The non-minimal coupling constant and the primordial de Sitter state

Orest Hrycyna

Theoretical Physics Division, National Centre for Nuclear Research,
Ludwika Pasteura 7, 02-093 Warszawa, Poland

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Orest Hrycyna[1,a]

[1] Theoretical Physics Division, National Centre for Nuclear Research, Ludwika Pasteura 7, 02-093 Warszawa, Poland

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Abstract Dynamical systems methods are used to investigate dynamics of a flat Friedmann–Robertson–Walker cosmological model with the non-minimally coupled scalar field and a potential function. Performed analysis distinguishes the value of non-minimal coupling constant parameter $\xi = \frac{3}{16}$, which is the conformal coupling in five dimensional theory of gravity. It is shown that for a monomial potential functions at infinite values of the scalar field there exist generic de Sitter and Einstein–de Sitter states. The de Sitter state is unstable with respect to expansion of the Universe for potential functions which do not change faster than linearly. This leads to a generic cosmological evolution without the initial singularity.

field in curved space and the renormalisation procedure also give rise to such term [11–14]. The non-minimal coupling is also interesting in the context of superstring theory [15] and induced gravity [16].

While the simplest inflationary model with a minimally coupled scalar field and a quadratic potential function is no longer favoured by the observational data [17–20] there is a need to extend this paradigm further. From the theoretical point of view and an effective theory approach the coupling constant becomes a free parameter in the model and should be obtained from some general considerations [21,22] or from a more fundamental theory. Taking a pragmatic approach its value should be estimated from the observational data [23–27].

Cosmology

General Theory of Relativity

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left(R-2\Lambda\right) + S_m$$

Friedmann-Robertson-Walker symmetry

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{\Lambda,0} + \Omega_{m,0}\left(\frac{a}{a_0}\right)^{-3} + \Omega_{r,0}\left(\frac{a}{a_0}\right)^{-4} + \dots$$

$$S_T = S - \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(\nabla^\alpha\phi\,\nabla_\alpha\phi + 2U(\phi)\right)$$

- inflationary epoch – inflaton
- current accelerated expansion – quintessence

The theory

We start from the total action of the theory

$$S = S_g + S_\phi,$$

consisting of the gravitational part described by the standard Einstein-Hilbert action integral

$$S_g = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\,R\,,$$

where $\kappa^2 = 8\pi G$, and the matter part of the theory is in the form of non-minimally coupled scalar field

$$S_\phi = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\Big(\varepsilon\nabla^\alpha\phi\,\nabla_\alpha\phi + \varepsilon\xi R\phi^2 + 2U(\phi)\Big)\,,$$

where $\varepsilon = +1, -1$ corresponds to the canonical and the phantom scalar field, respectively.

Asymptotically monomial potential functions

Inflationary paradigm:

"plateau-like" potential functions $\frac{U'(\phi)}{U(\phi)} \to 0$ as $\phi \to \infty$

Our assumptions:

$\lambda = -\phi \frac{U'(\phi)}{U(\phi)} \to$ const.,

not only potential function with $U(\phi) \to$ const. as $\phi \to \infty$ but all possible potential functions with an asymptotic behaviour $U(\phi) \propto \phi^{\alpha}$

Projective coordinates and analysis at infinity

projective coordinates

$$u \equiv \frac{x}{z} = \frac{\dot{\phi}}{H\phi}, \quad \bar{v} \equiv \frac{y^2}{z^2} = \frac{1}{2}\frac{U(\phi)}{H^2\phi^2}, \quad \bar{w} \equiv \frac{1}{z^2} = \frac{6}{\kappa^2}\frac{H_0^2}{H^2\phi^2},$$

u^*	$\bar{v}^*$	$\frac{\dot{H}}{H^2}\Big\|_*$
$-6\xi \pm \sqrt{-6\xi(1-6\xi)}$	0	$-2 + \frac{(u^*)^2}{6\xi}$
$-\frac{(4+\lambda)\xi}{1-(2-\lambda)\xi}$	$-\varepsilon\frac{(1-6\xi)\xi(6-(2-\lambda)(10+\lambda)\xi)}{(1-(2-\lambda)\xi)^2}$	$\frac{1}{2}\frac{(2+\lambda)(4+\lambda)\xi}{1-(2-\lambda)\xi}$

Non-minimal coupling and a monomial potential function

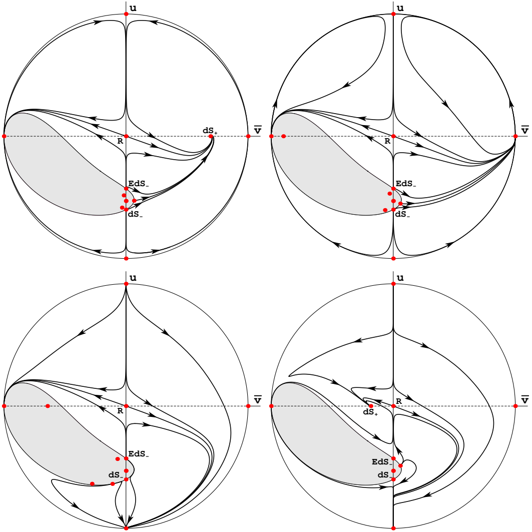


Figure: $\xi = \frac{3}{16}$: $\varepsilon = +1$, $\lambda = 2$, $\lambda = -\frac{1}{2}$, $\lambda = -\frac{5}{2}$, $\lambda = -20$

