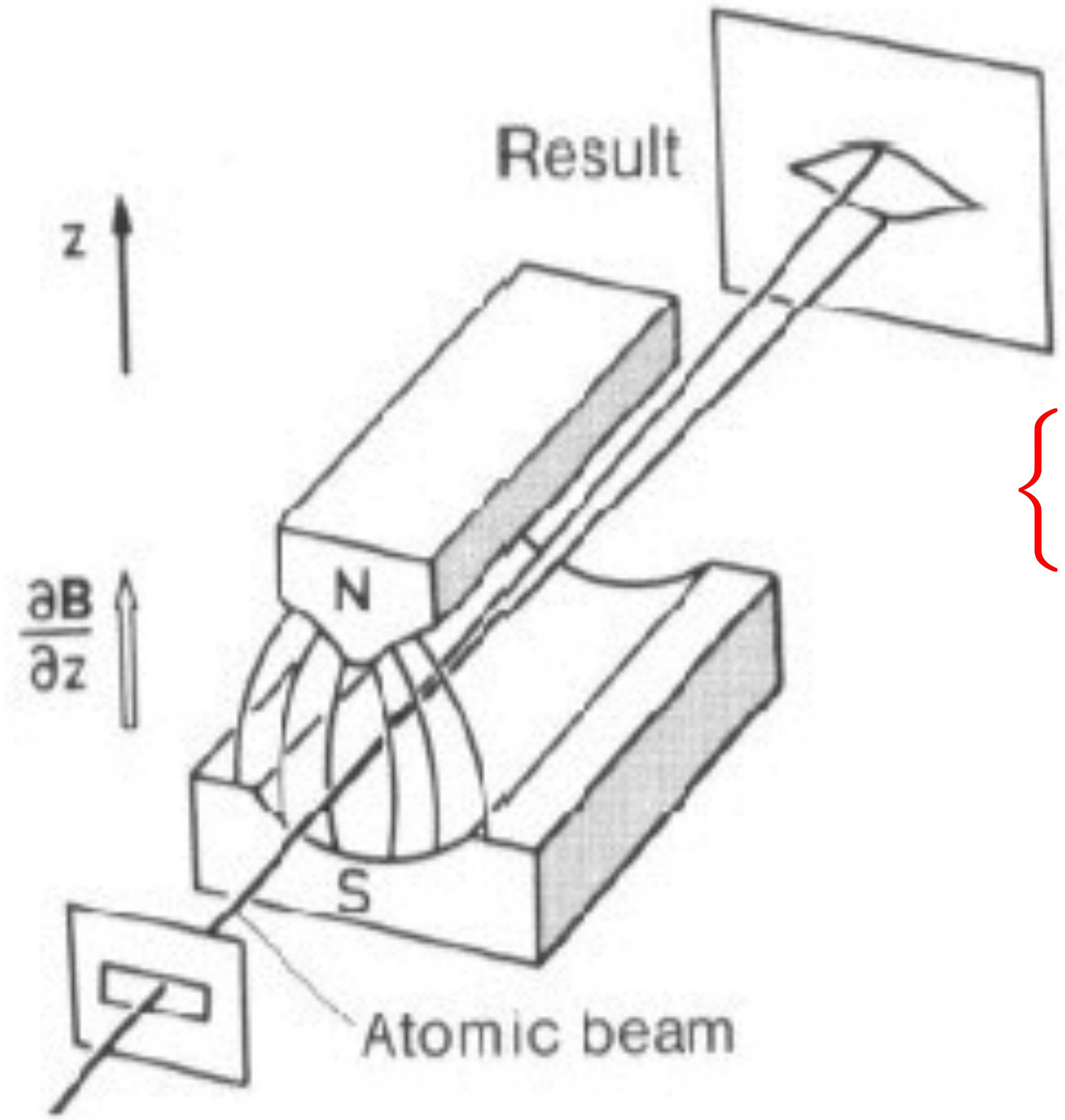


Preferred basis: no unique definition of measured observables

Definite outcome: we don’t measure superpositions

collapse of the wave function

The measurement problem in quantum mechanics

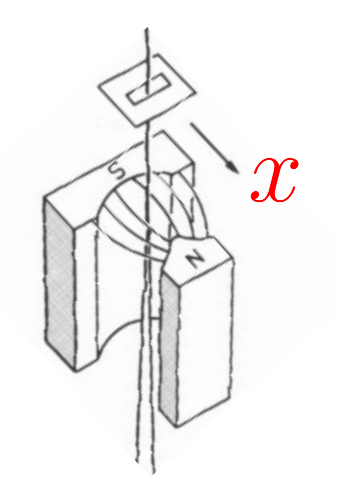


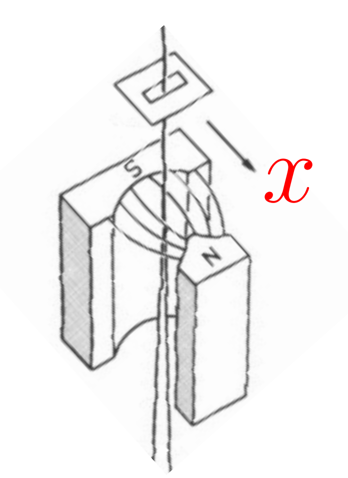
Statistical mixture

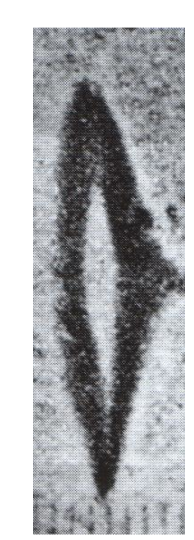
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\uparrow}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\downarrow}\rangle\right\}$$

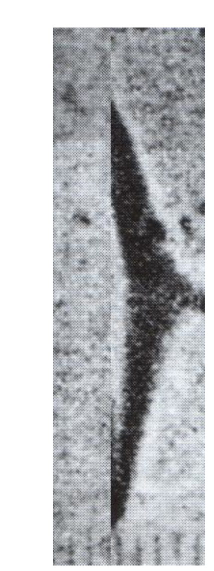
$$\left\{|\uparrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\} \cup \left\{|\downarrow\rangle \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$

$$\left\{(|\uparrow\rangle + |\downarrow\rangle) \otimes |\mathrm{SG}_{\mathrm{in}}\rangle\right\}$$



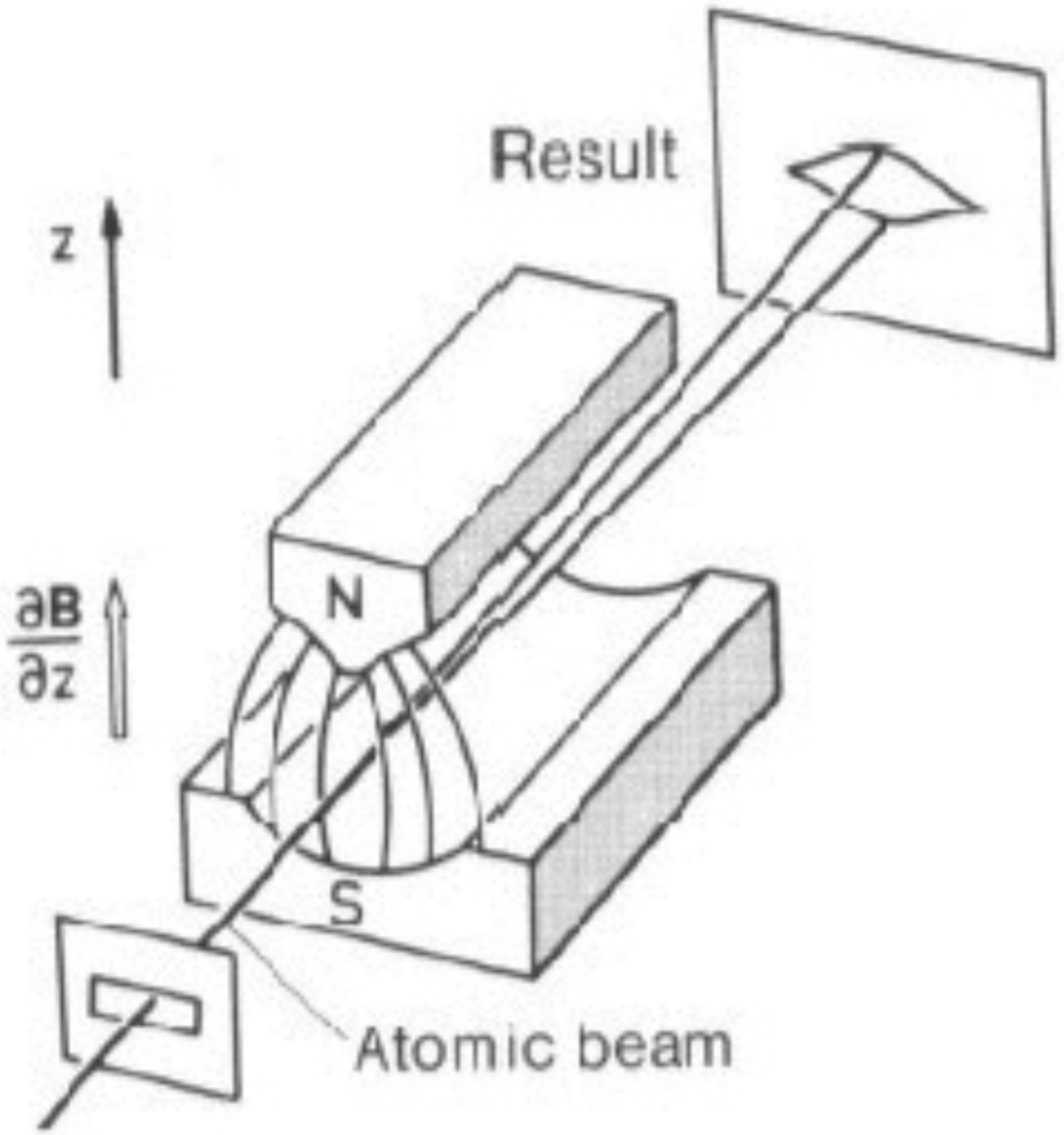






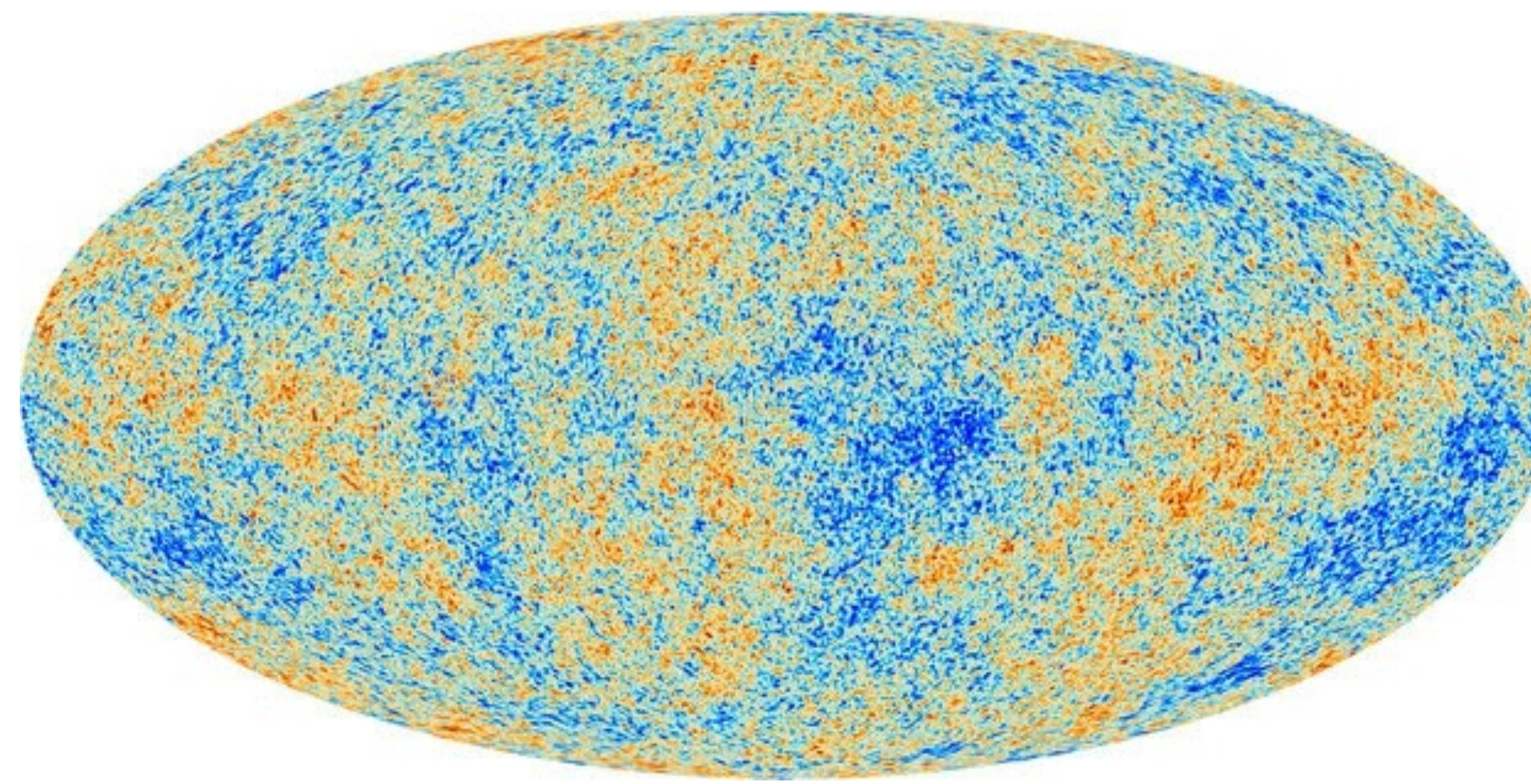
Stern-Gerlach

The measurement problem in quantum mechanics



What about the Universe itself?

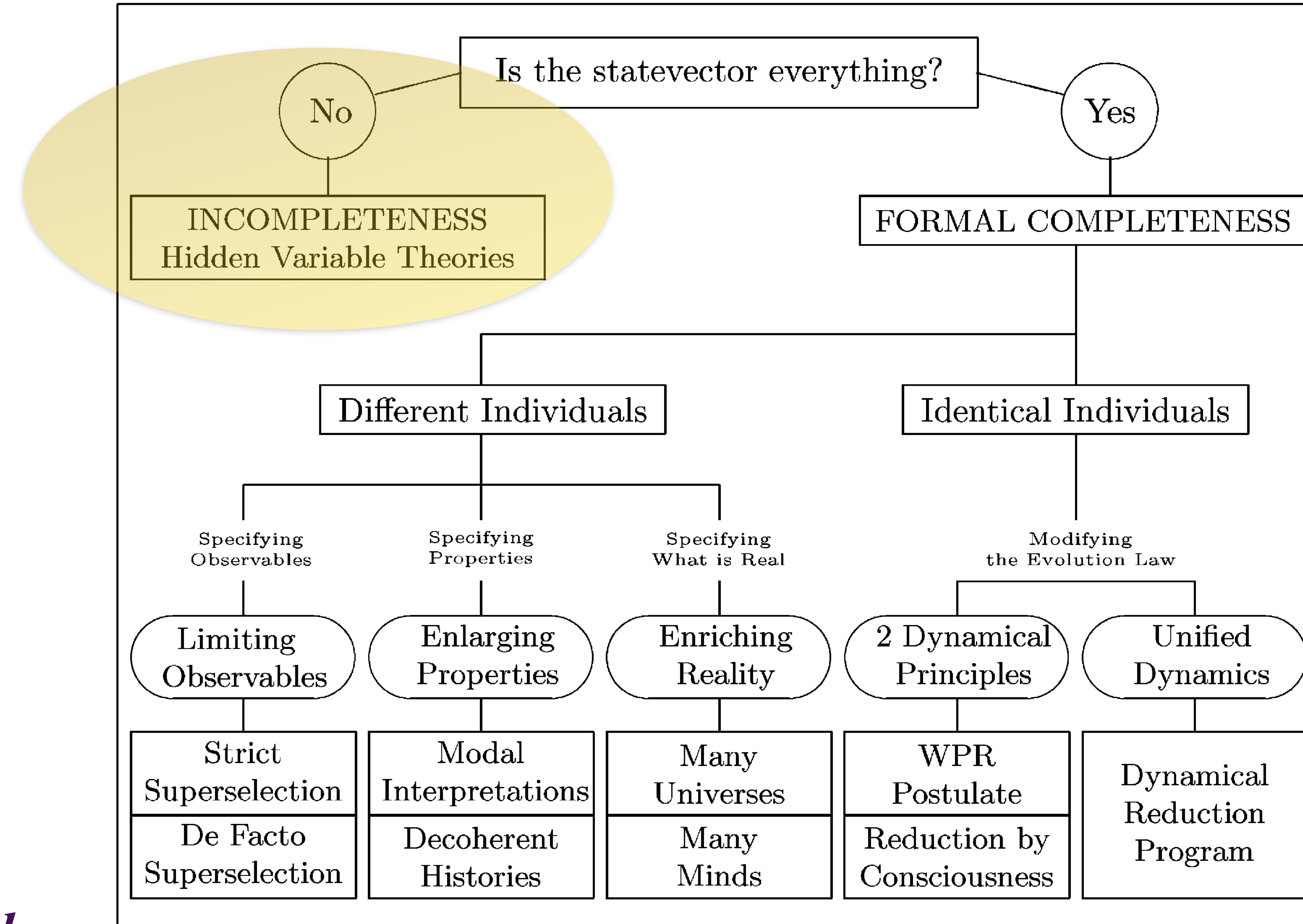
Stern-Gerlach



What about situations in which one has only one realization?

What do we do with the wave function of the Universe???

- Possible extensions and a criterion: the Born rule



A. Bassi & G.C. Ghirardi, *Phys. Rep.* **379**, 257 (2003)

▲ ***Superselection rules***

▲ ***Modal interpretation***

▲ ***Consistent histories***

▲ ***Many worlds / many minds***

▲ ***Hidden variables***

▲ ***Modified Schrödinger dynamics***

} Born rule not put by hand!

+ TESTABLE!

