Kosmologia wczesnego Wszechświata

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Seminarium sprawozdawczo-odbiorcze

Outline

- Modifications of the isotropic model describing the space-time geometry of the earliest Universe:
  - H. Bergeron, E. Czuchry, J.-P. Gazeau, P. Małkiewicz, *Integrable Toda system as a quantum approximation to the anisotropy of the mixmaster universe*, Phys. Rev. D **98** 083512 (2018).
  - E. Czuchry, *Quantum Toda-like regularisation of the Mixmaster anisotropy*, J. Phys. Conf. Ser (2018), Proceedings of the 32nd International Colloquium on Group Theoretical Methods in Physics, accepted.
- Going beyond General Relativity in order to avoid the initial singularity (the so-called Bing-Bang) problem:
  - N. A. Nilsson and E. Czuchry, *Horava-Lifshitz cosmology in light of new data*, Phys. Dark Universe (2018), accepted, in production.

ΛCDM model

The ΛCDM (Lambda cold dark matter) model is is successfully used to describe the data of observational cosmology, namely

- the cosmic microwave background fluctuations
- the large-scale structure in the distribution of galaxies
- the observed accelerating expansion of the Universe
- the abundances of hydrogen (including deuterium), helium, and lithium

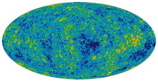


Figure: Microwave Sky: The detailed, all-sky picture created from nine years of WMAP data (NASA/WMAP Science Team).

FRW geometry

The ΛCDM model uses the Friedmann-Robertson-Walker (FRW) metric and the Friedmann equations to describe the observable universe. The FRW metric is an exact solution of Einstein's field equations of general relativity; it describes a homogeneous, isotropic expanding or contracting universe .

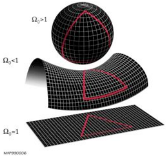


Figure: Closed, open and flat two-dimensional geometries (NASA/WMAP Science Team).

Limitations of the FRW geometry

The FRW model is used as a first approximation for the evolution of the real universe because it is simple to calculate.

It seems that the observable universe is well approximated by an almost FRW model, i.e. a model which follows the FRW metric apart from primordial density fluctuations. However

- The isotropy of space is dynamically unstable towards the big-bang singularity[1].
- Therefore an anisotropic model, comprising the Friedmann model as a particular case, is expected to be better suited for describing the earliest Universe.
- Mixmaster universe, Bianchi IX model, has sufficient generality.

[1] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, Adv. Phys. **19**, 525 (1970).

Mixmaster universe

The canonical description of the Bianchi-type IX model is given in terms of Misner's variables[2].

In this approach the dynamics resembles motion of a particle in a three-dimensional Minkowskian space-time and in a space-and-time-dependent confining potential.

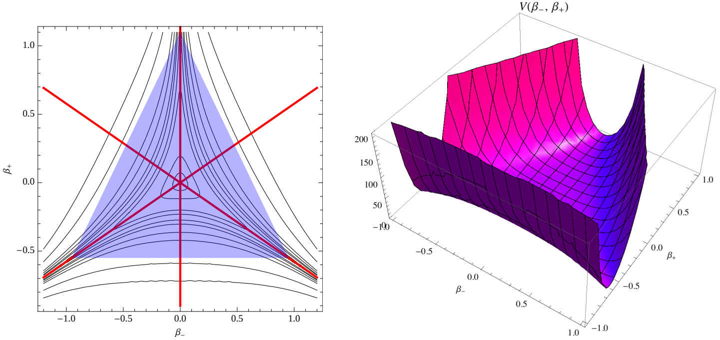


Figure: The plot of Bianchi IX potential near its minimum.

Mixmaster universe

- The respective Hamiltonian constraint in the Misner variables[3]. reads:

$$\mathrm{C} = \frac{\mathcal{N} e^{-3\Omega}}{24}\left(\frac{2\kappa}{\mathcal{V}_0}\right)^2\left(-p_\Omega^2 + \mathbf{p}^2 + 36\left(\frac{\mathcal{V}_0}{2\kappa}\right)^3 \mathfrak{n}^2 e^{4\Omega}[V(\boldsymbol{\beta}) - 1]\right),$$

  where the spacetime variables $(\Omega, \beta_+, \beta_-)$ have the cosmological interpretation.
- The potential that drives the motion of the geometry represents the spatial curvature ${}^3R$.:

$$V(\boldsymbol{\beta}) = \frac{e^{4\beta_+}}{3}\left[\left(2\cosh(2\sqrt{3}\beta_-) - e^{-6\beta_+}\right)^2 - 4\right] + 1\,.$$

[3] C. W. Misner, Phys. Rev. Lett. **22**, 1071 (1969); Phys. Rev. **186**, 1319 (1969).