Non-minimal coupling and a constant potential

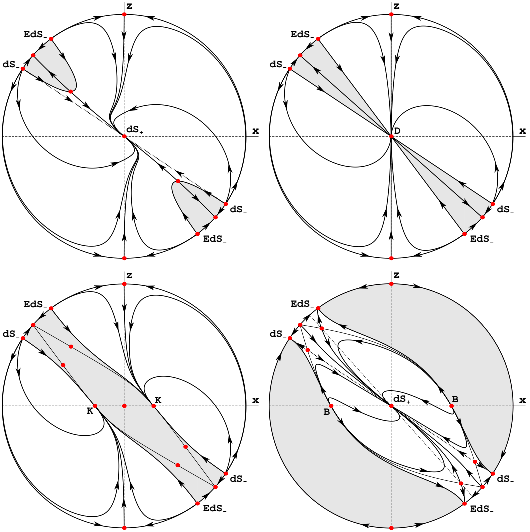


Figure: $\xi = \frac{3}{16}$: $\varepsilon = +1,\ U_0 > 0,\ U_0 = 0,\ U_0 < 0;\ \varepsilon = -1\ U_0 > 0$

Observational constraints

Working assumptions:
barotropic dust matter + $U(\phi) = U_0 = \text{const.}$

the energy conservation condition, Friedmann equation

$$\left(\frac{H(a)}{H(a_0)}\right)^2 = \Omega_{\Lambda,0} + \Omega_{m,0}\left(\frac{a}{a_0}\right)^{-3} + \varepsilon(1-6\xi)x^2 + \varepsilon 6\xi(x+z)^2\,,$$

where

$$\Omega_{m,0} \equiv \frac{\kappa^2 \rho_{m,0}}{3H_0^2}\,, \quad \Omega_{\Lambda,0} \equiv \frac{\kappa^2 U_0}{3H_0^2}\,.$$

Observational data:
Union2.1+H(z)+Alcock-Paczyński test

The general case

The Hubble function can be expanded in to the Taylor series

$$\left(\frac{H(a)}{H(a_0)}\right)^2 = h^2(a, \Omega_{bm,0}, \varepsilon, \xi, x_0, z_0)$$
$$\approx \Omega_{\Lambda,0} + \Omega_1 \left(\frac{a}{a_0}\right)^{-1} + \Omega_2 \left(\frac{a}{a_0}\right)^{-2} + \Omega_3 \left(\frac{a}{a_0}\right)^{-3} + \dots$$

where the density parameters are $\Omega_i = \Omega_i(\Omega_{bm,0}, \varepsilon, \xi, x_0, z_0)$. From the energy conservation condition we have

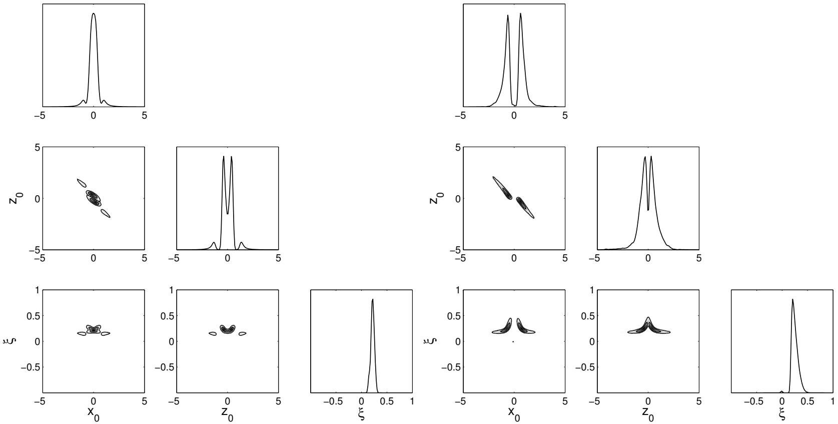
$$\Omega_1 + \Omega_2 + \Omega_3 + \cdots = \Omega_{bm,0} + \varepsilon(1 - 6\xi)x_0^2 + \varepsilon 6\xi(x_0 + z_0)^2 \,.$$

With the observational data used we can expect that $\Omega_i \approx 0$ for $i > 3$. Additionally, the ΛCDM model is favoured by the data and we can expect that $\Omega_1 \approx 0$ and $\Omega_2 \approx 0$. Thus we obtain that the leading term in the Taylor series above is the following

$$\Omega_3 \approx \Omega_{bm,0} + \varepsilon(1 - 6\xi)x_0^2 + \varepsilon 6\xi(x_0 + z_0)^2 \,.$$

The last terms in this formula can be interpreted as an effective dark matter in the model resulting from the present evolution of the scalar field.

Observational constraints



Conclusions

- In this paper we have investigated dynamics of a flat FRW cosmological model filled with the non-minimally coupled scalar field with a potential function. With assumption of the monomial type of behaviour at infinite values of the scalar field we were able to find the specific value of the non-minimal coupling constant $\xi = \frac{3}{16}$ for which there were de Sitter and Einstein-de Sitter states.
- For monomial scalar field potential functions $U(\phi) \propto U_0 \phi^\alpha$ with $\alpha < 1$ both the Einstein-de Sitter and the de Sitter states are unstable with respect to the expansion of the universe
- We have found that the asymptotic unstable with respect to expansion of the universe de Sitter and Einstein-de Sitter states exist both for negative and positive potential functions.

Conclusions

- Global dynamical analysis of model with a constant potential function and the non-minimal coupling constant $\xi = \frac{3}{16}$ was performed. For the positive cosmological constant there is an open and dense set of initial conditions giving rise to non-singular evolution of the universe from an unstable de Sitter state toward a stable one.
- The value of the non-minimal coupling constant $\xi = \frac{3}{16}$ corresponds to conformal coupling value in a $5-$dimensional theory of gravity. Presented analysis and results might point toward a new fundamental symmetry in the matter sector of the theory.