Hidden Variable Theories

Schrödinger $$i\frac{\partial\Psi}{\partial t}=\left[-\frac{\nabla^2}{2m}+V\left(\boldsymbol{r}\right)\right]\Psi$$

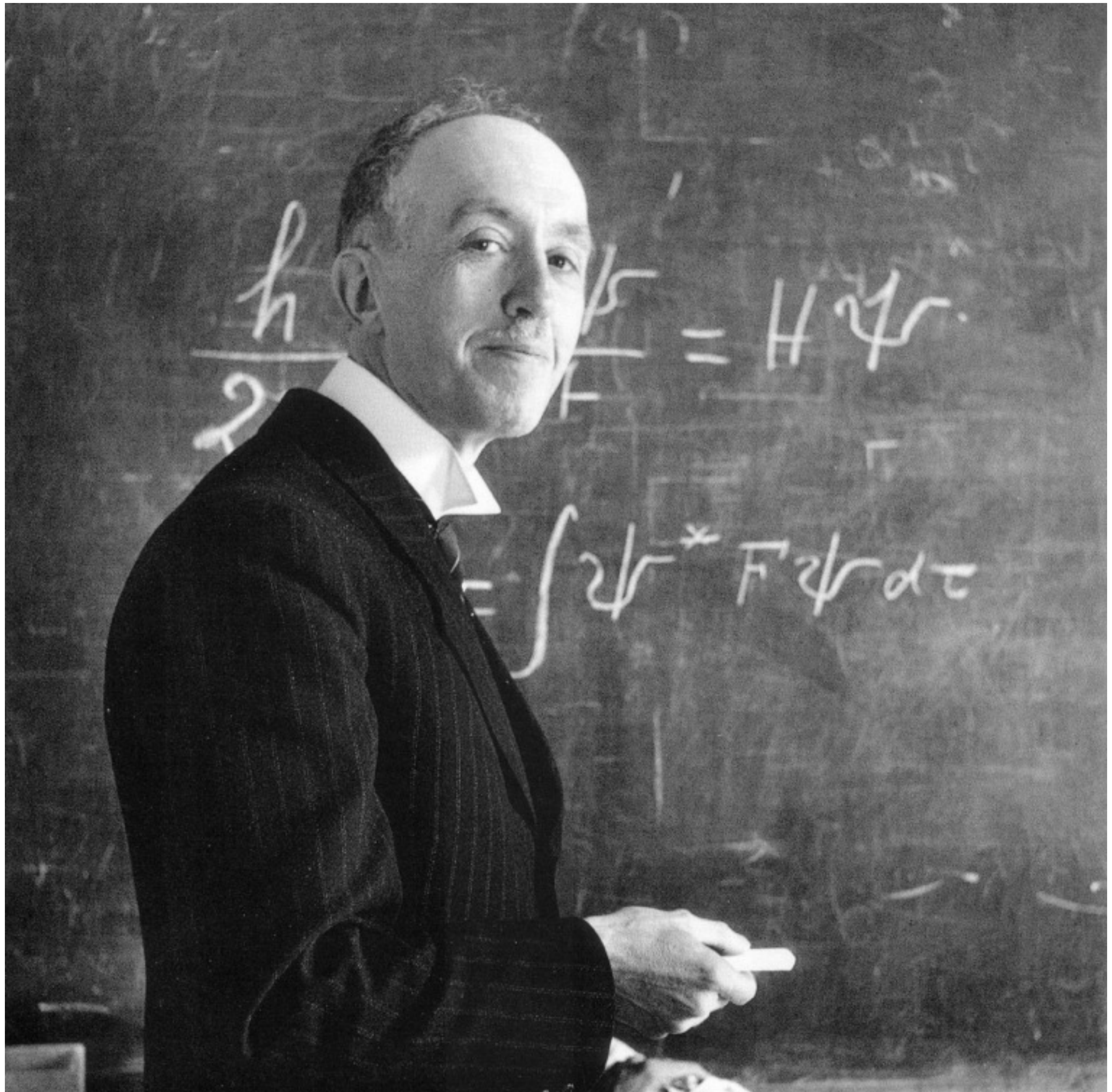
Polar form of the wave function $$\Psi=A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t}+\frac{\left(\nabla S\right)^2}{2m}+V\left(\boldsymbol{r}\right)+Q\left(\boldsymbol{r},t\right)=0$$

quantum potential

$$\equiv-\frac{1}{2m}\frac{\nabla^2 A}{A}$$

Ontological *formulation* (dBB)



Louis de Broglie (duke)

David Bohm (communist)

1927 Solvay meeting and von Neuman mistake ...   ‘*In 1952, I saw the impossible done*’ (J. Bell)

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS\left(\boldsymbol{r},t\right)}$$

Ontological *formulation* (dBB) $\exists\,\boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = \quad \boldsymbol{\nabla}S$$

$$\Psi = A(\boldsymbol{r}, t)\, \mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (BdB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (Bohm) $$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V + Q) \qquad Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$

$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$

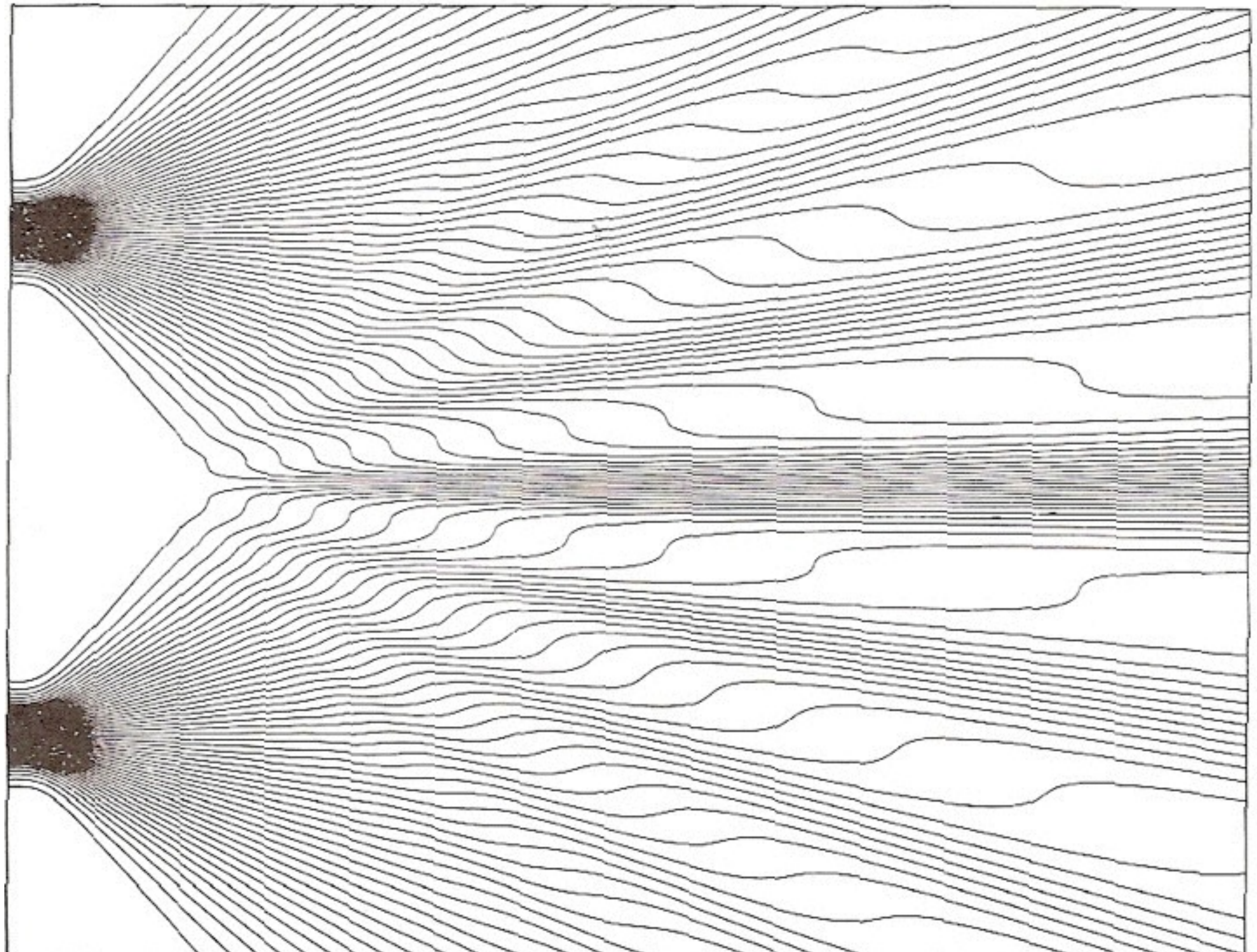
Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^*\boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = \boldsymbol{\nabla}S$$

Properties:

- strictly equivalent to Copenhagen QM
  - probability distribution (attractor) $\exists t_0; \rho\left(\boldsymbol{x},t_0\right) = \left|\Psi\left(\boldsymbol{x},t_0\right)\right|^2$
- classical limit well defined $Q \longrightarrow 0$ $\left(Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}\right)$
- state dependent
- $\exists$ intrinsic reality
  - non local …
- no need for external classical domain/observer!

The two-slit experiment:



Surrealistic trajectories?

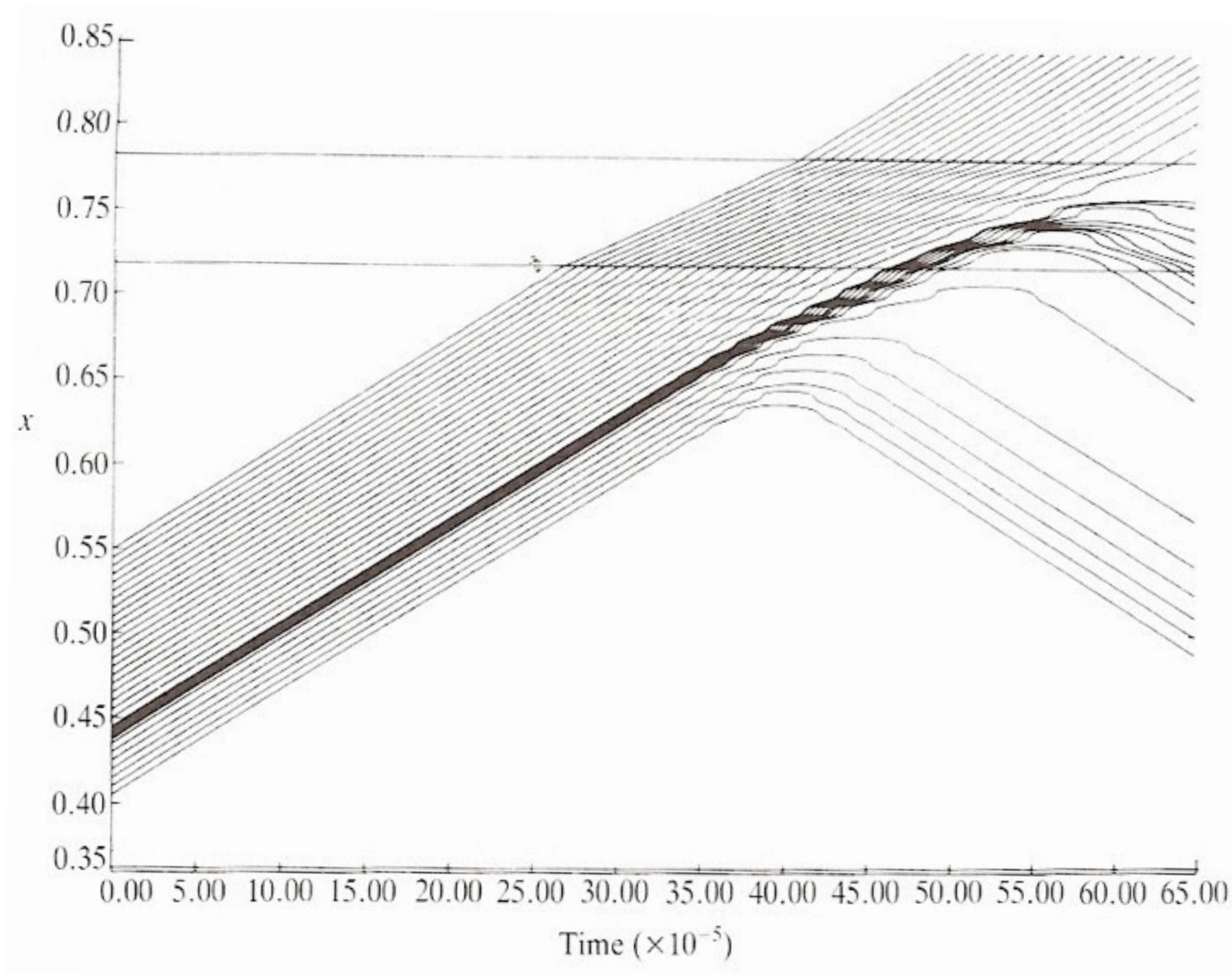
Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(X + Q\right)$$

... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

R. P. Feynman (1961)

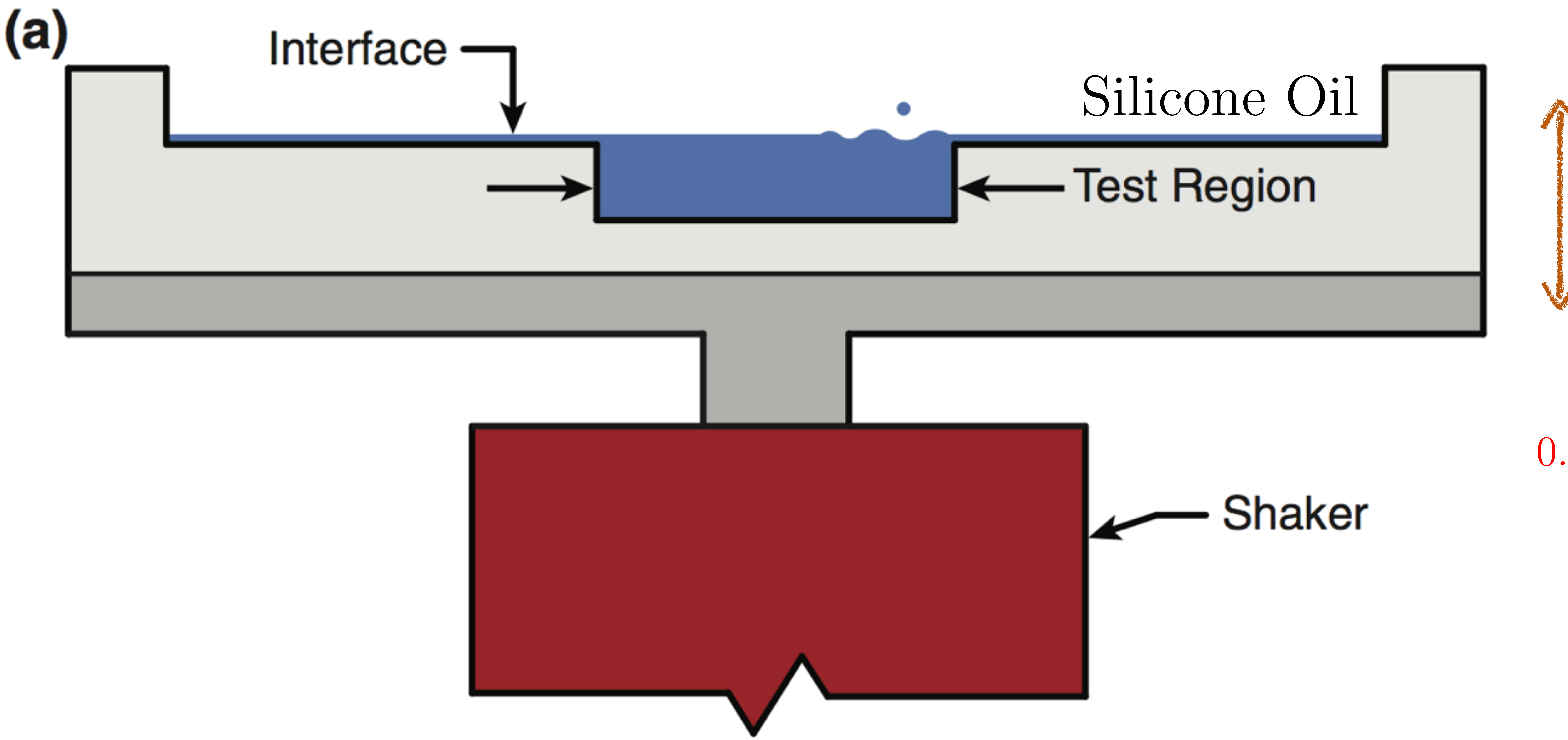
Diffraction by a potential simple understanding of tunnelling ...



Aside: a nice hydrodynamical analogy

Faraday waves… (1831)

forced standing surface waves



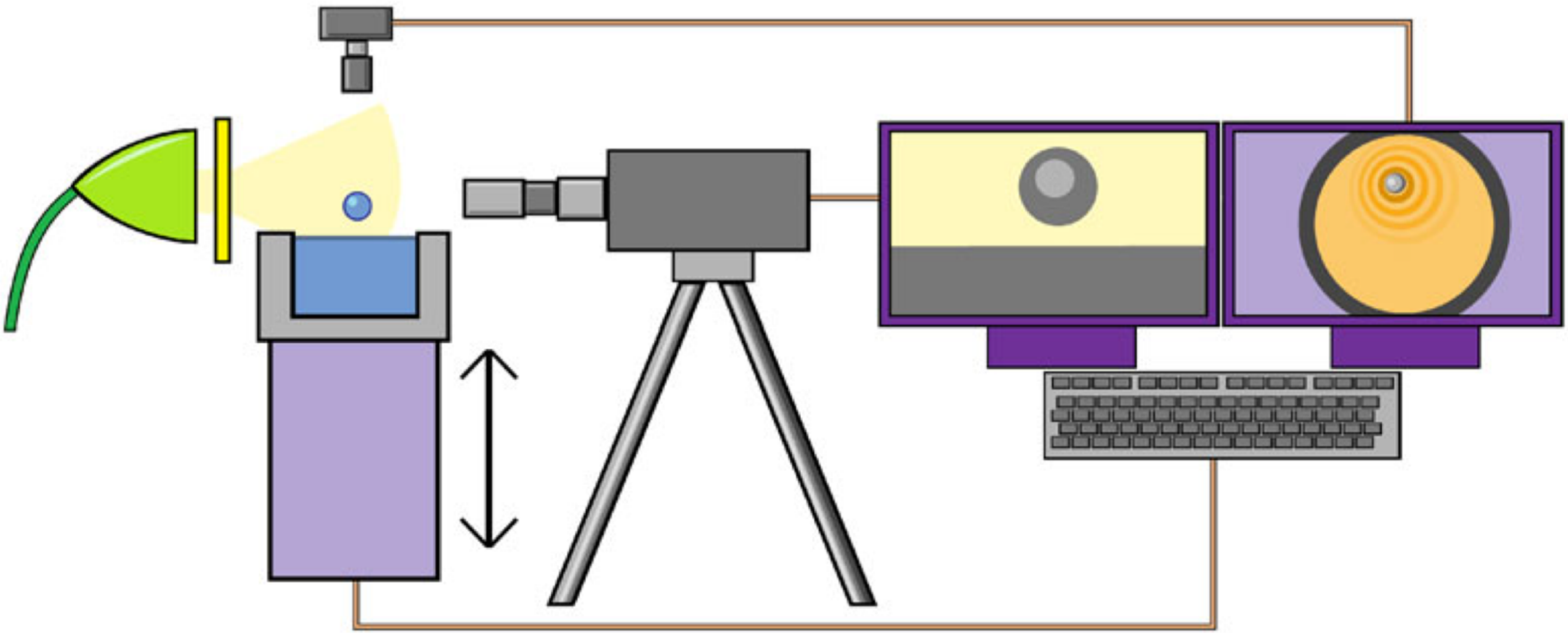
$A_0 \sin(2\pi f t)$

0.19 mm 70 Hz

Just above Faraday wave threshold

J. Walker (1978)
Y. Couder et al. (>2006)

http://math.mit.edu/~bush/?page_id=252



Typical values for the experiment

R_0	Drop radius	0.07–0.8 mm
ρ	Silicone oil density	949–960 kg m^{-3}
ρ_a	Air density	1.2 kg m^{-3}
σ	Drop surface tension	20–21 mN m^{-1}
g	Gravitational acceleration	9.81 m s^{-2}
V_{in}	Drop incoming speed	0.1–1 m s^{-1}
V_{out}	Drop outgoing speed	0.01–1 m s^{-1}
μ	Drop dynamic viscosity	10^{-3}–10^{-1} kg m^{-1} s^{-1}
μ_a	Air dynamic viscosity	1.84×10^{-5} kg m^{-1} s^{-1}
ν	Drop kinematic viscosity	10–100 cSt
ν_a	Air kinematic viscosity	15 cSt
T_C	Contact time	1–20 ms
C_R	$= V_{in}/V_{out}$ Coefficient of restitution	0–0.4
f	Bath shaking frequency	40–200 Hz
γ	Peak bath acceleration	0–70 m s^{-2}
ω	$= 2\pi f$ Bath angular frequency	250–1250 rad s^{-}1
ω_D	$= (\sigma/\rho R_0^3)^{1/2}$ Characteristic drop oscillation frequency	300–5000 s^{-1}
We	$= \rho R_0 V_{in}^2/\sigma$ Weber number	0.01–1
Bo	$= \rho g R_0^2/\sigma$ Bond number	10^{-3}–0.4
Oh	$= \mu(\sigma\rho R_0)^{-1/2}$ Drop Ohnesorge number	0.004–2
Oh_a	$= \mu_a(\sigma\rho R_0)^{-1/2}$ Air Ohnesorge number	10^{-4}–10^{-3}
Ω	$= 2\pi f\sqrt{\rho R_0^3/\sigma}$ Vibration number	0–1.4
Γ	$= \gamma/g$ Peak non-dimensional bath acceleration	0–7