Weyl-Heisenberg integral quantization

- The so-called canonical quantization works sufficiently well for the basic observables and many typical Hamiltonians.
- It it less suitable for more complex observables, in particular, when the latter admit some sort of singularities.
- The three channels of the anisotropy potential, which narrow to the zero width, are in a sense singular and one may expect that the quantization should smooth out those singular features.
- Weyl-Heisenberg quantization scheme is a general procedure that in some circumstances provides conceptually better justified quantization prescriptions than 'canonical prescription'.

W-H integral quantization

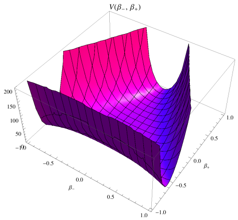
- The Weyl-Heisenberg integral quantizationIt results in the quantized form of the Bianchi IX potential (as a multiplication operator)

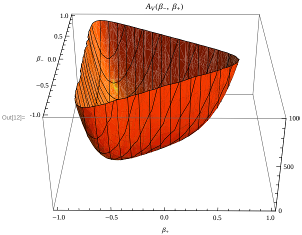
$$A_{V(\beta)} = \frac{1}{3}\Big(2D_+^4 D_-^{12} e^{4\beta_+}\cosh 4\sqrt{3}\beta_- - 4D_+ D_-^3 e^{-2\beta_+}\cosh 2\sqrt{3}\beta_- + D_+^{16} e^{-8\beta_+} - 2D_+^4 e^{4\beta_+}\Big) + 1,$$

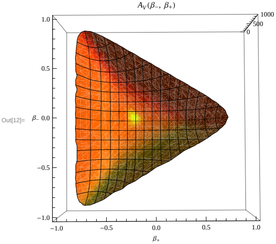
  where $D_\pm := e^{\frac{2}{\sigma_\pm^2}}$

- The original Bianchi IX potential $V(\beta) \equiv V(\beta_+, \beta_-)$ is recovered for $D_+ = 1 = D_-$, thus for weights $\sigma_+,\ \sigma_- \to \infty$ (the "limit" Weyl-Wigner case).

Regularized BIX potentials after quantization







- Plot of the original Bianchi IX potential $V(\beta)$ and its regularized version after quantization, near its minimum, for sample values $D_+ = 1.1$, $D_- = 1.4$.
- The original escape channels became regularized and the whole potential is now fully confining.

Regularized BIX potentials after W-H quantization

- Our quantization procedure should preserve the basic properties of the classical potential by requiring (i) the isotropy around its minimum and (ii) keeping the position of the minimum at the point $(0, 0)$.
- Imposing those conditions yields the same result in both cases: $D_+ = D_- + =: D$.
- The resulting potential reads as

$$A_{V(\beta_+,\beta_-)} = \frac{1}{3}\left(D^{16}\left(2e^{4\beta_+}\cosh 4\sqrt{3}\beta_- + e^{-8\beta_+}\right)\right.$$
$$\left. -D^4\left(4e^{-2\beta_+}\cosh 2\sqrt{3}\beta_- - 2e^{4\beta_+}\right)\right) + 1$$

- The introduction of new variables $q_1,\ q_2,\ q_3$ such that $q_1 - q_2 = 4\sqrt{3}\beta_- + 4\beta_+$ and $q_2 - q_3 = -4\sqrt{3}\beta_- + 4\beta_+$ leads to

$$A_{V(\beta_+,\beta_-)} = \frac{D^{16}}{3}\left(e^{q_1-q_2} + e^{q_2-q_3} + e^{q_3-q_1}\right) + \\ - \frac{D^4}{3}\left(e^{-\frac{1}{2}(q_1-q_2)} + e^{-\frac{1}{2}(q_2-q_3)} + e^{-\frac{1}{2}(q_3-q_1)}\right) + 1.$$

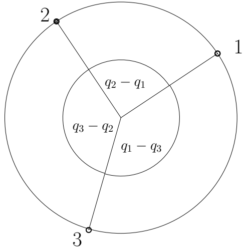
- Therefore, the above potential consists of the dominating part $\sim D^{16}$, corresponding to the periodic 3-particle Toda system[4] perturbed by another 3-particle Toda potential.

[4] M. Berry, Topics in Nonlinear Mechanics, ed. Ss Jorna, Am. Inst. Ph. Conf. Proc. **46**, 16 (1978).

- The periodic Toda system is a system of N equal-mass particles interacting via exponential forces, described by the Hamiltonian:

$$H = \frac{1}{2}\sum_{k=1}^{N} p_k^2 + \sum_{k=1}^{N} e^{-(q_k - q_{k+1})}$$

with periodicity condition $q_0 \equiv q_N$ and $q_1 \equiv q_{N+1}$.
- The periodic 3-particle Toda system is the simplest nontrivial periodic crystal consisting of three particles. The particles on the lattice interact with the neighbor ones via exponential potential.



- It is known that the Toda systems are integrable and solutions can be derived.
- The periodic 3-body Toda system has been analysed both on the classical and quantum levels. In the literature, one may find ways to construct classical solutions as well as the corresponding quantum eigenfunctions and eigenvalues.
- This provides an analytically solvable approximation to the anisotropic potential of the Bianchi IX'
- New possibilities to studies of the quantum dynamics of Mixmaster.

Results described in

- H. Bergeron, E. Czuchry, J.-P. Gazeau, P. Małkiewicz, *Integrable Toda system as a quantum approximation to the anisotropy of the mixmaster universe*, Phys. Rev. D **98** 083512 (2018).
- E. Czuchry, *Quantum Toda-like regularisation of the Mixmaster anisotropy*, J. Phys. Conf. Ser (2018), Proceedings of the 32nd International Colloquium on Group Theoretical Methods in Physics, accepted.

Modifications of General Relativity

- General Relativity is not renormalizable:
  A theory is said to be *power-counting renornalizable* if all of its interaction terms scale like momentum to some non positive power, as then Feynman diagrams are expected to be convergent or have at most logarithmic divergence.
  Renormalization at one-loop demands that GR should be supplemented by higher-order curvature terms. However such theories are not viable as they contain ghost degrees of freedom
- The observed value for cosmological constant is smaller than the value derived form particle physics by at best 60 orders of magnitude.
- Dark energy and dark matter: recent observational data provides that about 95% of the Universe is made from unknown components.

Going beyond General Relativity

- Higher dimensional spacetimes (Kaluza-Klein type theories):
  One can expect that for any higher-dimensional theory, a 4-dimensional effective field theory can be derived in the low energy limit.
- Adding extra fields (or higher-order derivatives):
  One can modify the gravitational action by considering more degrees of freedom. This can be achieved by adding extra dynamical fields or equivalently considering theories with higher-order derivatives.
- Giving up diffeomorphism invariance:
  Lorentz symmetry breaking can lead to a modification of the graviton propagator in the UV, thus rendering the theory power-counting renormalizable.

Existence of bounce in Hořava-Lifshitz cosmology

Hořava-Lifshitz gravity – a proposal for a UV-complete renormalizable gravity theory – may lead to a bouncing cosmology.

- Hořava proposed a model for quantum gravity which is power-counting renormalizable and hence potentially ultra-violet (UV) complete.
- In the UV the theory possesses a fixed point with an anisotropic, Lifshitz scaling between time and space, therefore it is referred to as Hořava-Lifshitz gravity.
- This model does not have the complete diffeomorphism invariance of General Relativity, but the action has a fixed point in the infrared (IR) which corresponds to GR.
- The analog of the Friedmann equation in HL gravity contains a term which scales in the same way as dark radiation in braneworld scenarios and gives a negative contribution to the energy density. Thus, at least in principle it is possible to obtain non-singular cosmological evolution within HL theory.