Bouncing droplet…

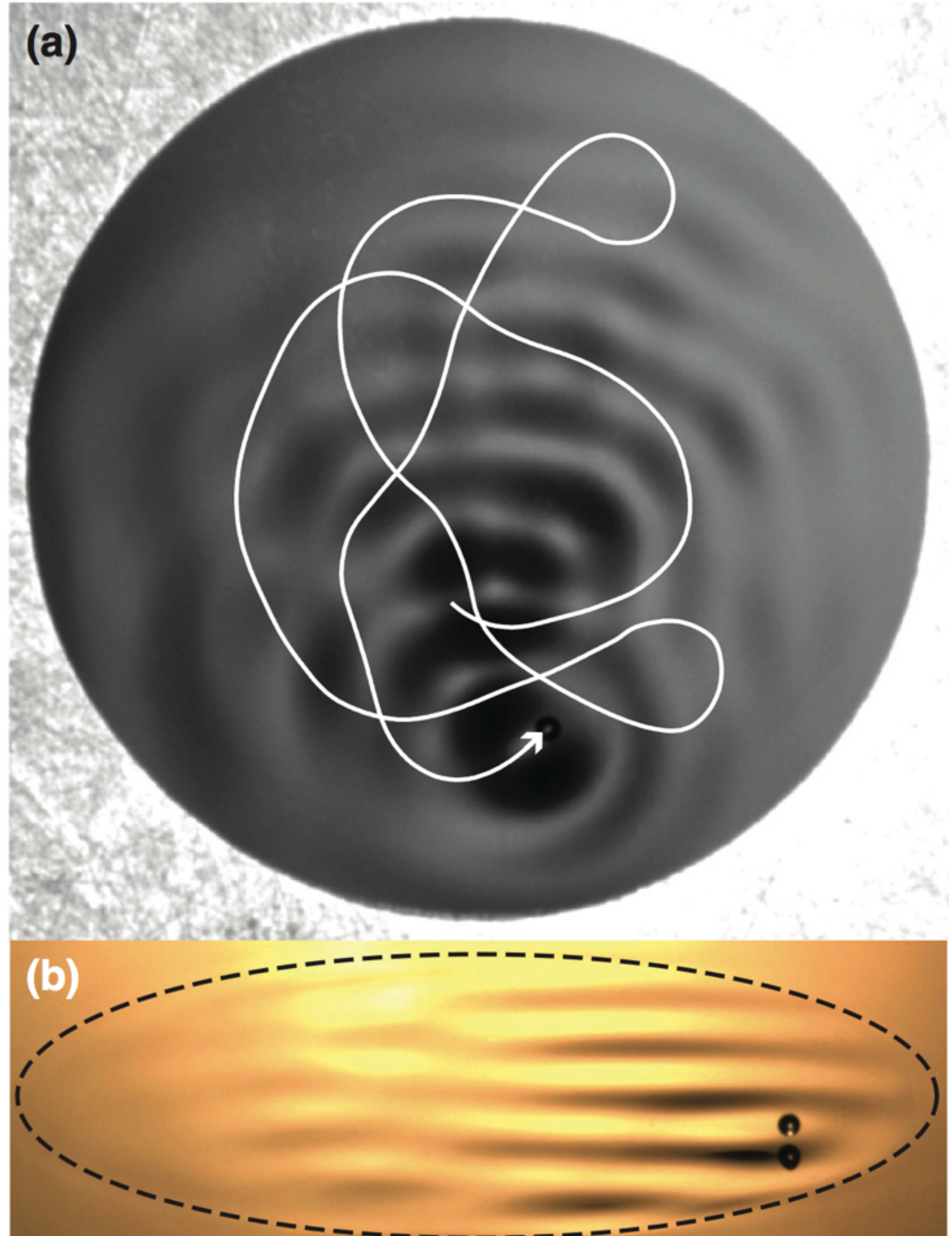
Bouncing droplet…

or bouncing droplets…

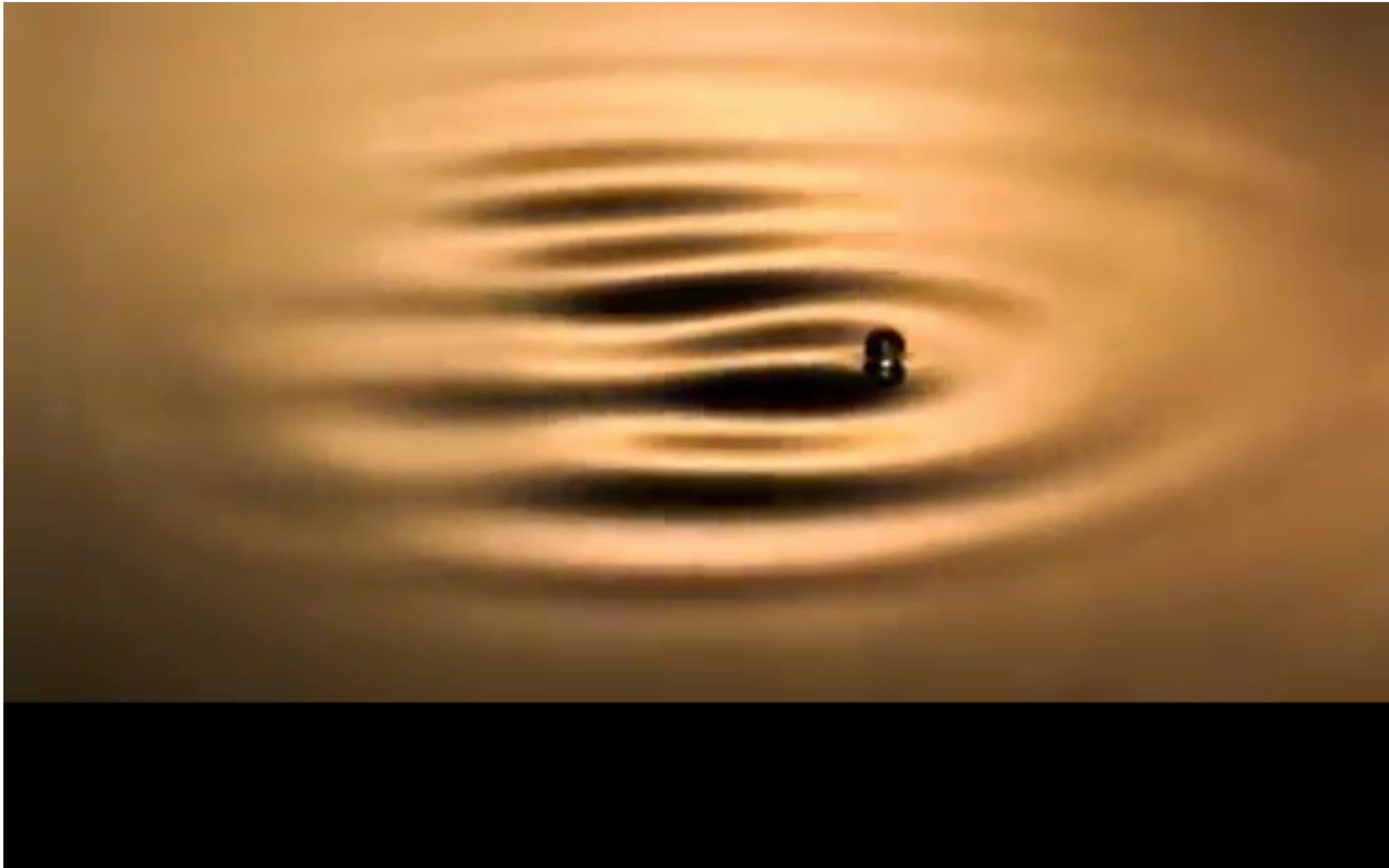
+ subharmonic modulation (larger forcing amplitude) => instability => motion!!!

droplets become walkers…

one image per bounce => suppress vertical motion => horizontal mode only



apparent randomness
of the motion…



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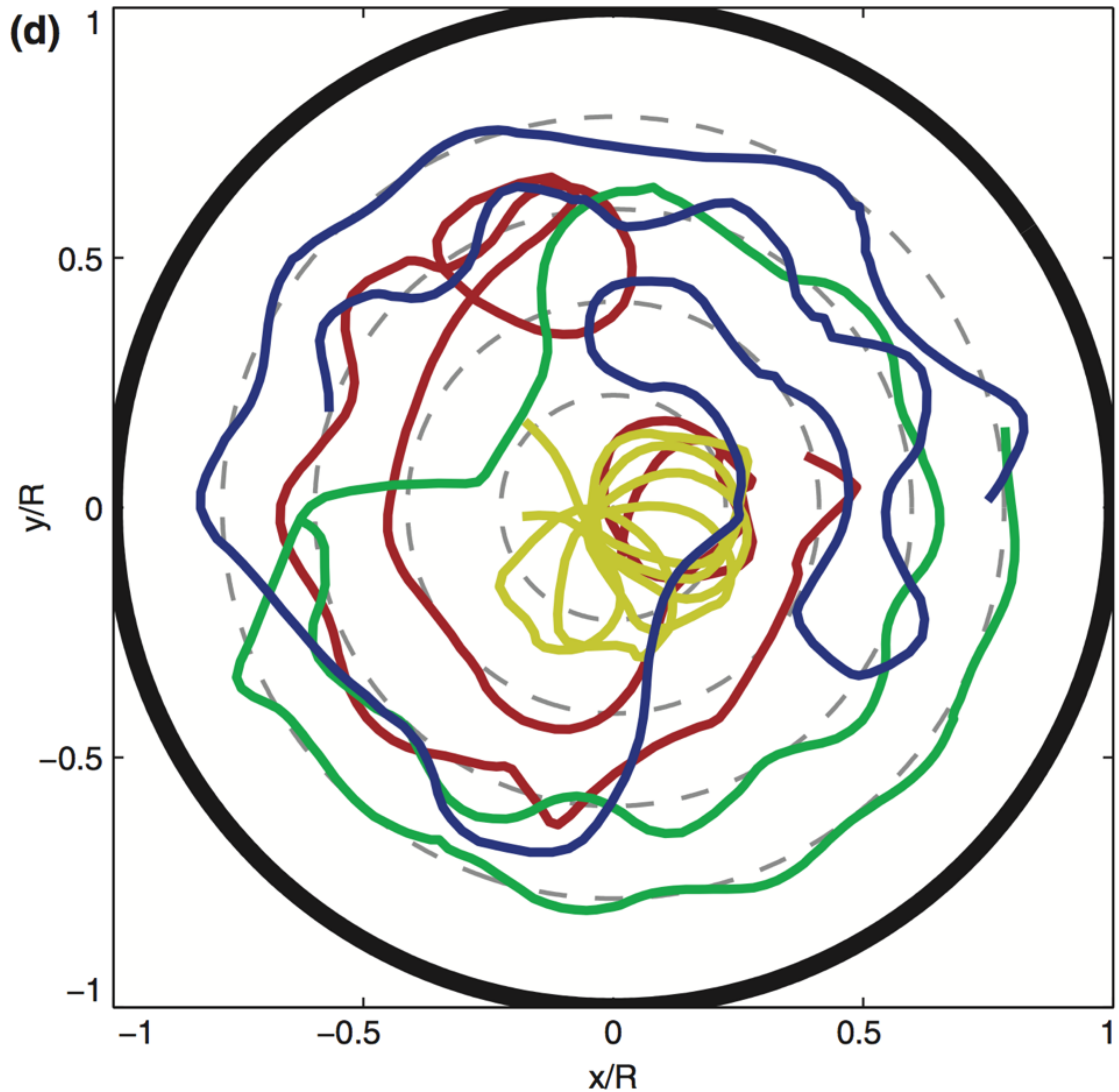
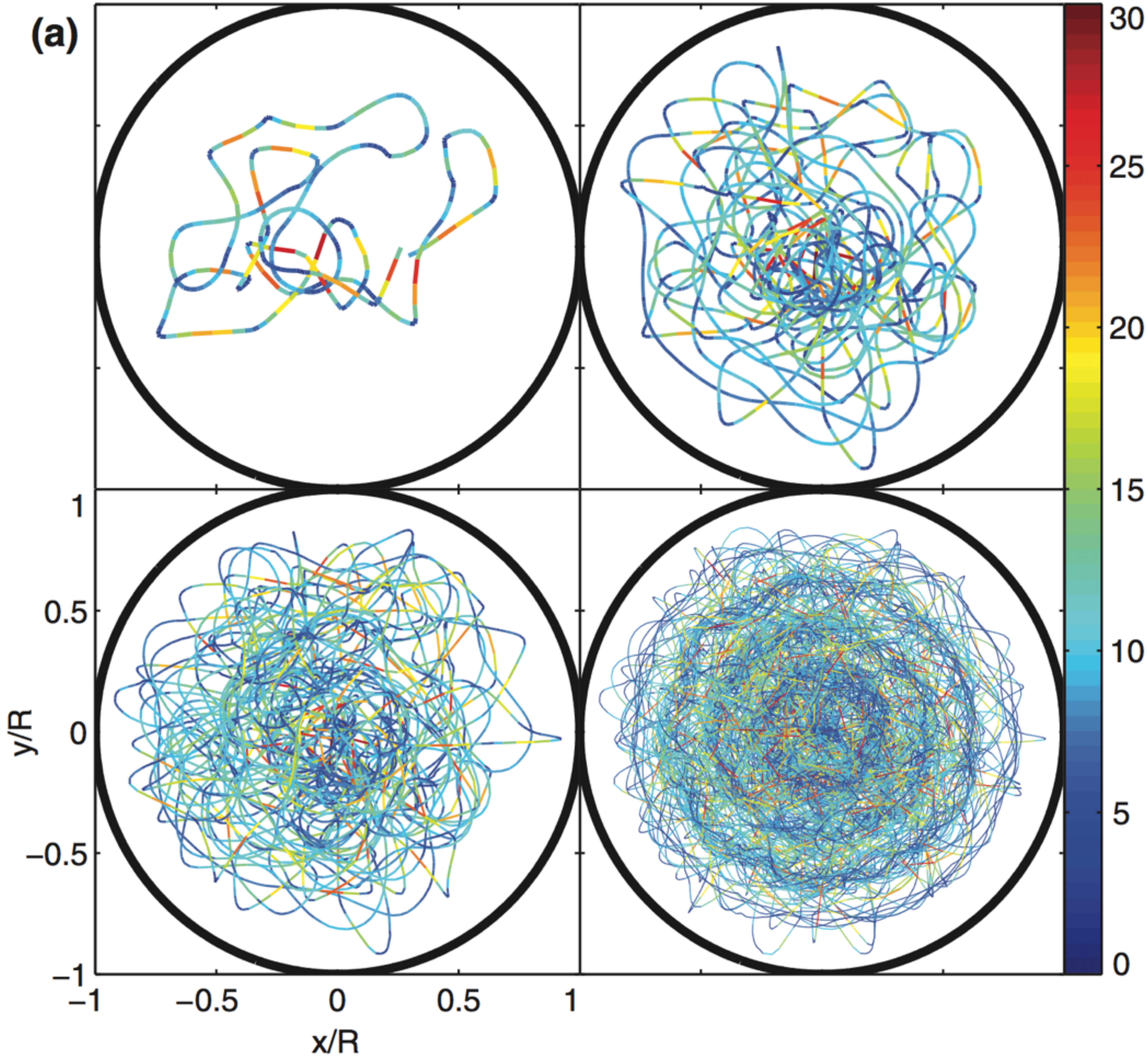
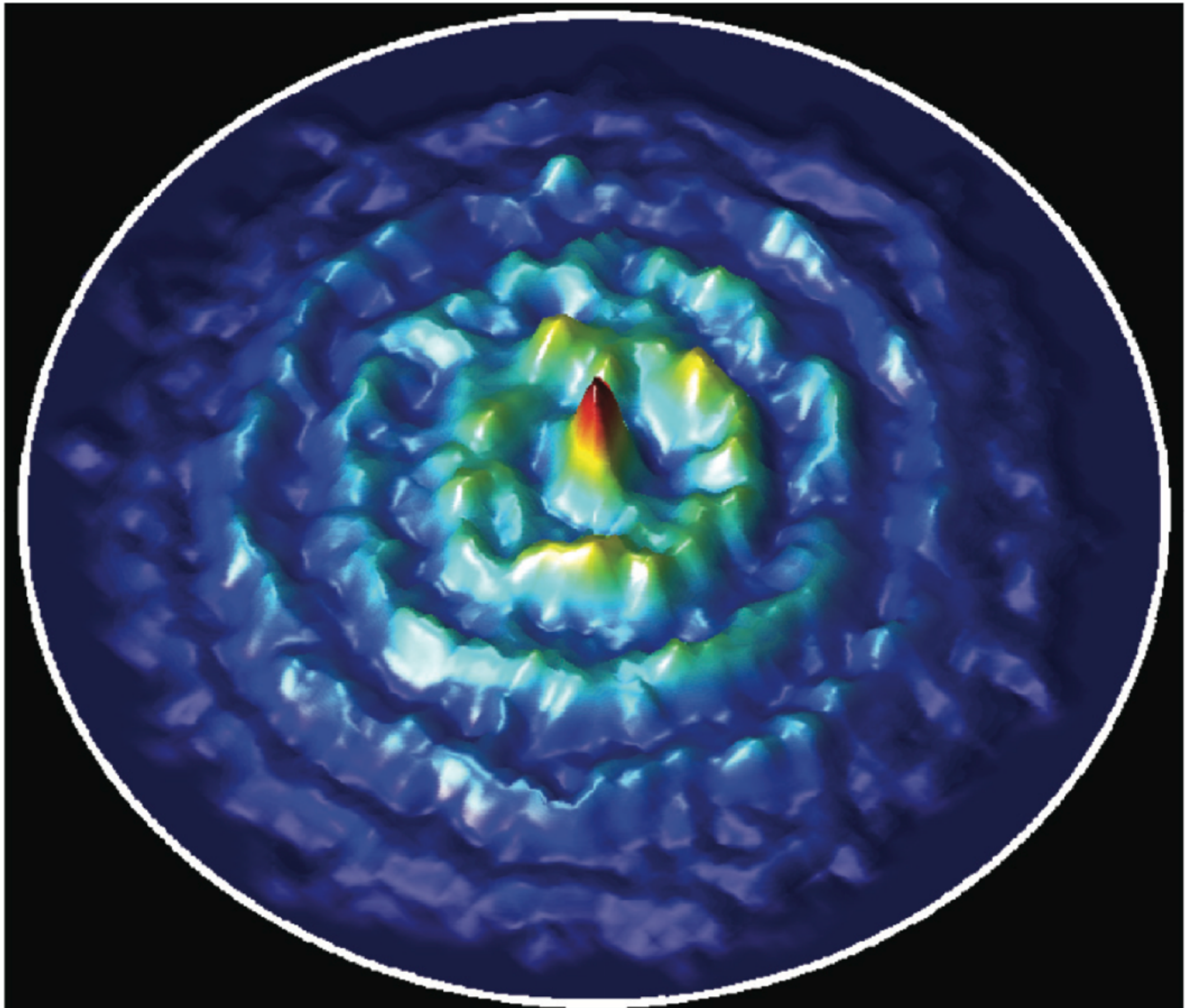


integrate over time…

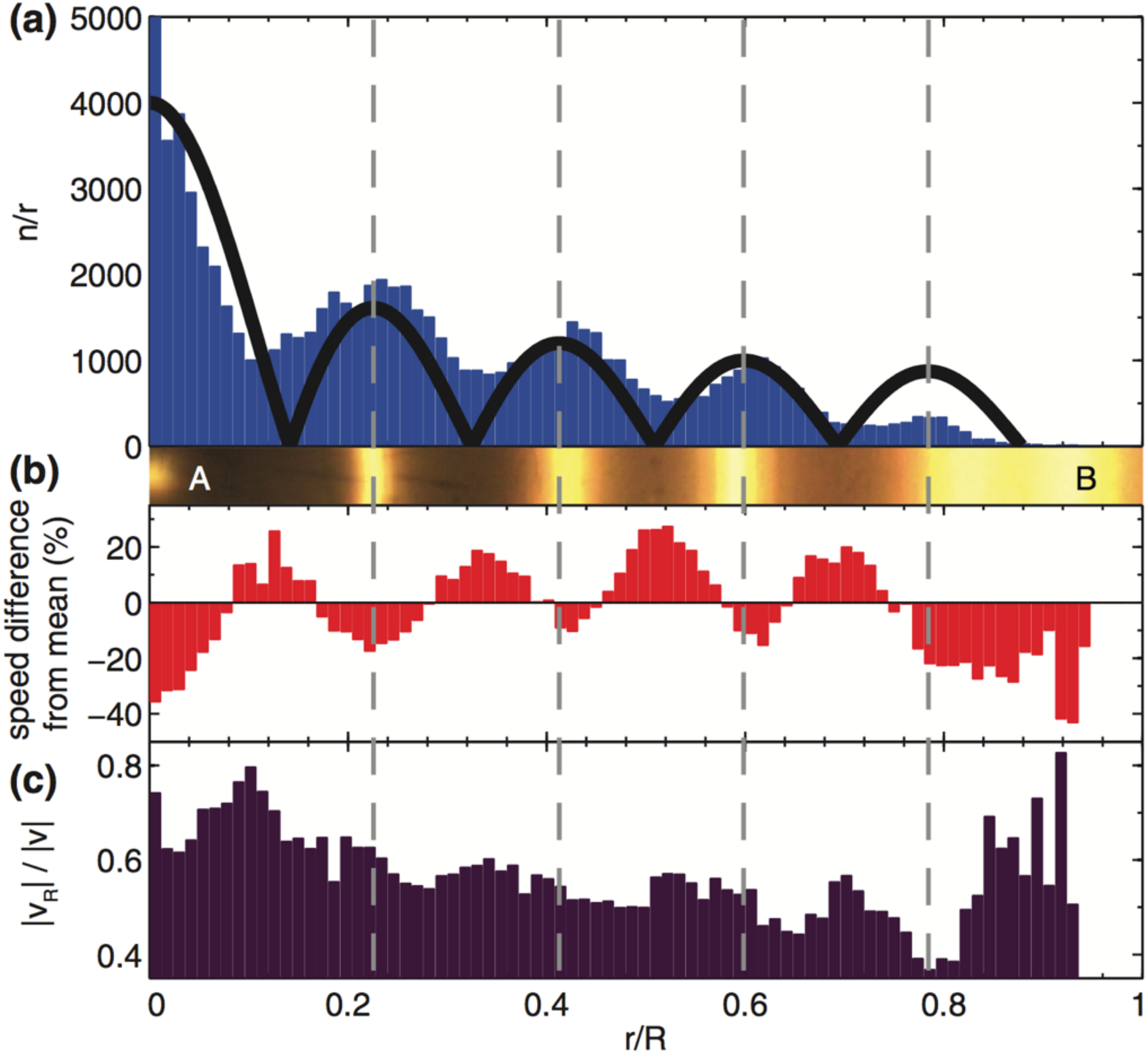


longer times…

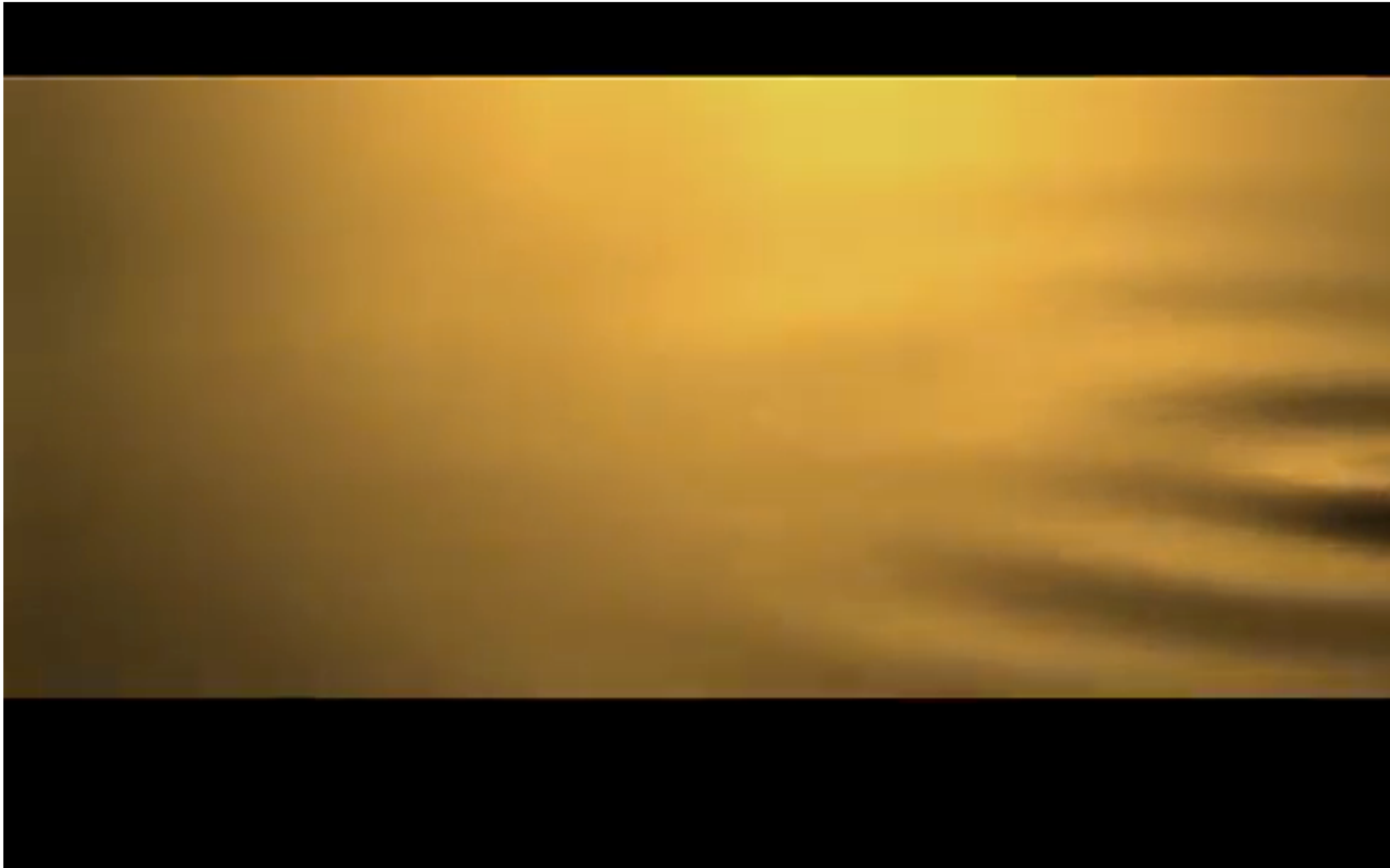
and reconstruct the standing wave pattern!

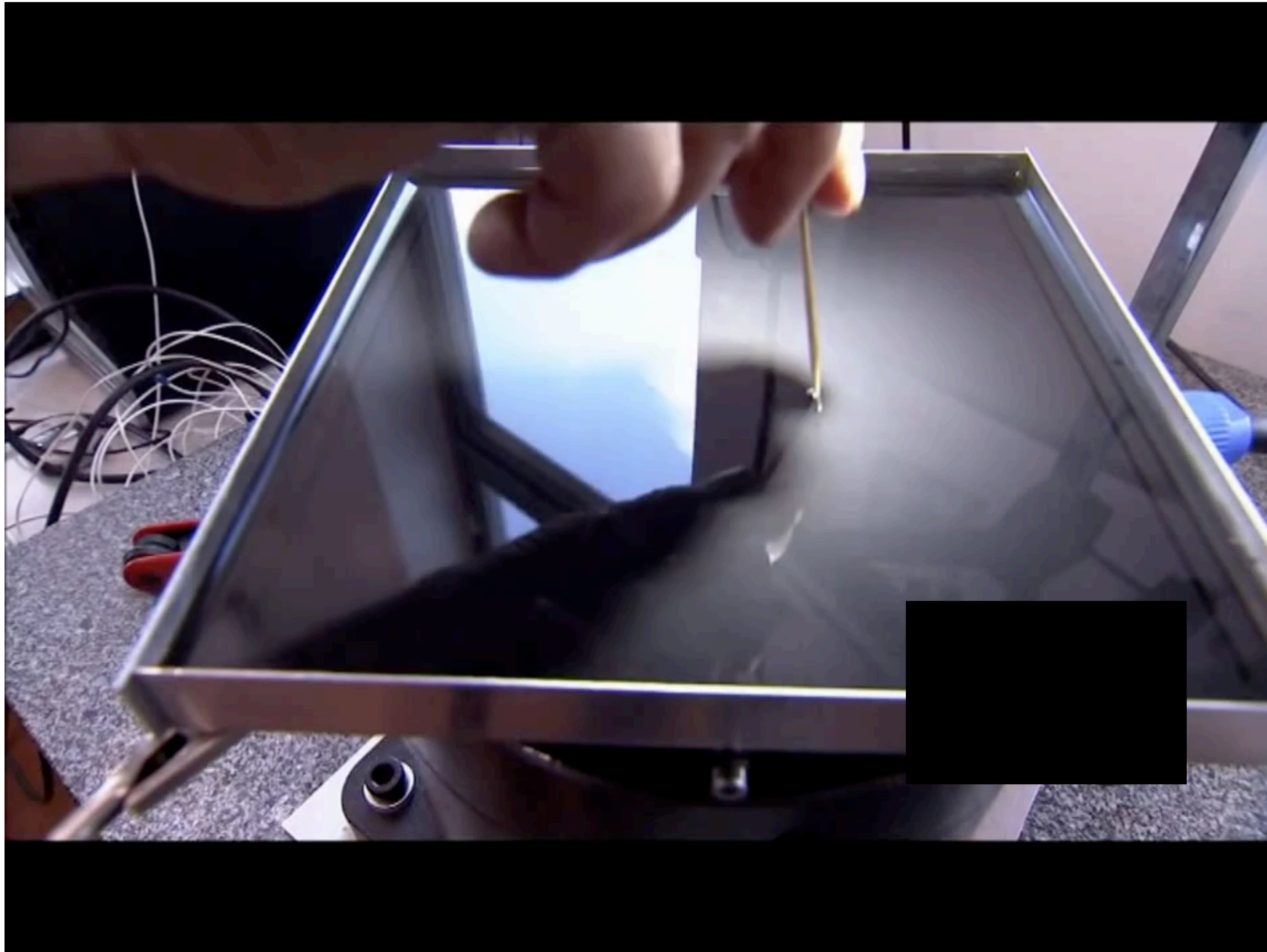


Probability
Distribution
Function



Comparison with actual Faraday wave pattern forms quantised bound states





Morgan Freeman's "Through the Wormhole" / Science Channel
Season II, episode 6 - How Does The Universe Work?

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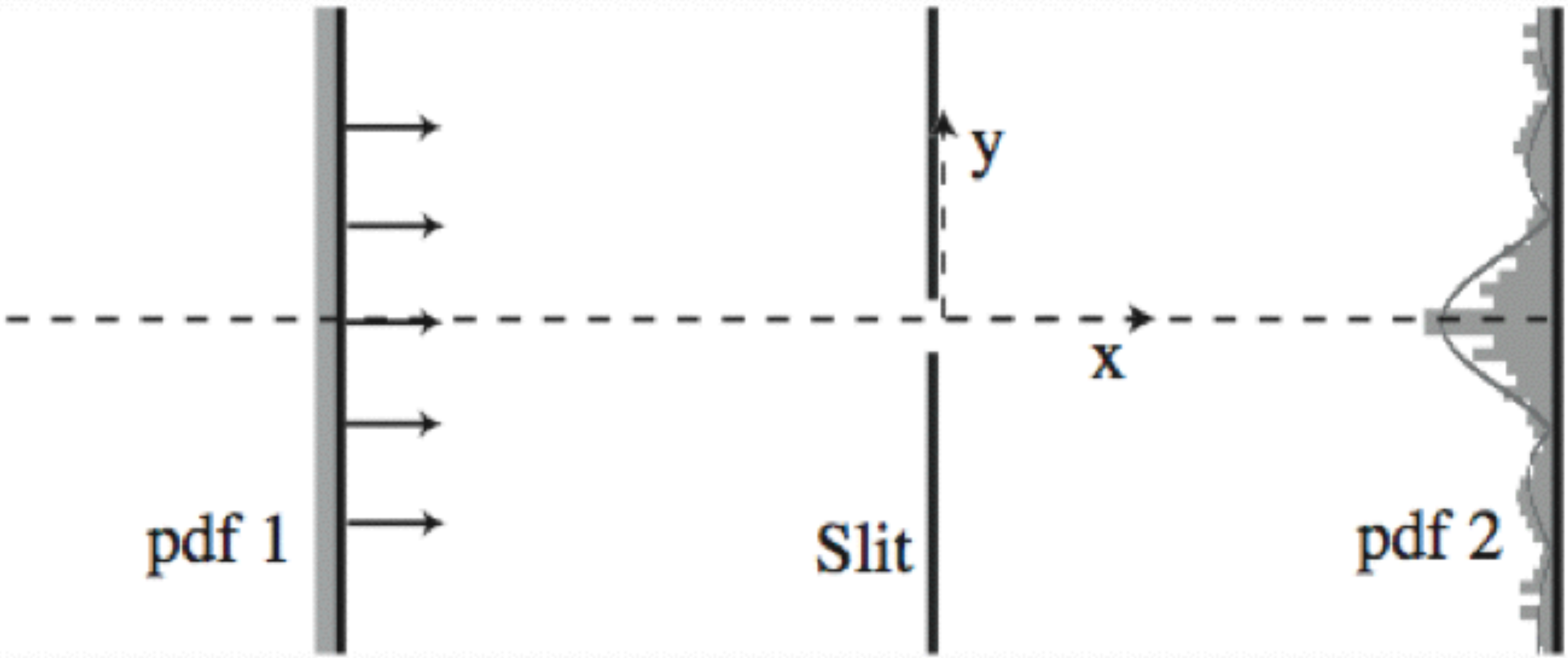
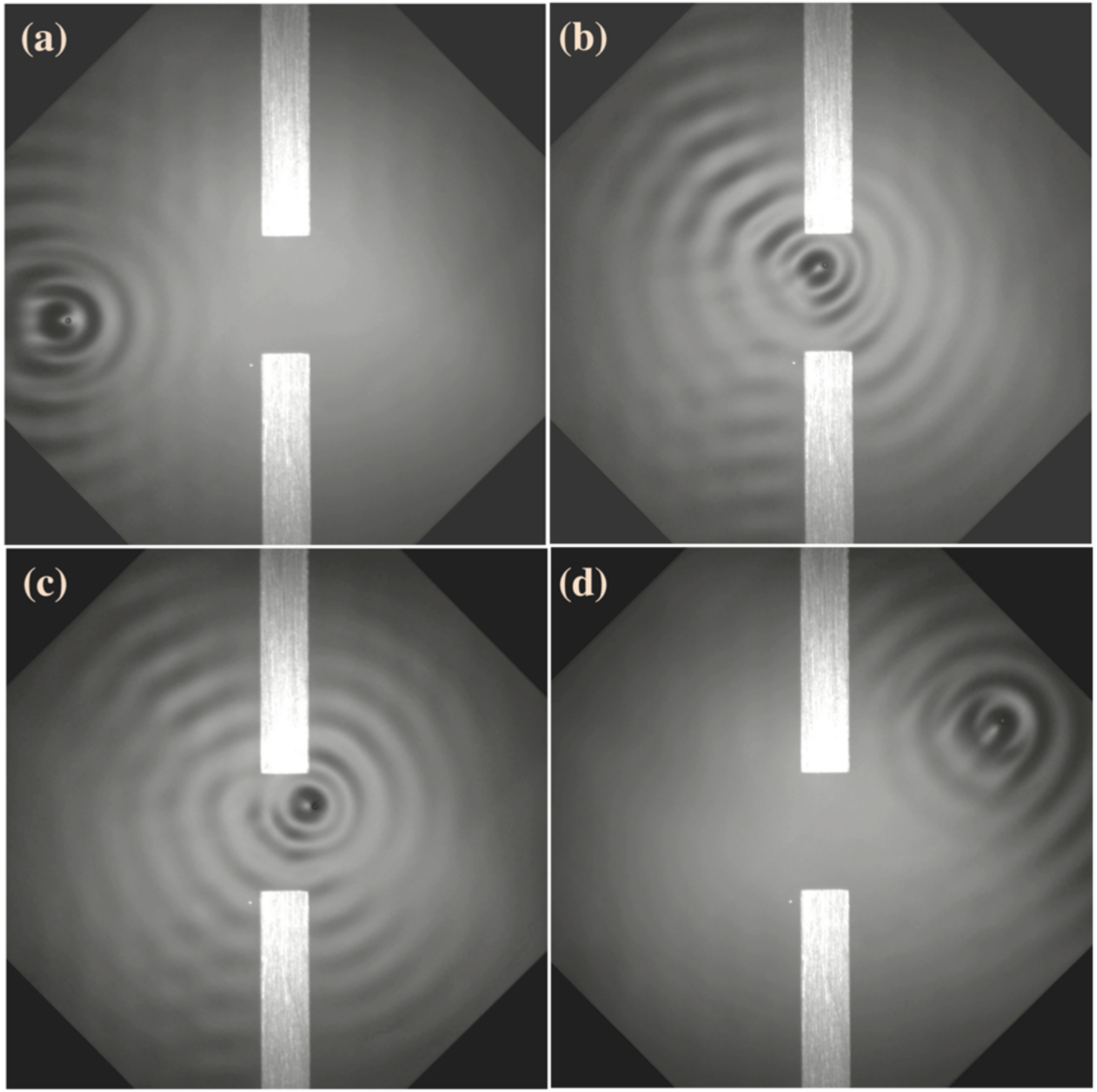
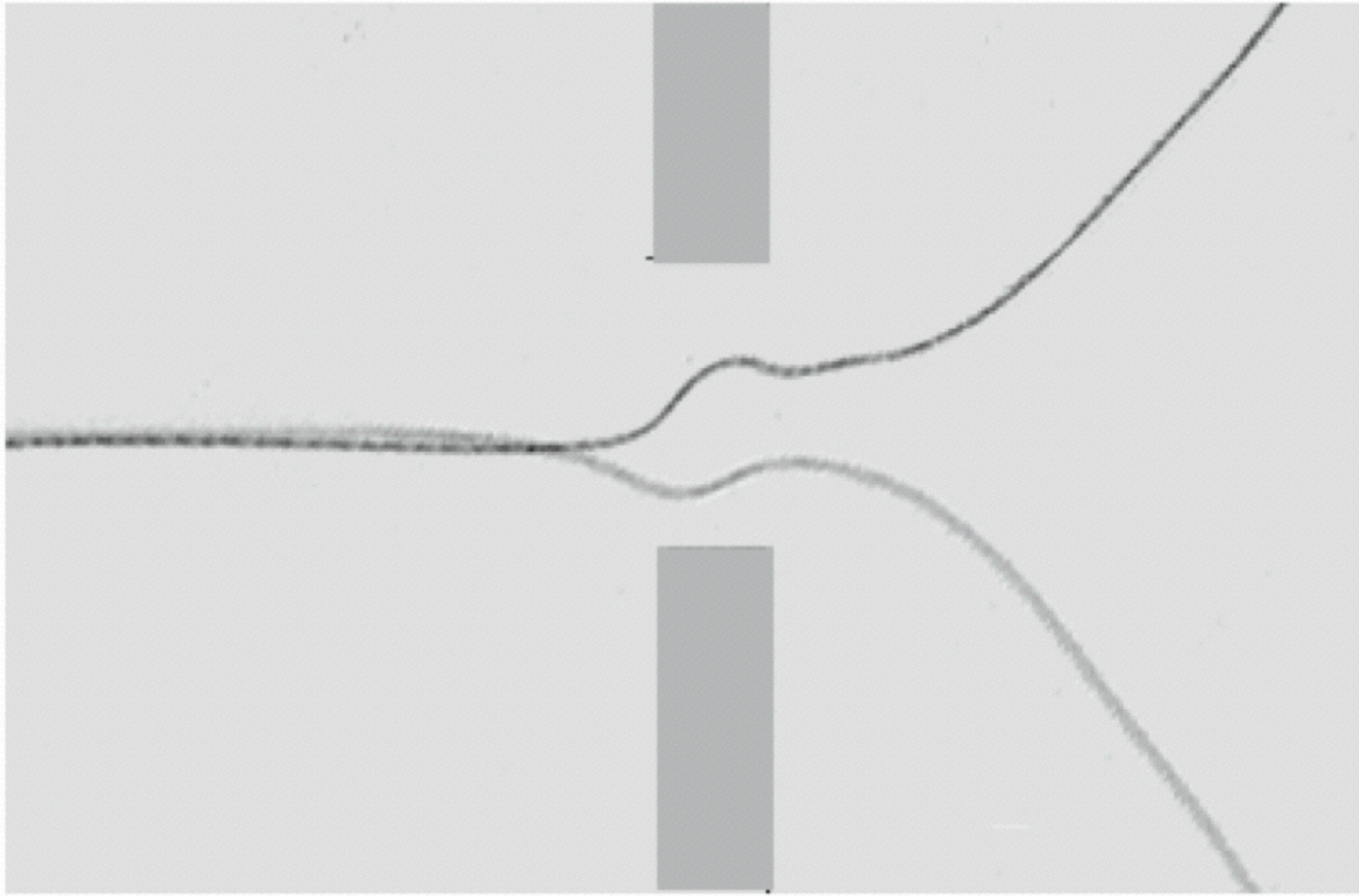
self-interfering classical particle!



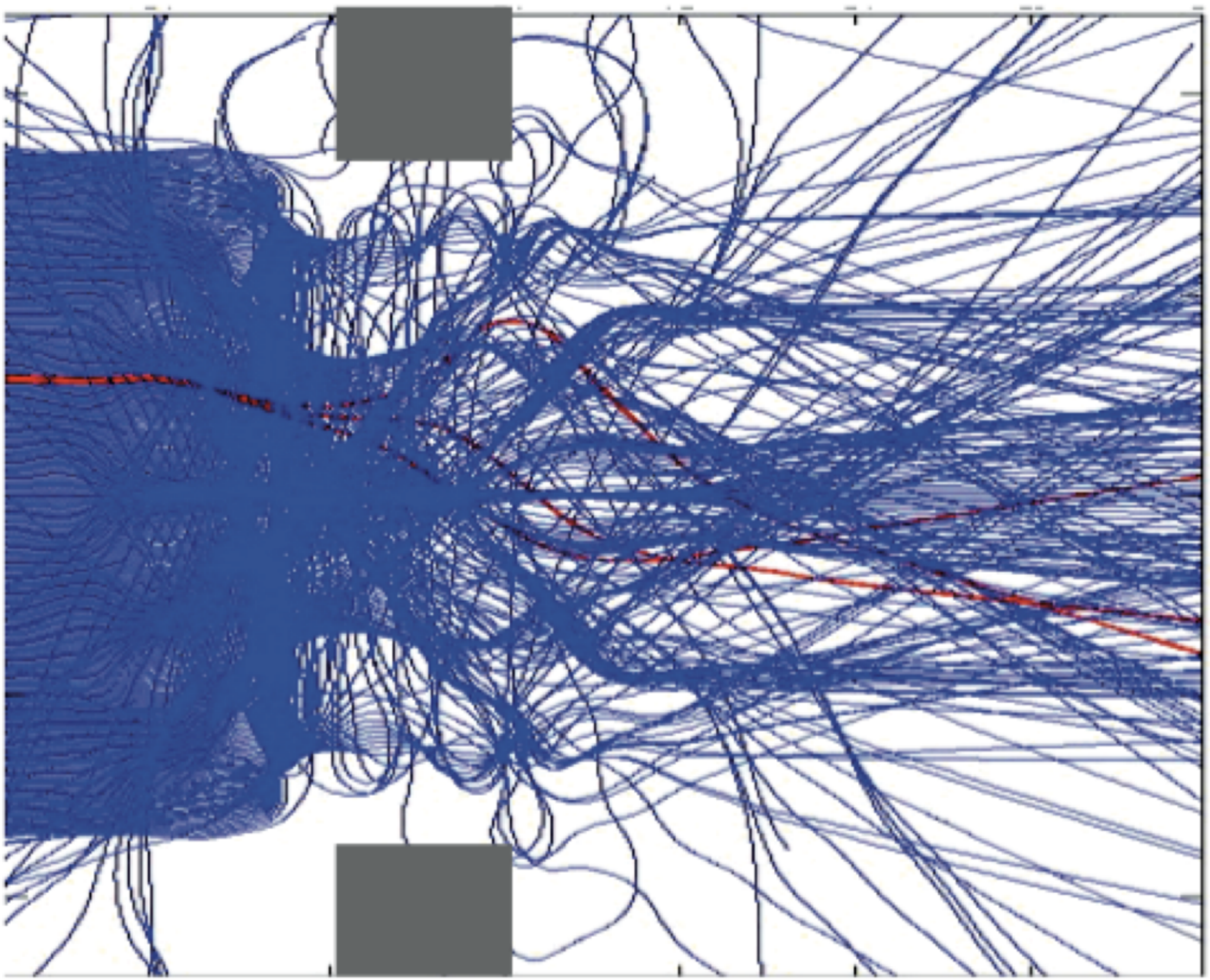
experimental setup



actual snapshots



a couple of trajectories…



apparently random again

more trajectories!

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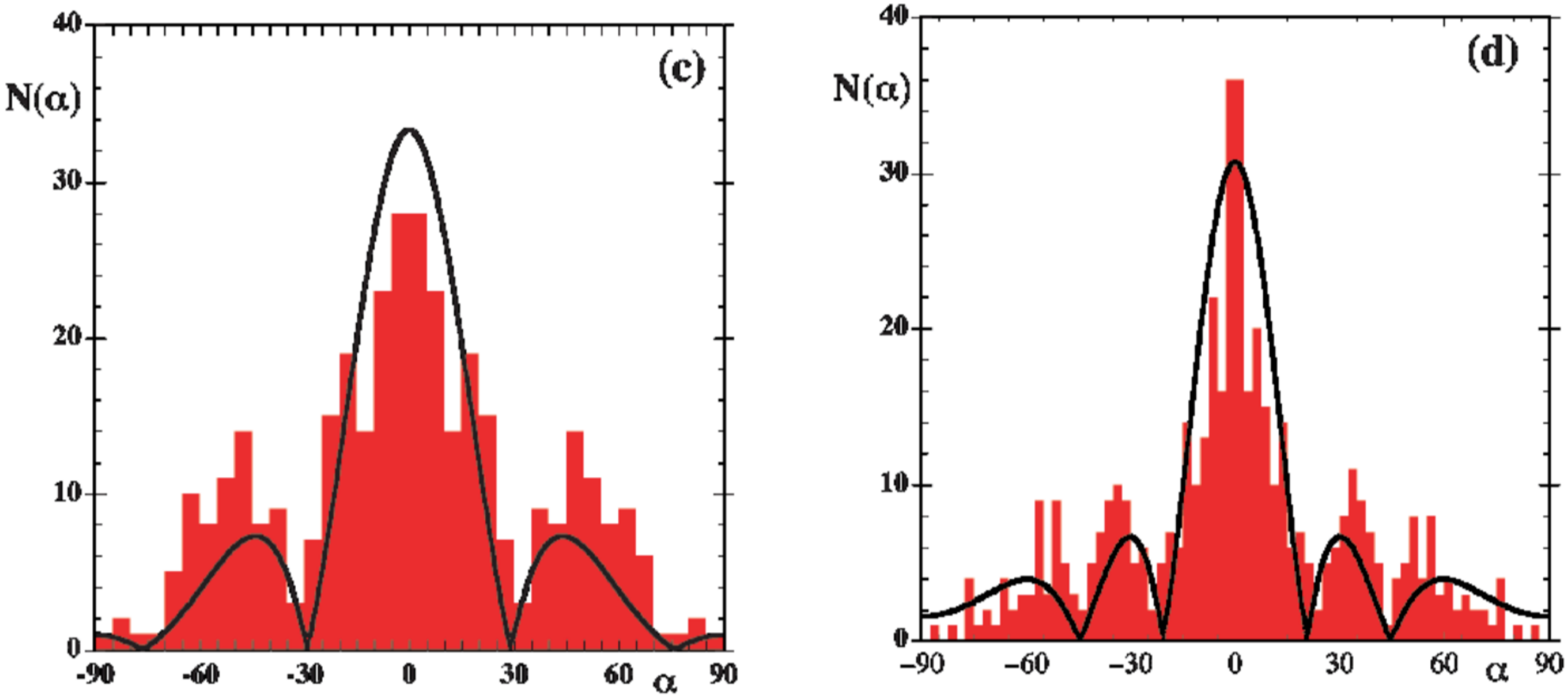
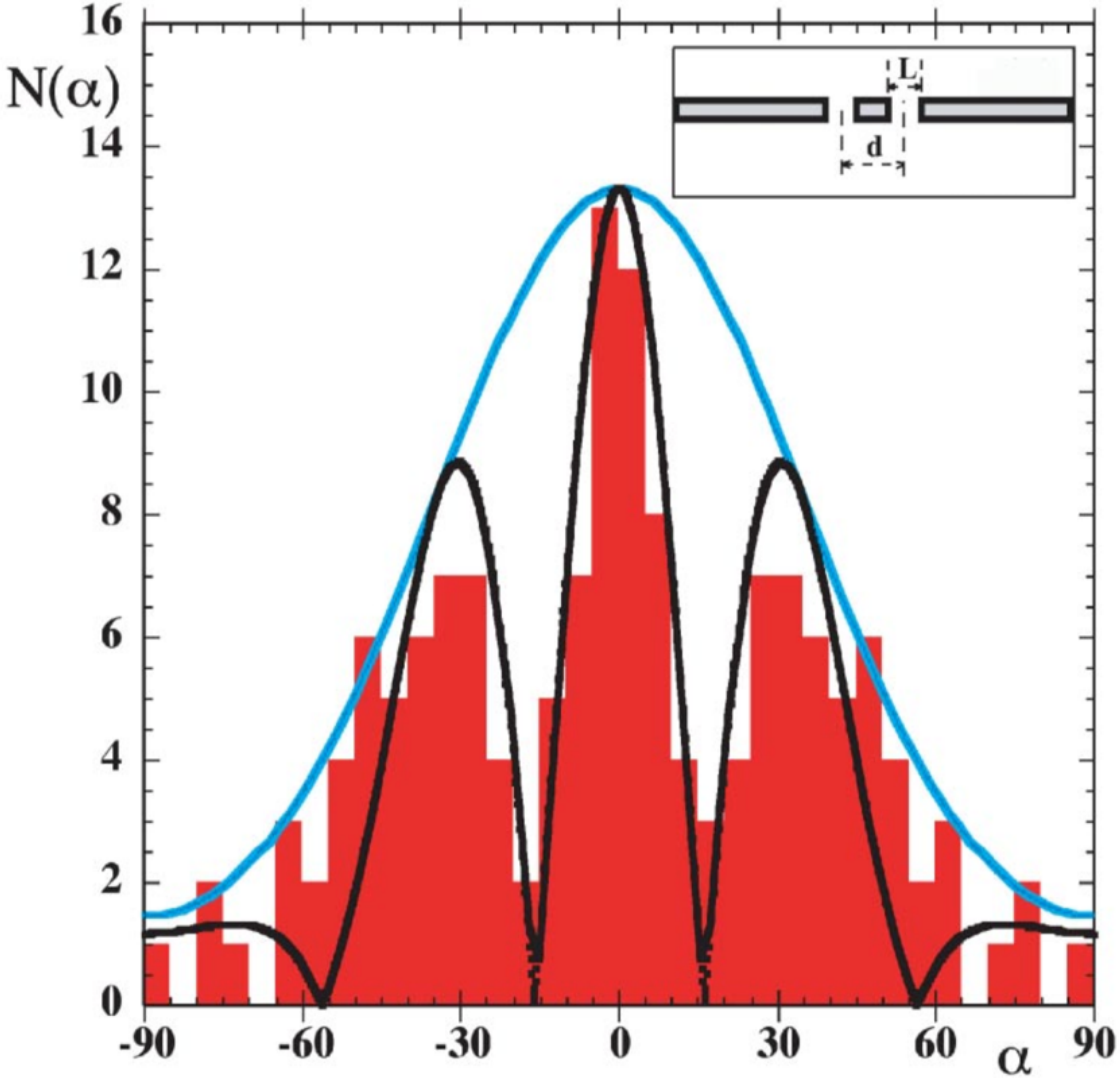
statistical determinacy



One slit + fit

Y. Couder and E. Fort, *Phys. Rev. Lett.* **97**, 154101 (2006)



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Tao slit + fit

... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

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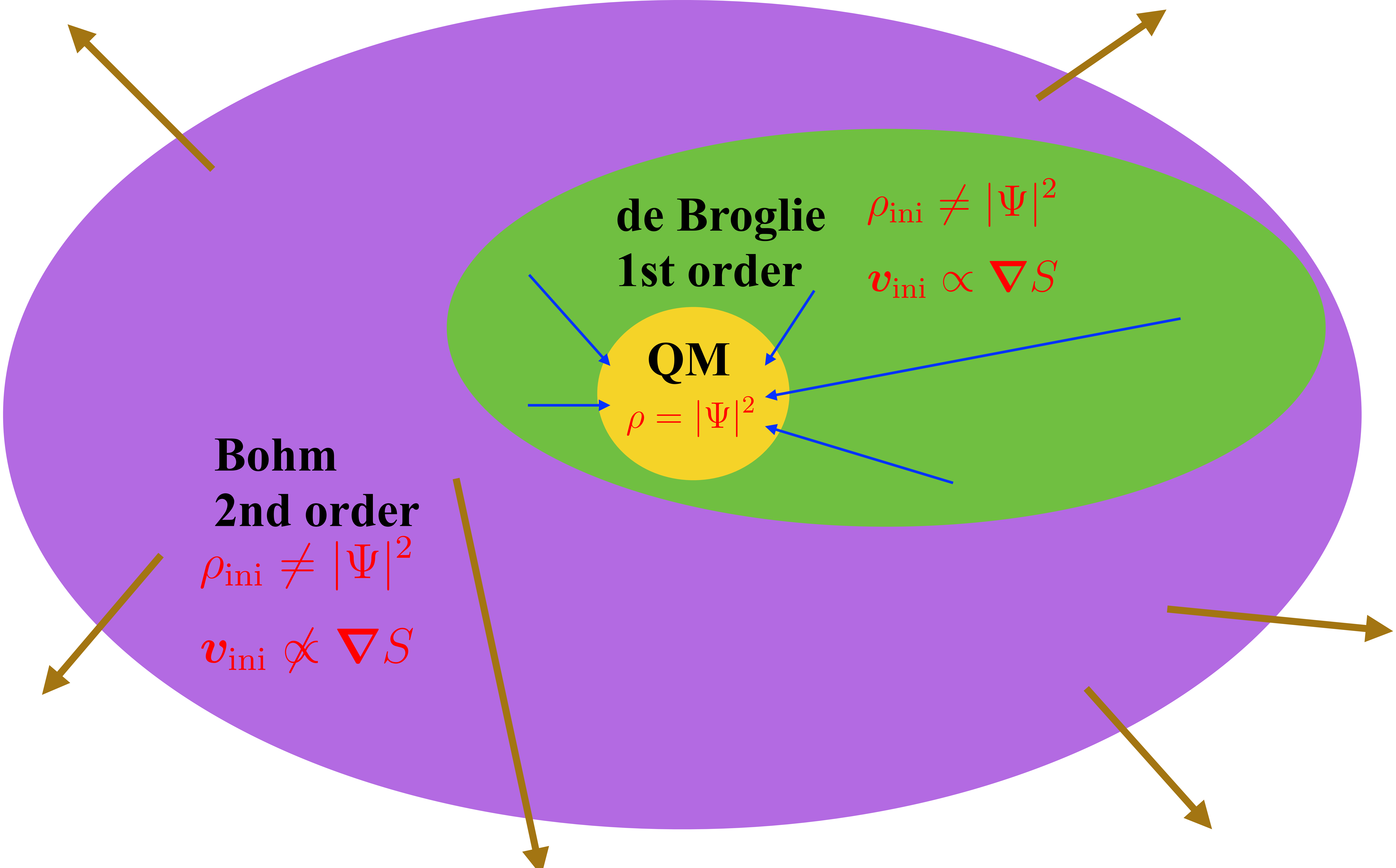

de Broglie
1st order
$\rho_{\text{ini}} \neq |\Psi|^2$
$\boldsymbol{v}_{\text{ini}} \propto \boldsymbol{\nabla} S$
QM
$\rho = |\Psi|^2$
Bohm
2nd order
$\rho_{\text{ini}} \neq |\Psi|^2$
$\boldsymbol{v}_{\text{ini}} \not\propto \boldsymbol{\nabla} S$

Quantum equilibrium

(Valentini & Westman, 2005)

$$i\frac{\mathrm{d}}{\mathrm{d}t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Particle in a box - 2D

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} - \frac{1}{2}\frac{\partial^2\psi}{\partial y^2} + V\psi$$

infinite square well - size π

Density of actual configurations

$$\rho\left(x,y,t\right) \quad \Longrightarrow \frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho\dot{x}\right) + \frac{\partial}{\partial y}\left(\rho\dot{y}\right) = 0$$

continuity equation

Energy eigenfunctions $\phi_{mn}\left(x,y\right) = \frac{2}{\pi}\sin\left(mx\right)\sin\left(ny\right)$

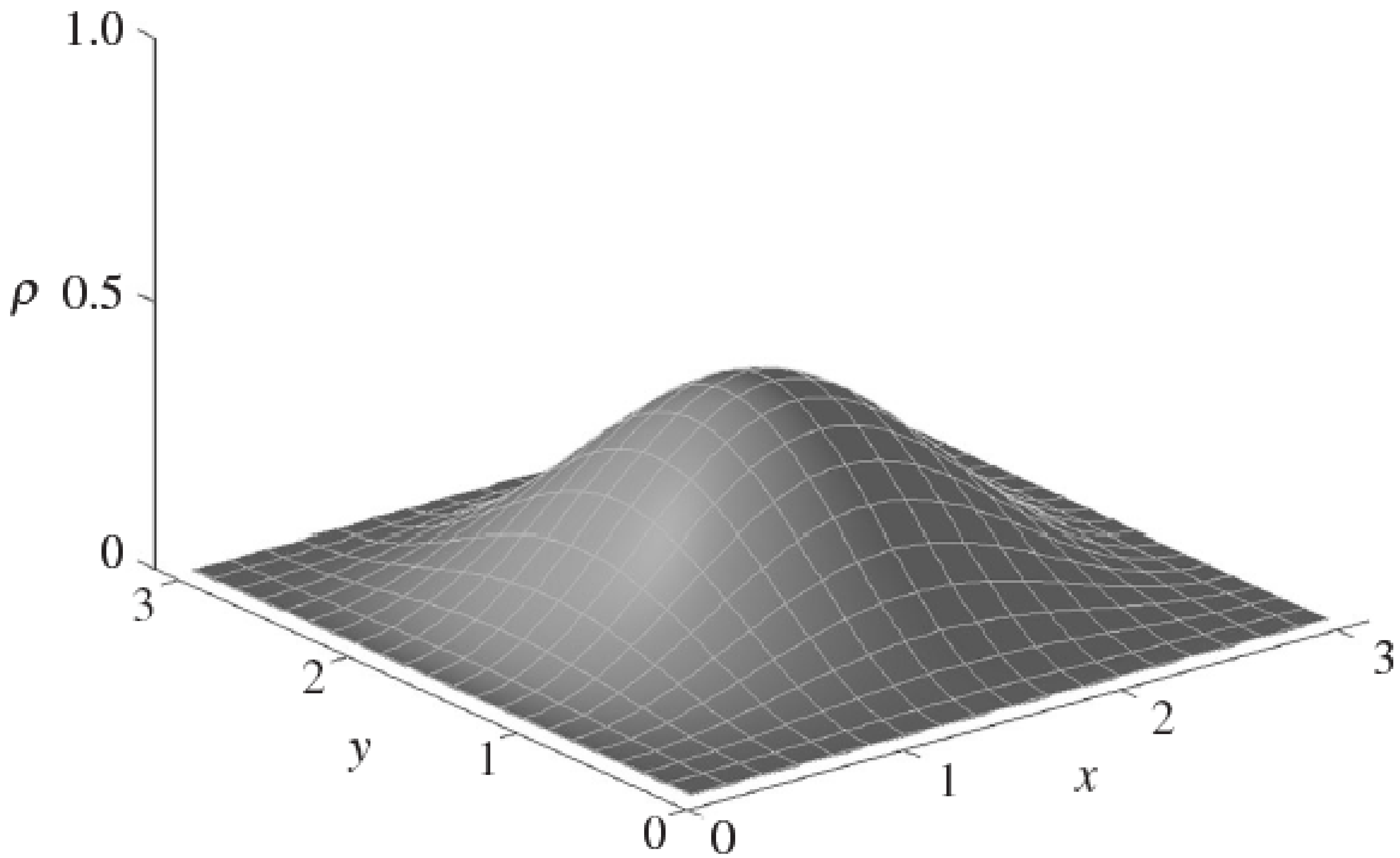
Energy levels $E_{mn} = \frac{1}{2}\left(m^2+n^2\right)$

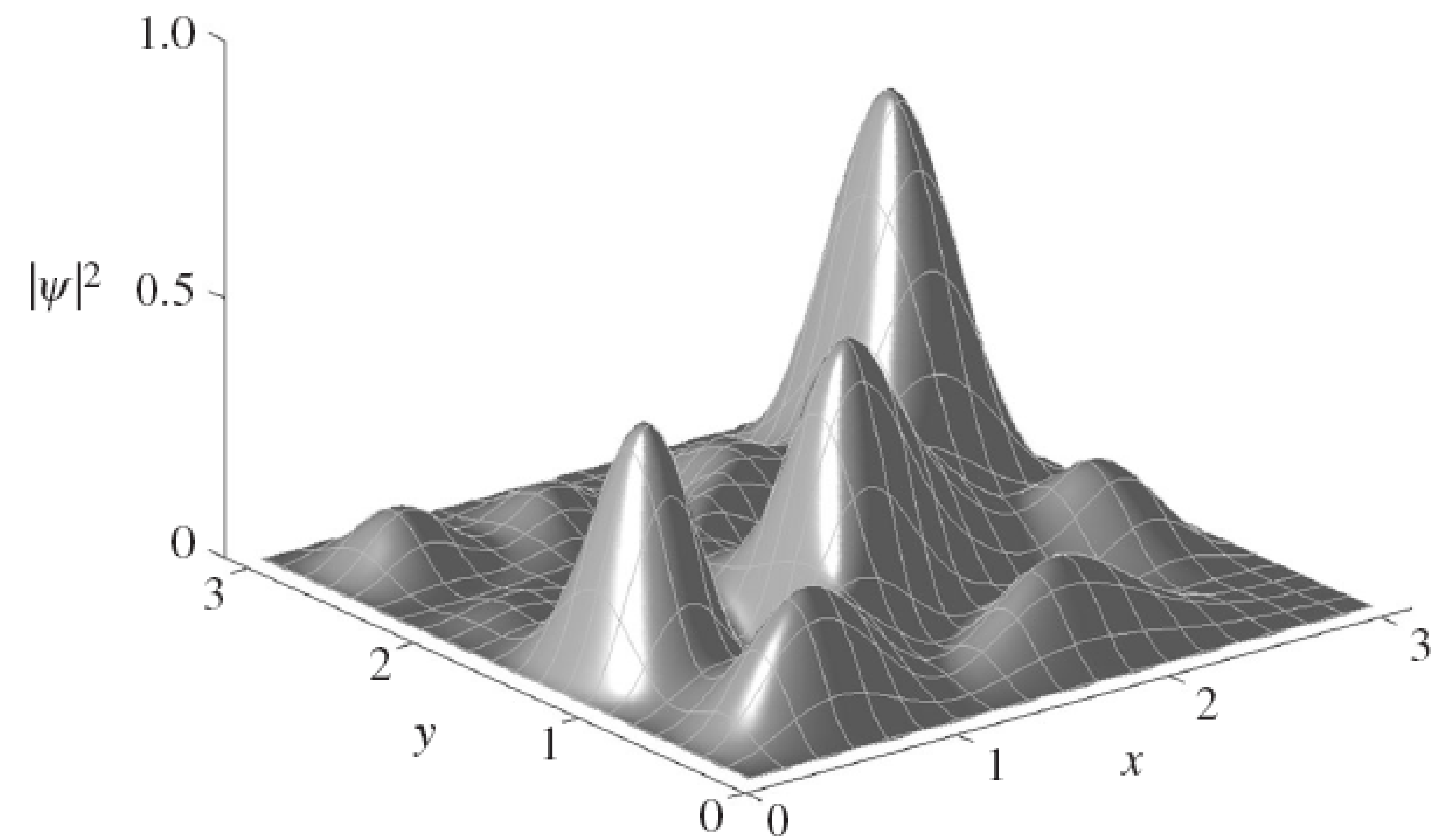
Initial configuration

$$\rho(x,y,0) = |\phi_{11}(x,y)|^2$$

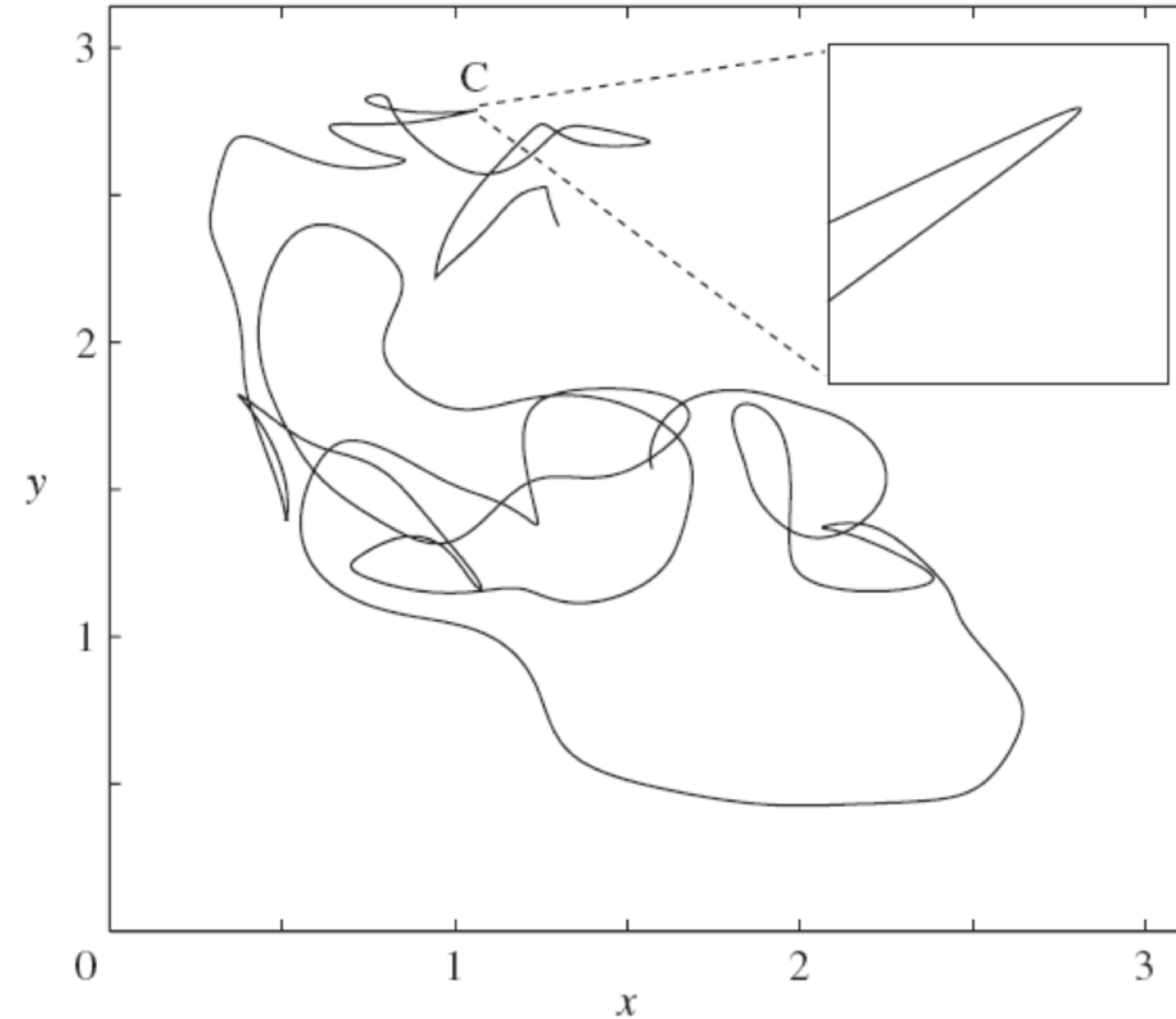
$$\psi(x,y,0) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp(i\theta_{mn})$$

$$\psi(x,y,t) = \sum_{m,n=1}^{4} \frac{1}{4}\phi_{mn}(x,y)\exp i(\theta_{mn} - E_{mn}t)$$

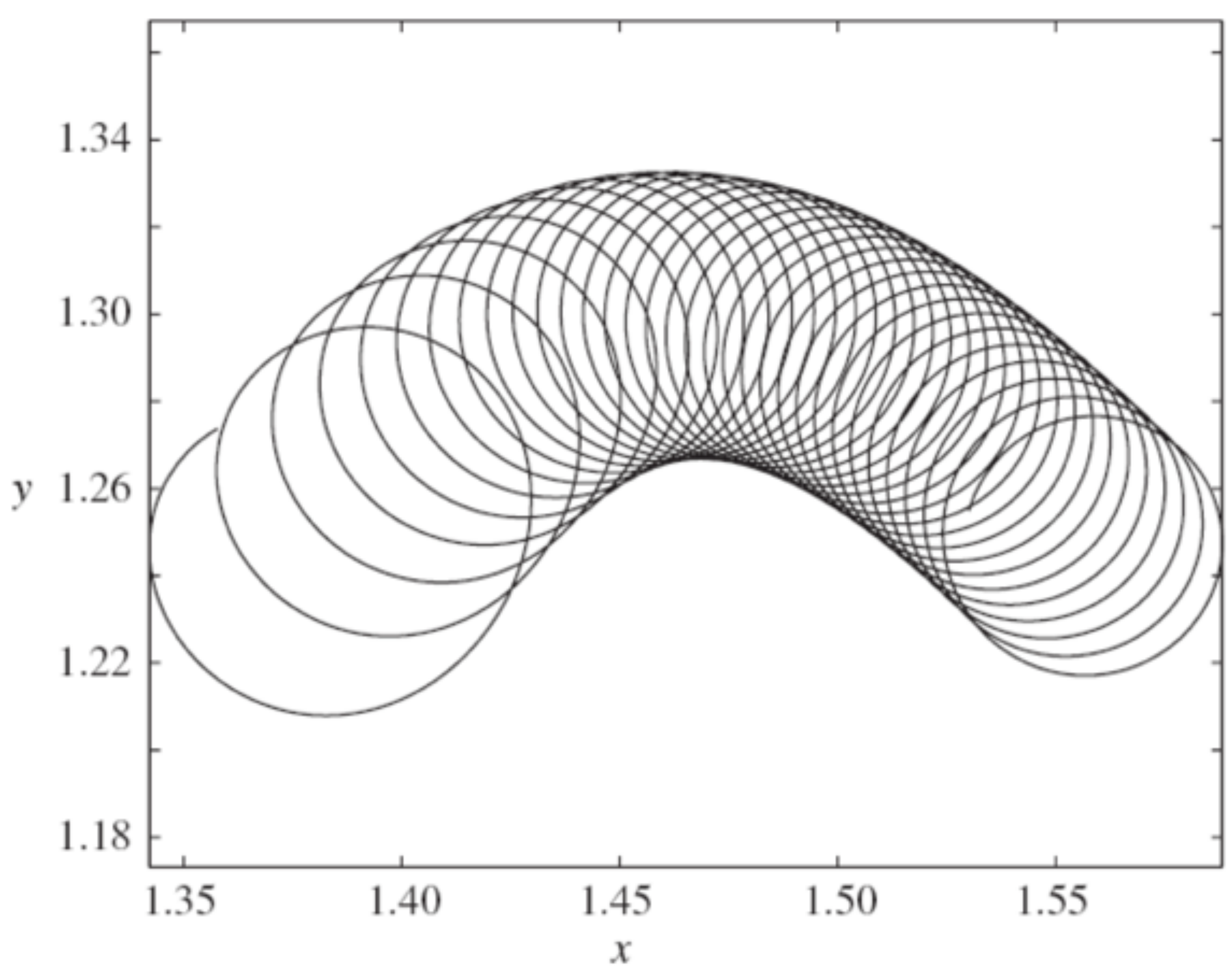


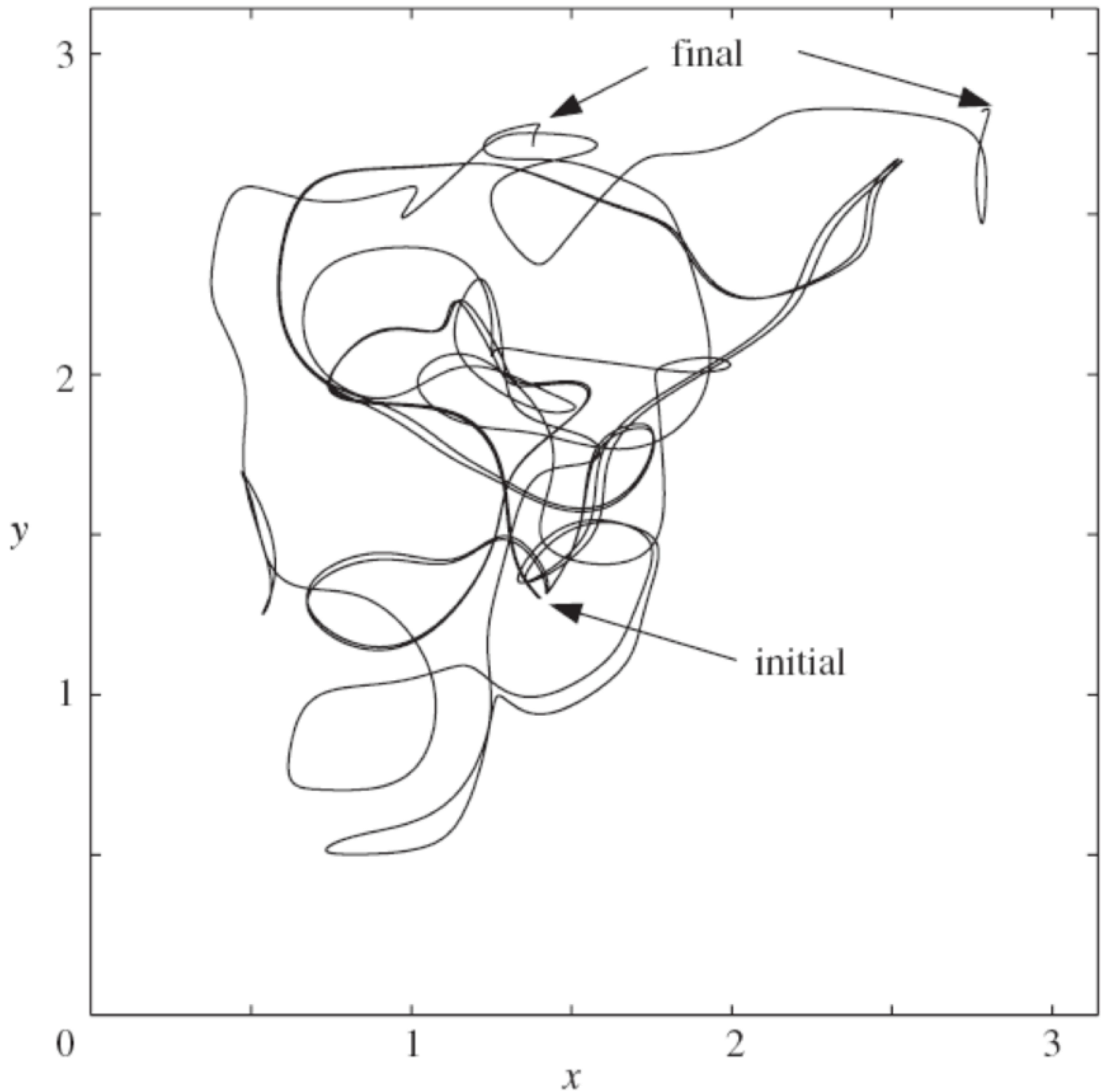


Typical quantum trajectory…



Close-up of a trajectory near a node



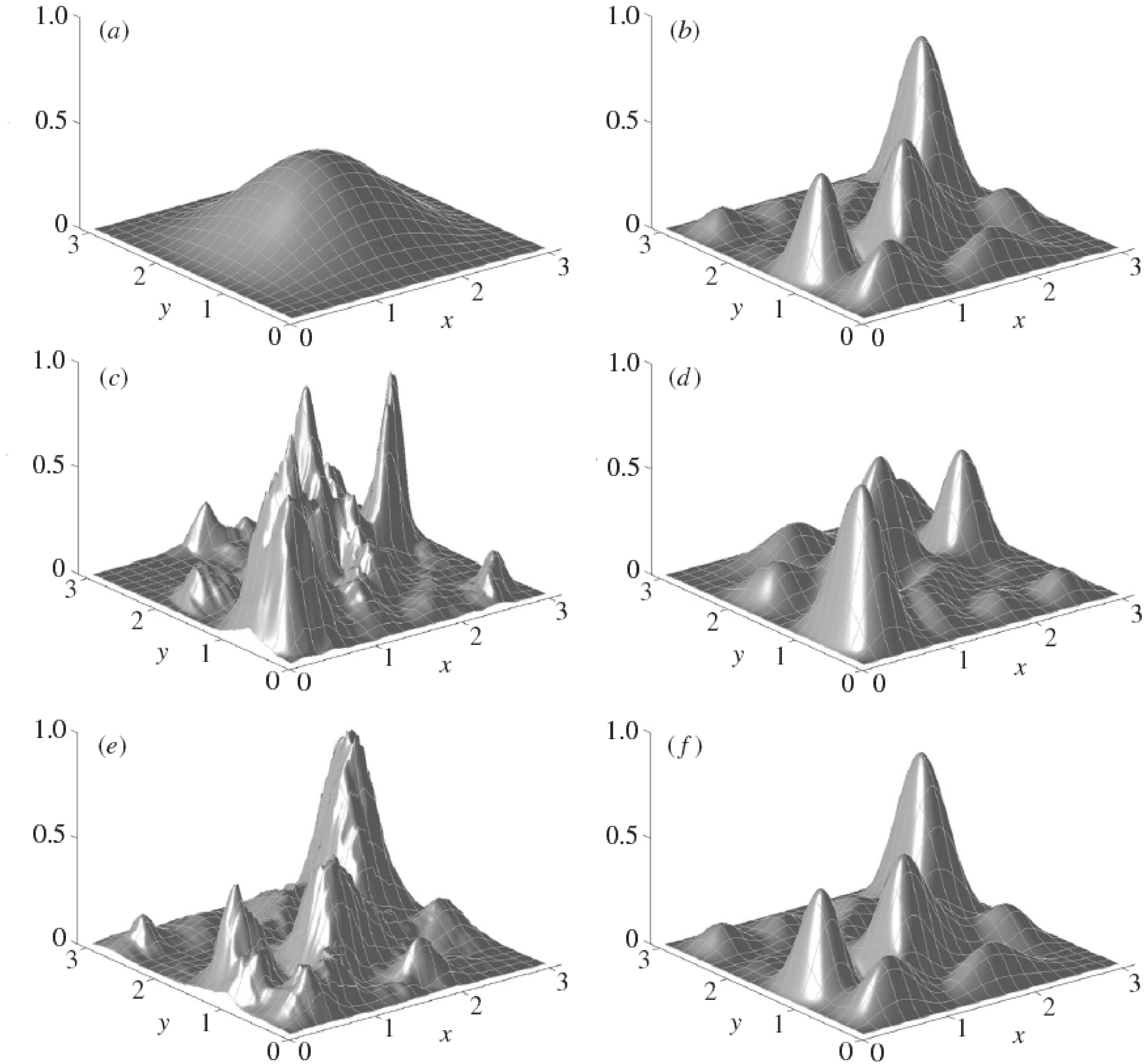


chaotic mixing…

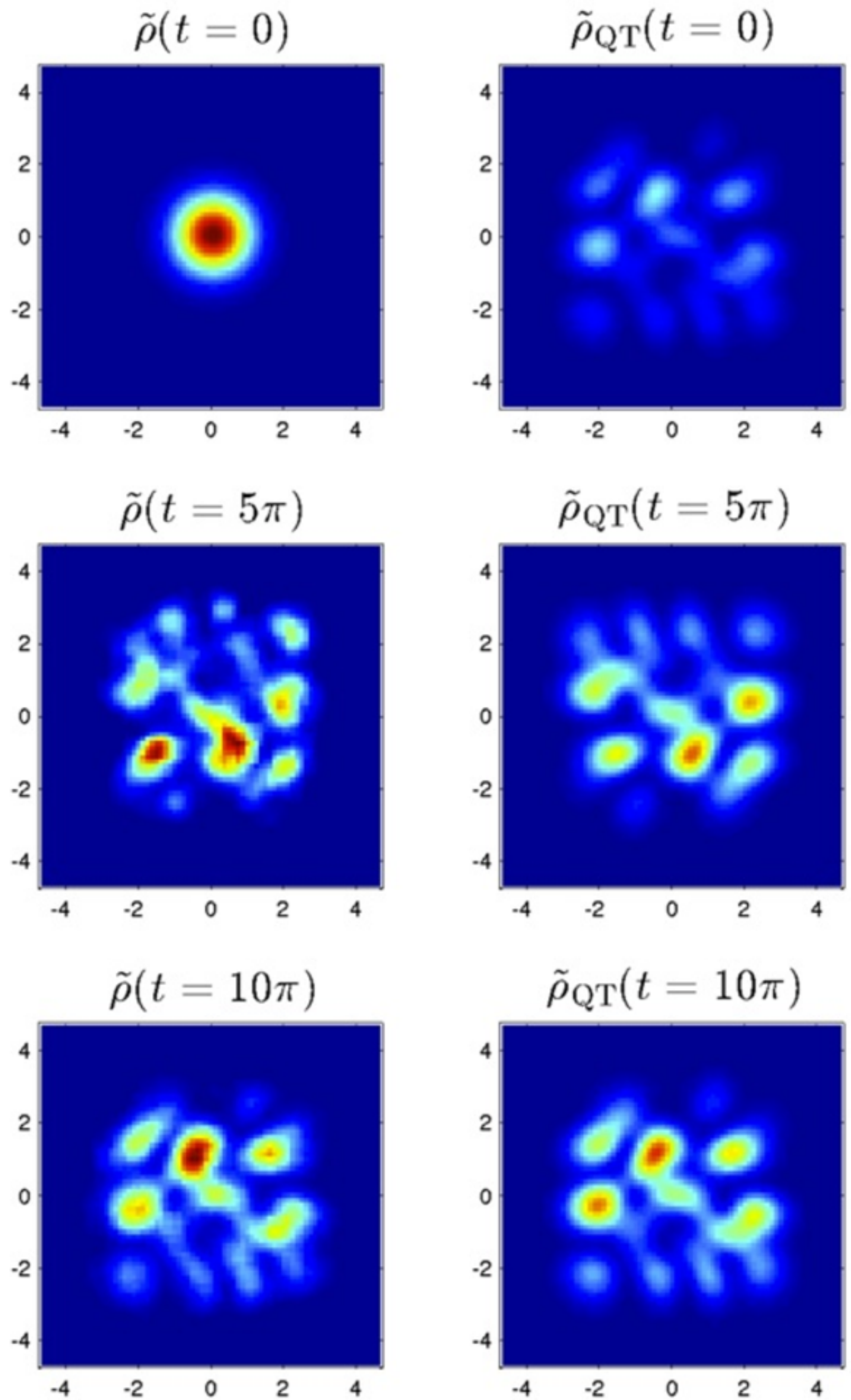
Dynamical evolutions

time

ρ



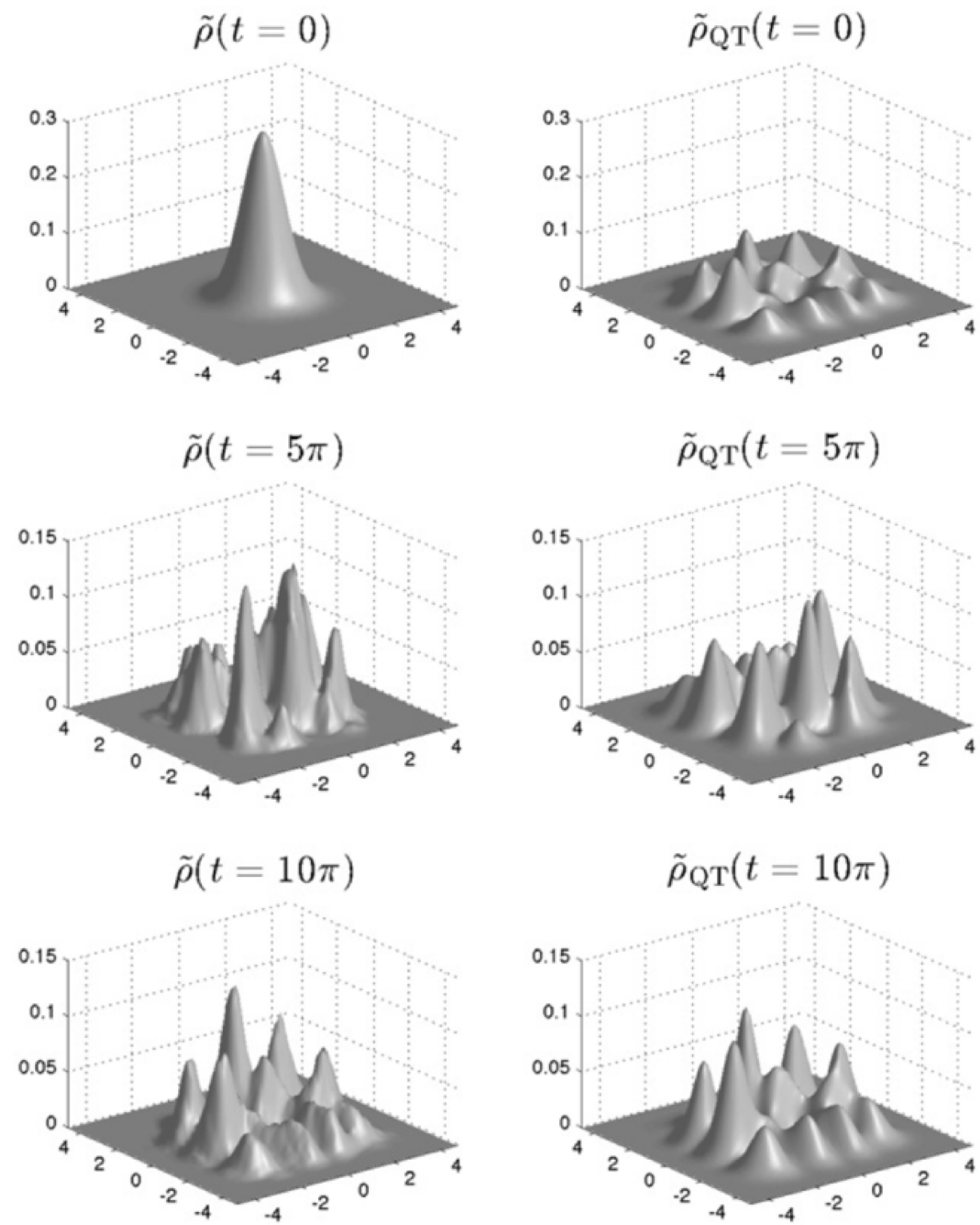
$|\Psi|^2$



chaotic mixing…

relaxation towards equilibrium

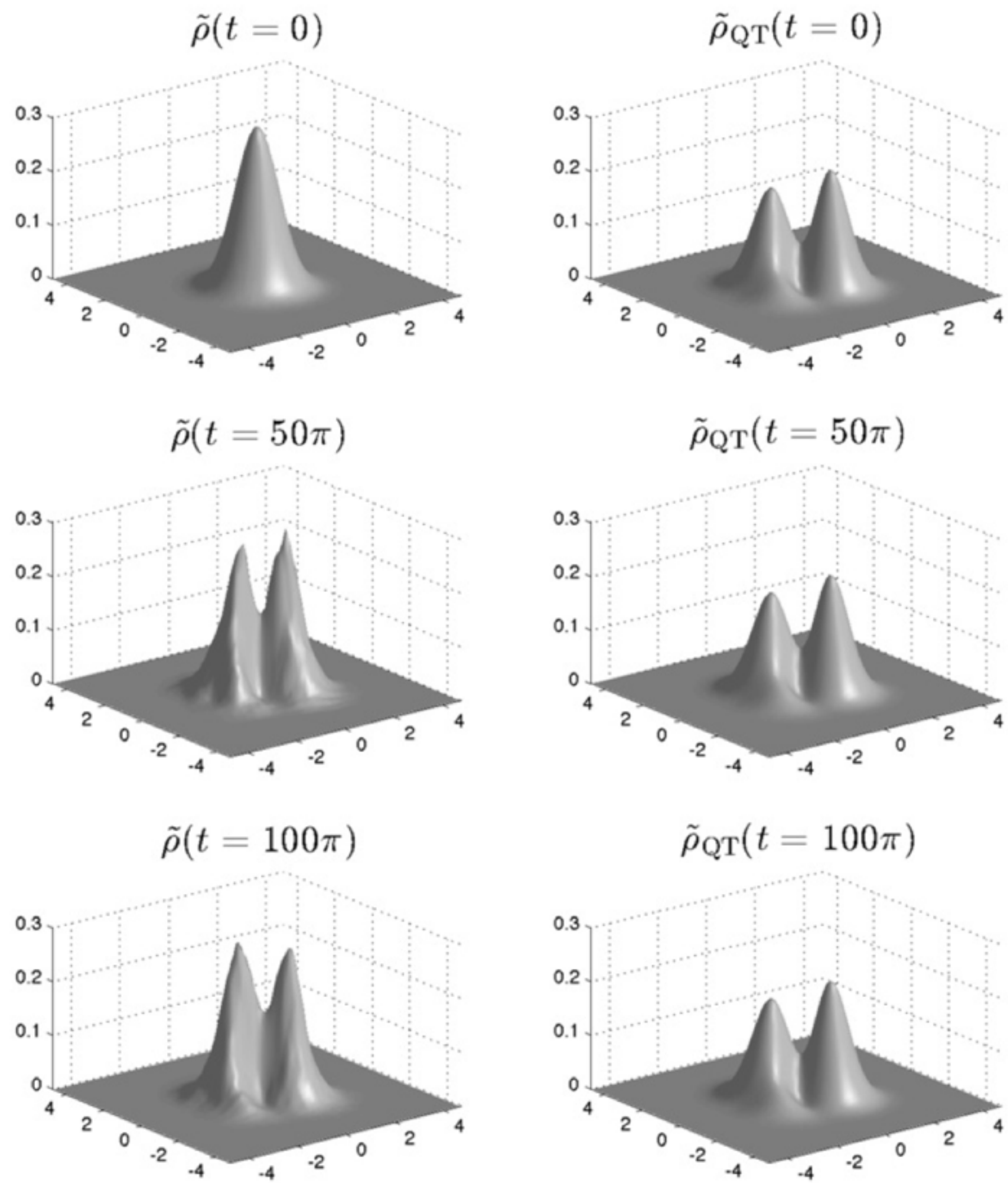
just like ordinary thermal equilibrium



chaotic mixing...

relaxation towards equilibrium

just like ordinary thermal equilibrium



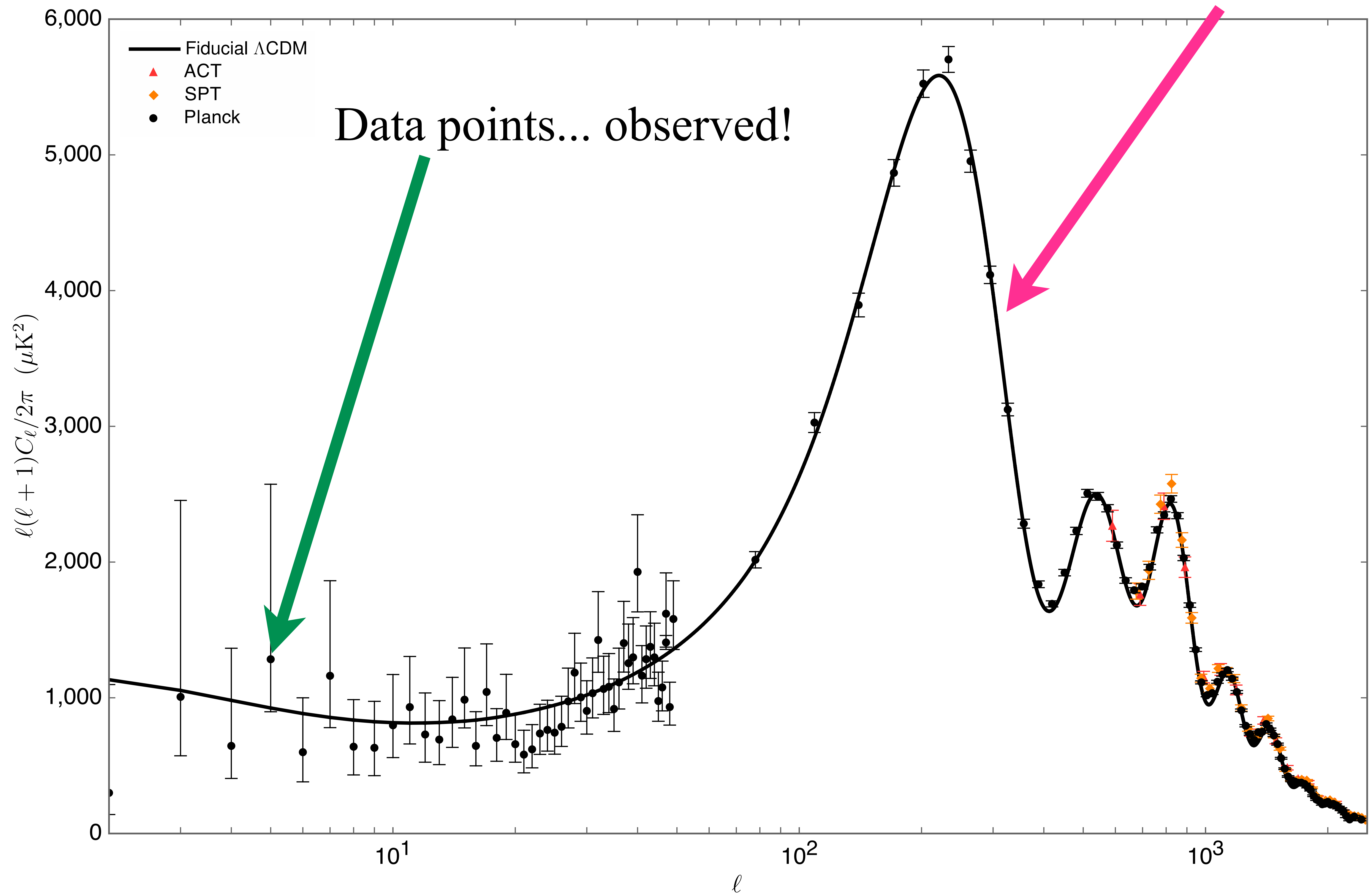
chaotic mixing…

relaxation towards equilibrium

just like ordinary thermal equilibrium

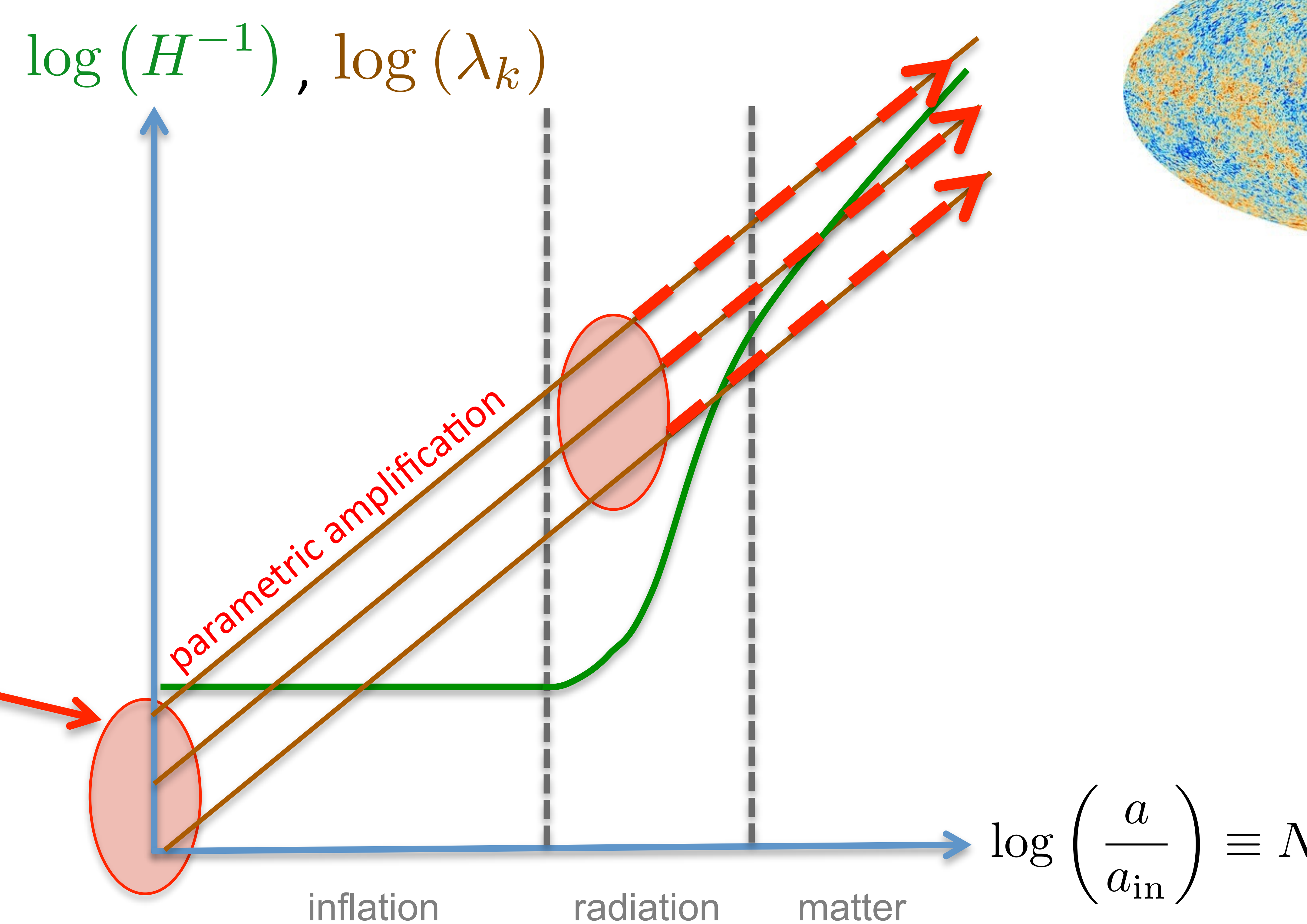
Back to cosmology

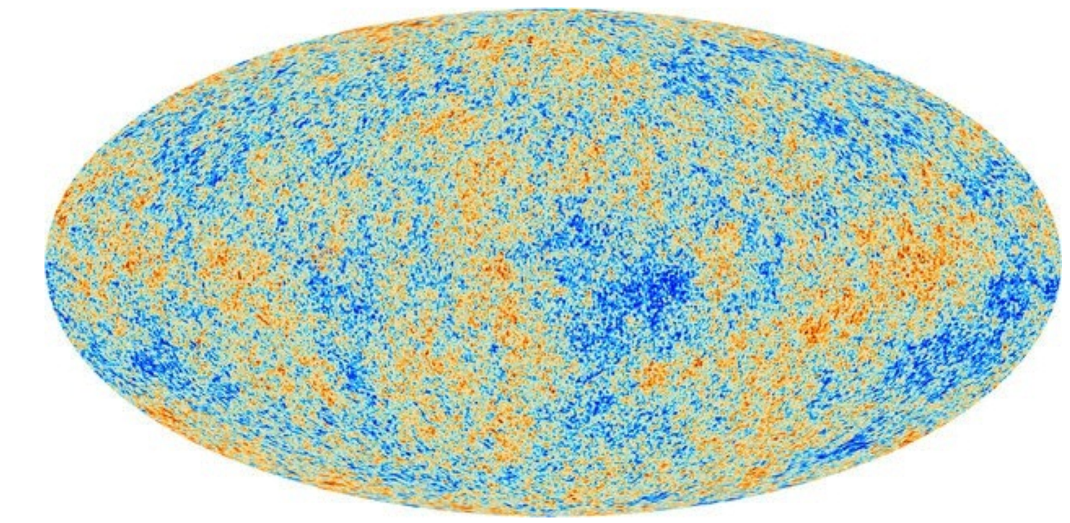
quantum vacuum fluctuations



Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

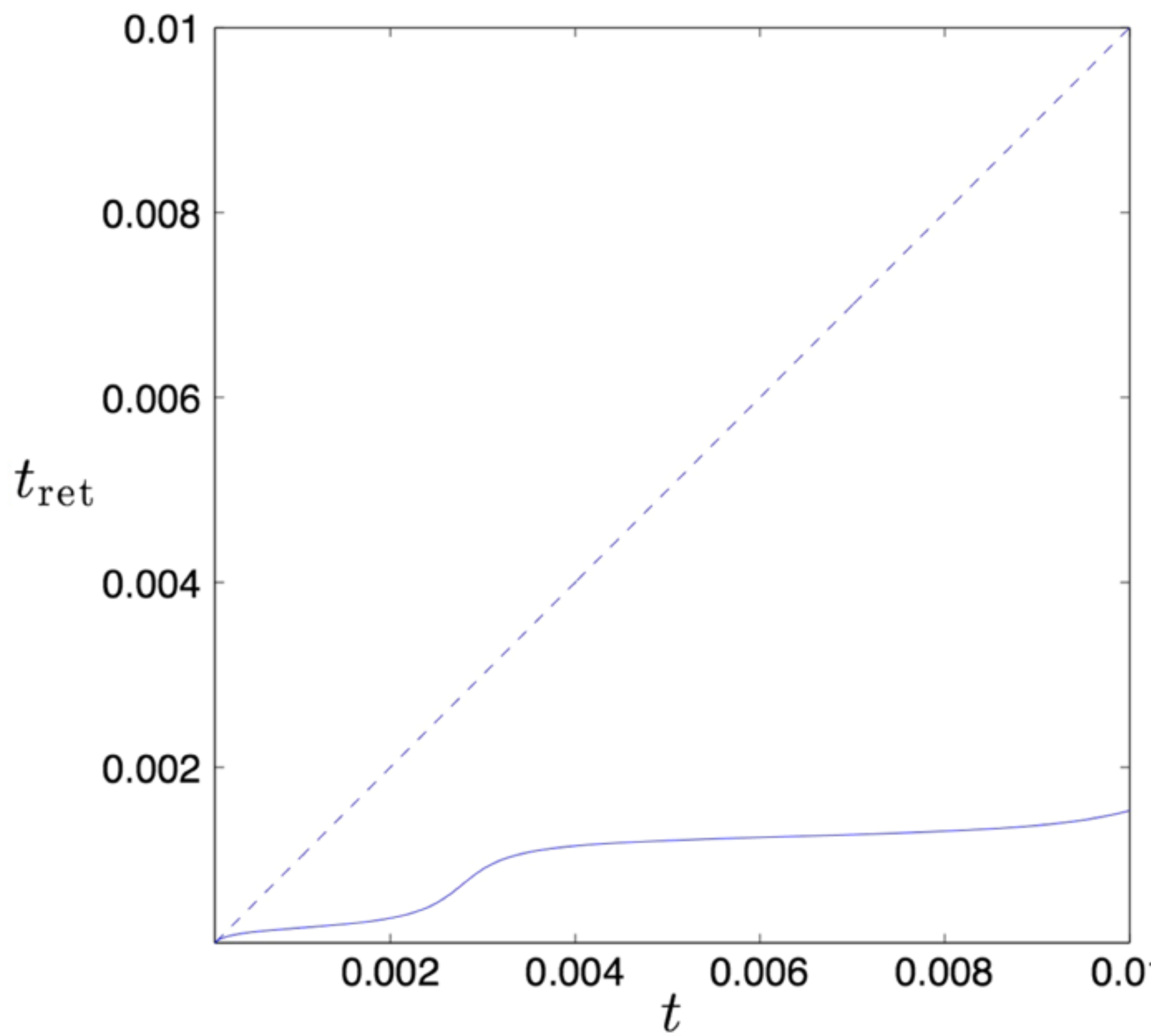




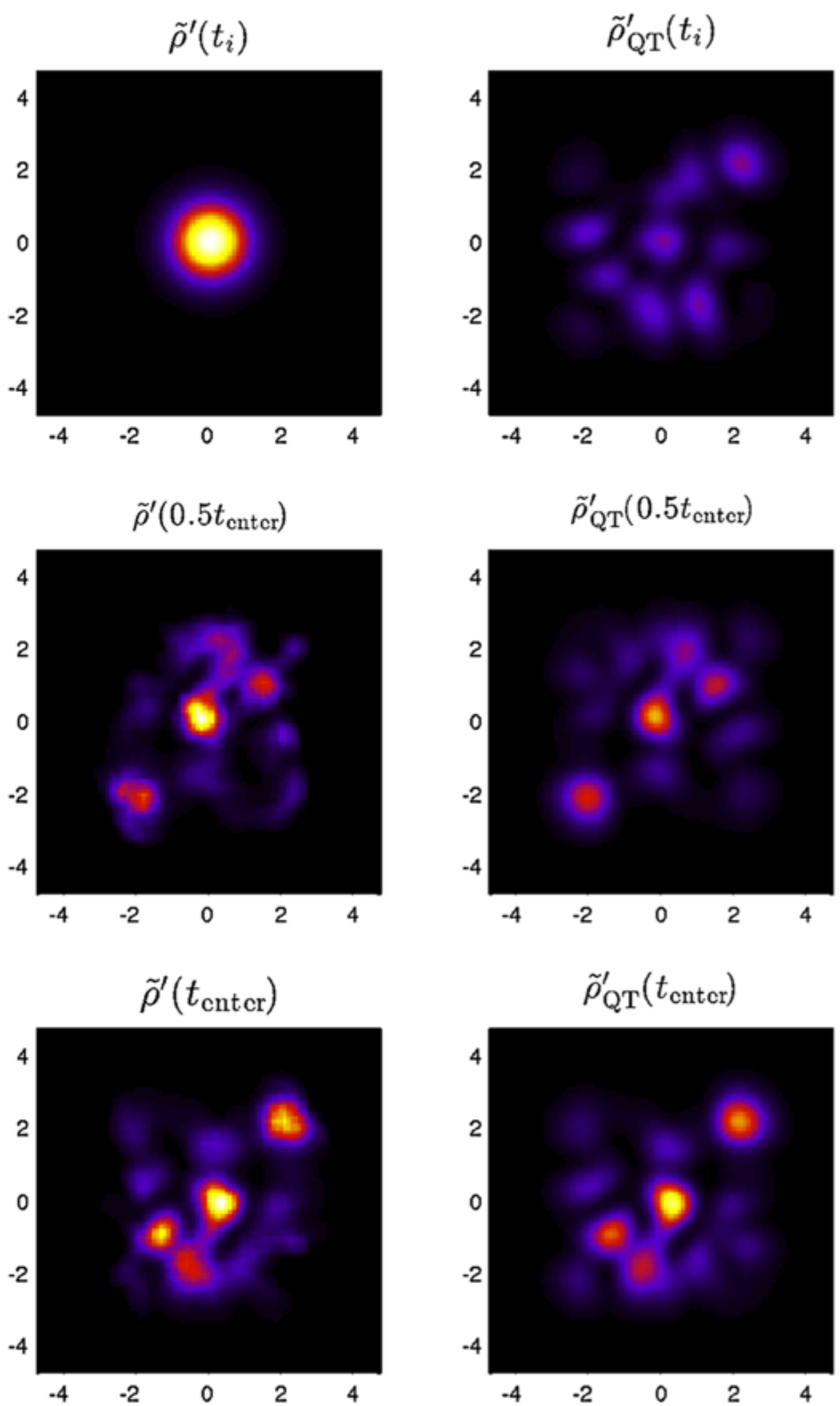
Out-of-equilibrium time evolution

- Usual behaviour = evolves towards equilibrium (Minkowski or slowly expanding Universe)

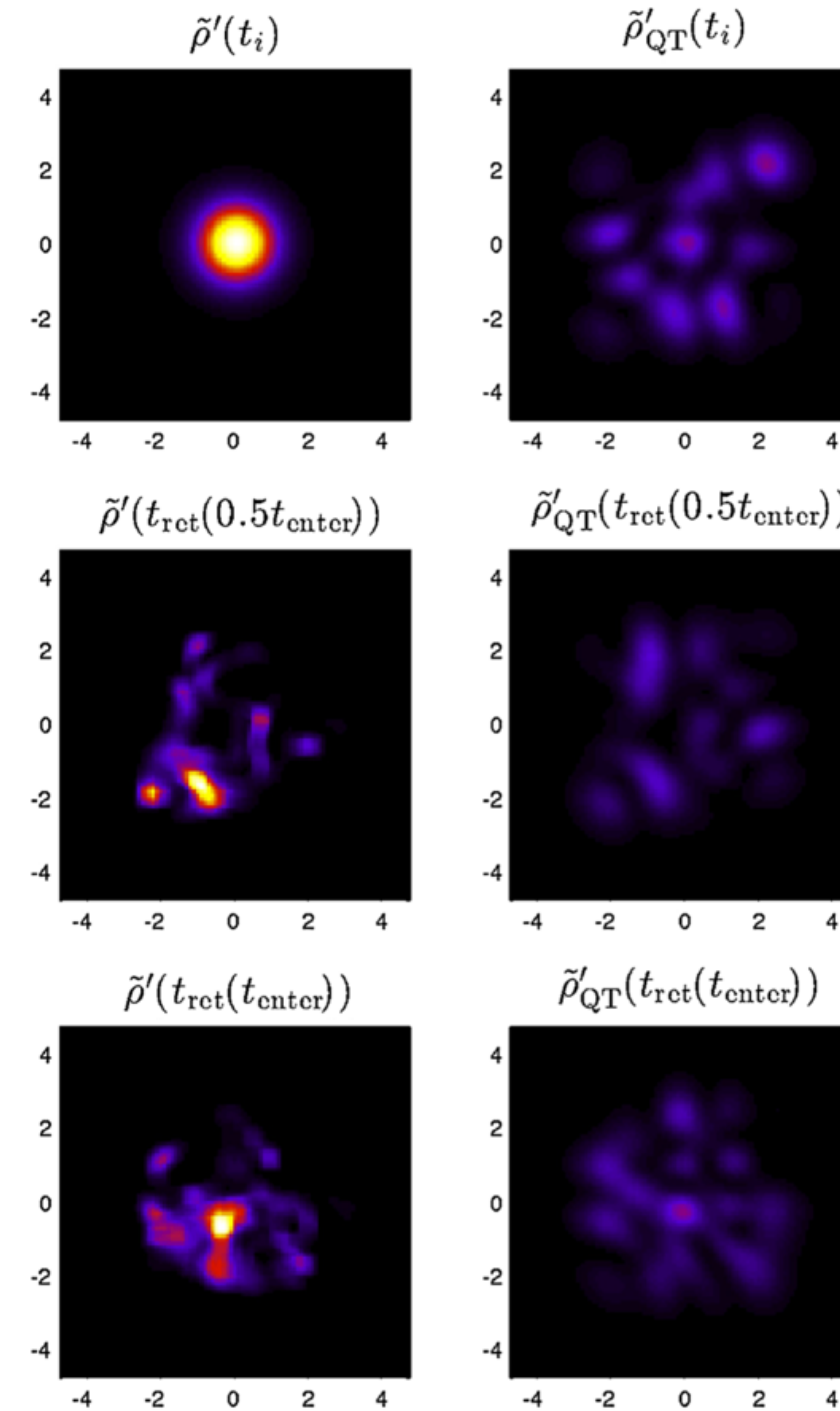
- Inflation: there is a retarded time…



Freezing the pdf out of equilibrium

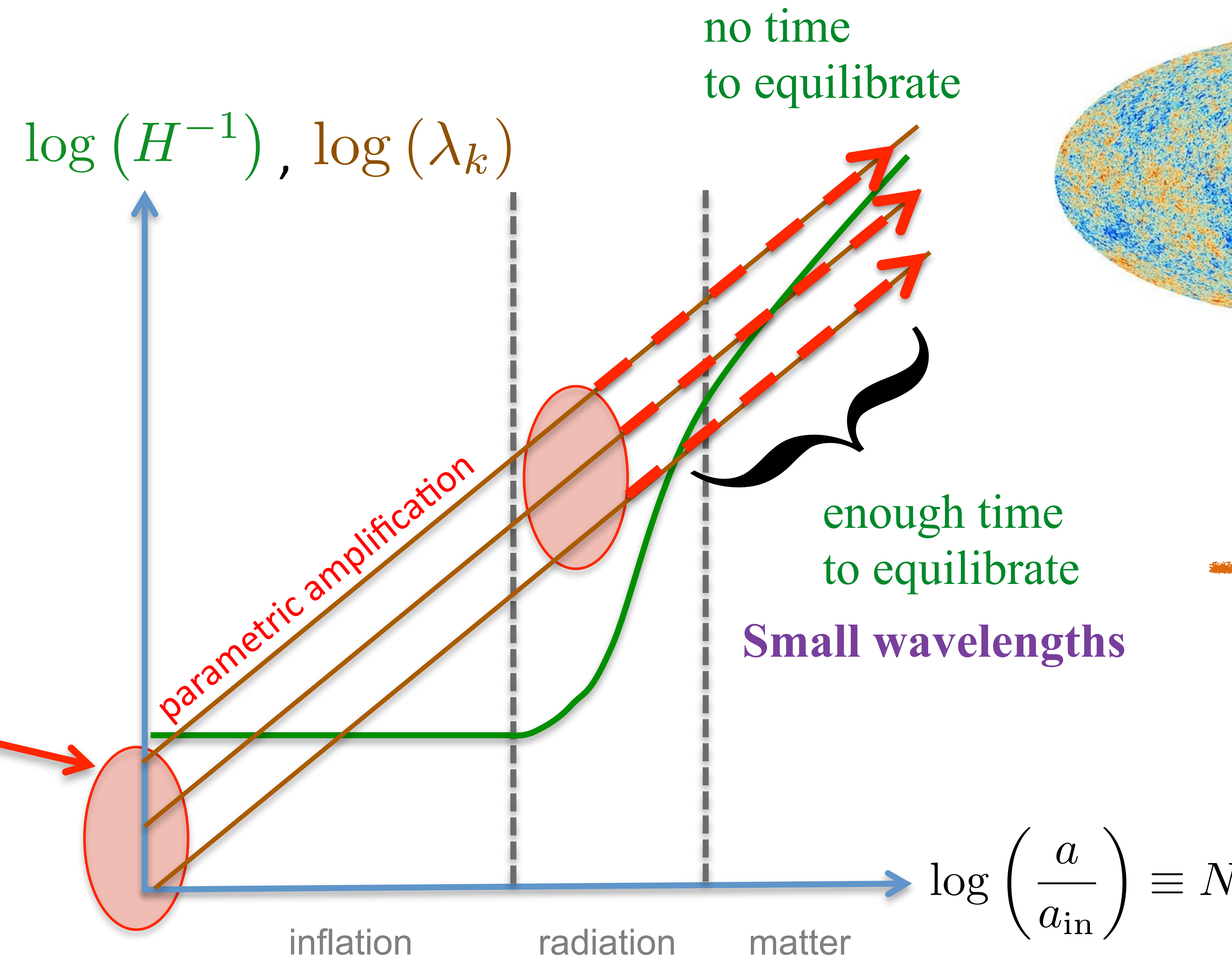


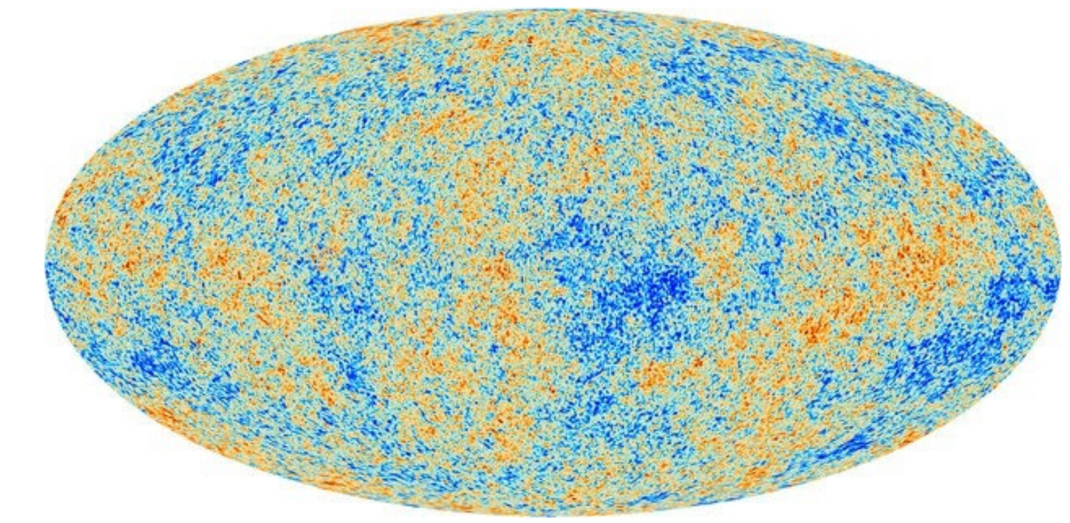
without expansion
EQUILIBRIUM



with expansion
OUT OF EQUILIBRIUM

less power ← Very long wavelengths



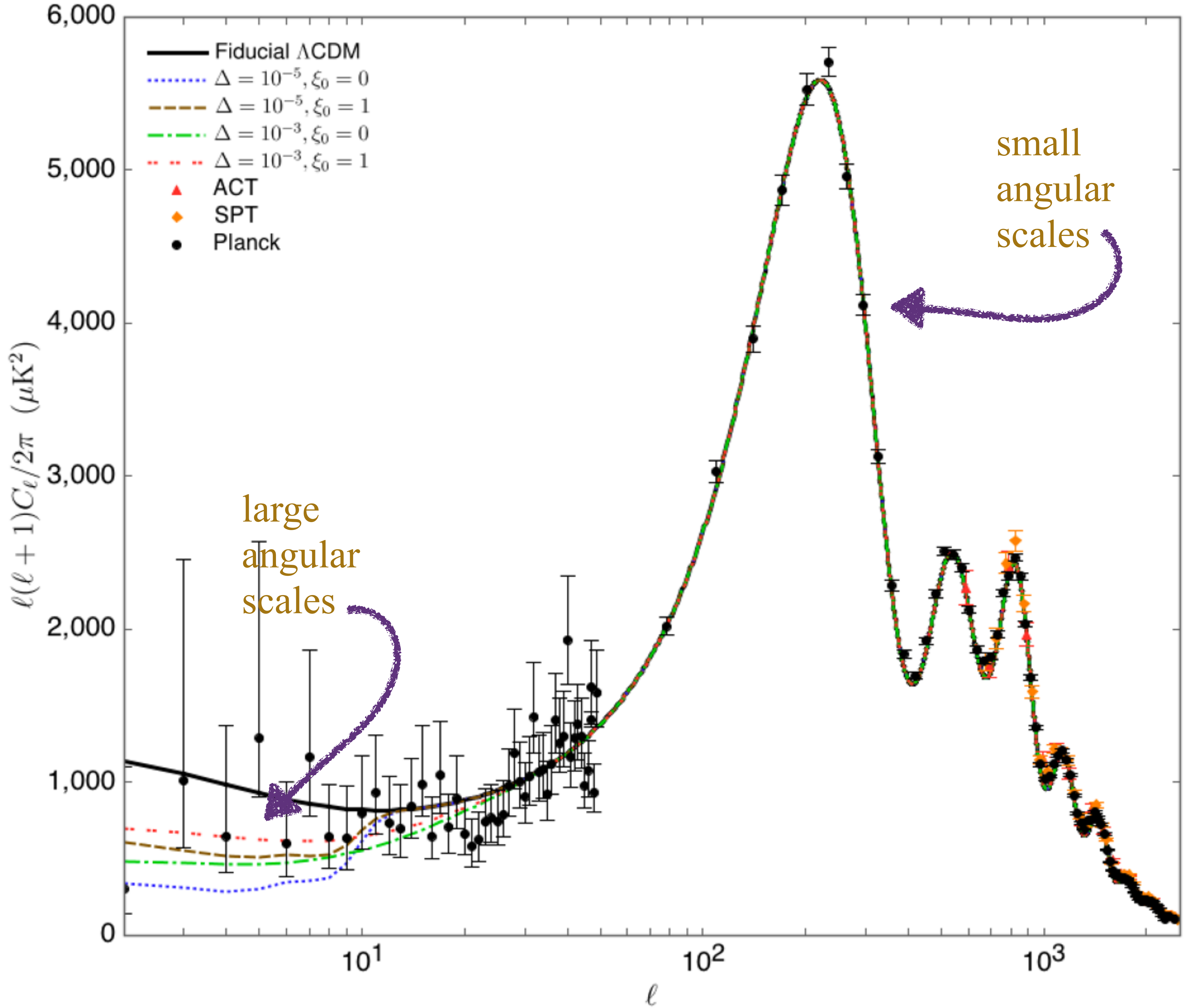


→ usual spectrum

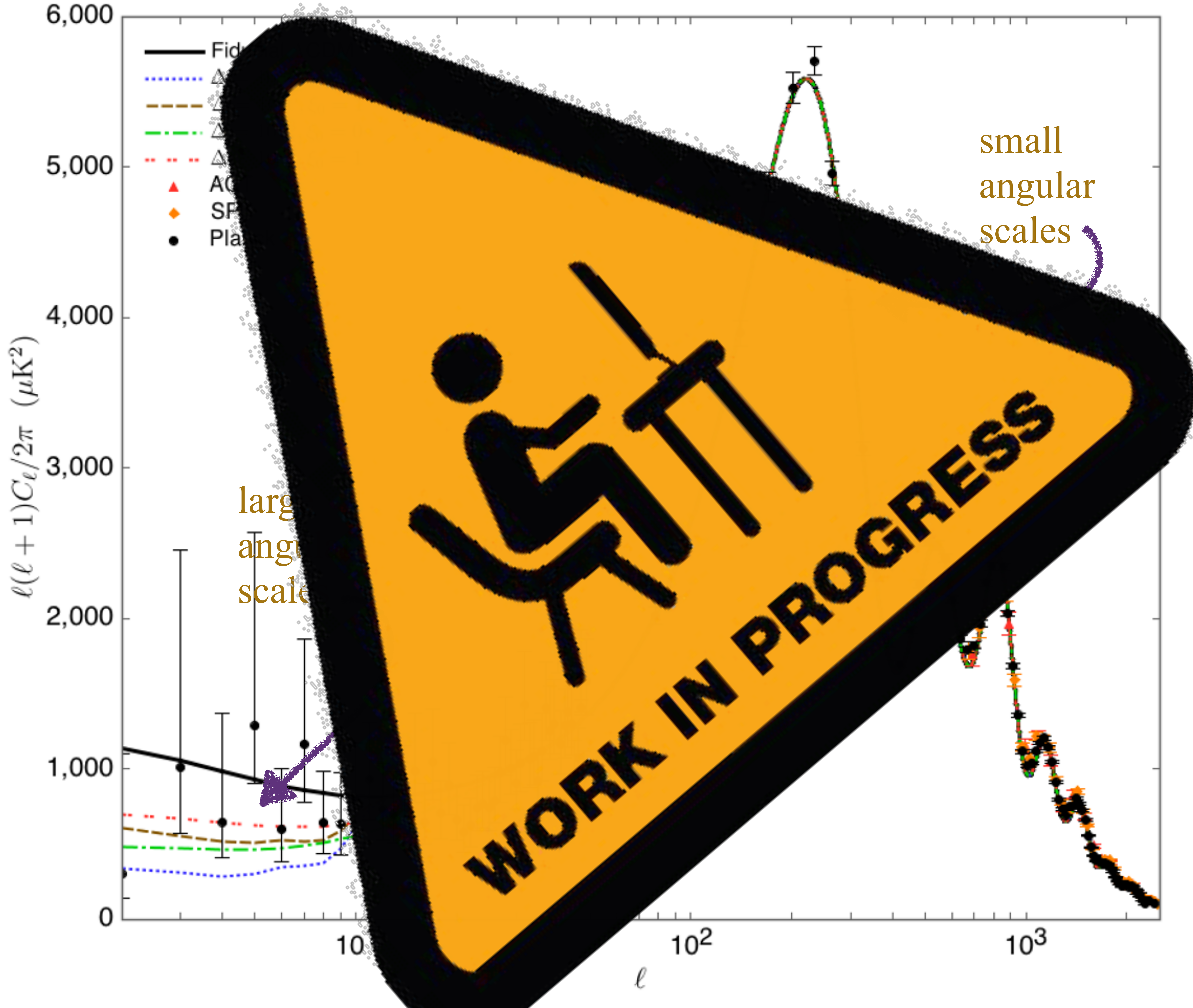
Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Out-of-Equilibrium
initial density:
less quantum noise


Fiducial ΛCDM
$\Delta = 10^{-5}, \xi_0 = 0$
$\Delta = 10^{-5}, \xi_0 = 1$
$\Delta = 10^{-3}, \xi_0 = 0$
$\Delta = 10^{-3}, \xi_0 = 1$
ACT
SPT
Planck
small angular scales
large angular scales
$\ell(\ell+1)C_\ell/2\pi \ (\mu K^2)$
ℓ

Better fit?



6,000
5,000
4,000
3,000
2,000
1,000
0
$\ell(\ell+1)C_\ell/2\pi$ (μK^2)
10^2
10^3
ℓ
small angular scales
WORK IN PROGRESS

Better fit?

Conclusions

- cosmology works amazingly well
- perturbation theory fits the data
- demands initial conditions from quantum vacuum fluctuations
- described by the Schrödinger equation

- quantum mechanics works amazingly well
- can be described by actual particle trajectories
- there exists an analog hydrodynamical system
- theory can be tested if out-of equilibrium

- only known regime possible = cosmology
- demands initial condition with less quantum noise
- predicts less amplitude on large scales
- TO BE CONTINUED…