Hořava-Lifshitz cosmology

The equations for Hořava-Lifshitz cosmology are obtained in the low energy limit and imposing condition of homogeneity and isotropy of the metric.

$$H^2 = \frac{\kappa^2\rho}{12} + \frac{\kappa^4\mu^2\Lambda}{32}\frac{k}{a^2} - \frac{\kappa^4\mu^2}{64}\left(\Lambda^2 + \frac{k^2}{a^4}\right), \quad (1)$$

$$\dot{H} = -\frac{\kappa^2(\rho+p)}{8} - \frac{\kappa^4\mu^2\Lambda}{32}\frac{k}{a^2} + \frac{\kappa^4\mu^2}{32}\frac{k^2}{a^4}. \quad (2)$$

Matter is described by the scalar field:

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0.$$

The significant new terms in the above equations of motion are the $(1/a^4)$-terms on the right-hand sides of (1) and (2). They are reminiscent of the dark radiation term in braneworld cosmology and are present only if the spatial curvature of the metric is non-vanishing.

Hořava-Lifshitz cosmology

Introducing natural units $8\pi G = 1 = c$ and taking the IR limit $\lambda = 1$ reduces the analog of Friedmann equation to:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\rho_m + \rho_r\right) + \frac{1}{3}\left(\frac{3K^2}{2\Lambda a^4} + \frac{3\Lambda}{2}\right) - \frac{K}{a^2}.$$

We also define in the IR limit the canonical density parameters for the current universe $a_0 = 1$ (subscript 0 indicates the value as measured today) as follows

$$\Omega_m^0 = \frac{\rho_m}{3H_0^2}, \quad \Omega_r^0 = \frac{\rho_r}{3H_0^2}, \quad \Omega_k^0 = -\frac{K}{H_0^2 a_0^2},$$

where H_0 is the Hubble parameter. Using these parameter we rewrite Friedmann equation as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_m^0 a^{-3} + \Omega_r^0 a^{-4} + \Omega_k^0 a^{-2} + \frac{(\Omega_k^0)^2 H_0^2}{2\Lambda} a^{-4} + \frac{\Lambda}{2H_0^2}\right].$$

Hořava-Lifshitz cosmology

In the above Friedmann equation we encounter the term $\Omega_k^2 H_0^2/2\Lambda a^4$, which is the coefficient of dark radiation.
We can conveniently express this in terms of the effective number of neutrino species present in the BBN epoque: $\frac{(\Omega_k^0)^2 H_0^2}{2\Lambda} = 0.135\Delta N_\nu \Omega_r^0$.

Now, since all the density parameters have to add up to unity we have:

$$\Omega_m^0 + \Omega_r^0 + \Omega_k^0 + \frac{(\Omega_k^0)^2}{4 \cdot 0.135\Delta N_\nu \Omega_r^0} + 0.135\Delta N_\nu \Omega_r^0 = 1,$$

and we can rewrite the Friedmann equation as:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_m^0 a^{-3} + \Omega_r^0 a^{-4} + \Omega_k^0 a^{-2} + \frac{(\Omega_k^0)^2}{4 \cdot 0.135\Delta N_\nu \Omega_r^0} + 0.135\Delta N_\nu \Omega_r^0 a^{-4}\right],$$

and this is the equation that we have used in our MCMC analysis of detailed balance.

Observational constraints

- Using the Friedmann equations as a starting point we wanted to find the parameter set which best fits the data. We used a large updated data set with CMB (Planck), SN1a, BAO and more.
- We also used a Markov-Chain Monte Carlo (MCMC) method. The parameters were completely unconstrained but were given initial guesses, which speeded up computation.
- We introduced a Gaussian prior on one parameter: H_0, derived from the Hubble constant value, $H_0 = (69.6 \pm 0.7)$ km s^{-1} Mpc^{-1}.
- During every step in the computation, the MCMC method calculated the χ^2, and in the end returns the parameter set which minimized the χ^2 function. This way, we were able to obtain information about the posterior probability distribution without knowing it explicitly.

Observational constraints

Parameter	DB: 1σ limits
Ω_m^0	0.316 ± 0.0054
Ω_k^0	$(-2.27 \pm 0.25) \cdot 10^{-3}$
Ω_r^0	$(9.08 \pm 0.10) \cdot 10^{-4}$
Ω_{DE}^0	0.686 ± 0.0053
H_0	68.530 ± 0.413
ΔN_ν	0.155 ± 0.033
Λ	$\left(0.676^{+0.125}_{-0.128}\right) \cdot 10^{-35}$

Table: Constraints on the parameters, the units of H_0 are km$\cdot$s$^{-1}\cdot$Mpc^{-1} and of Λ are s^{-2}).

Observational constraints

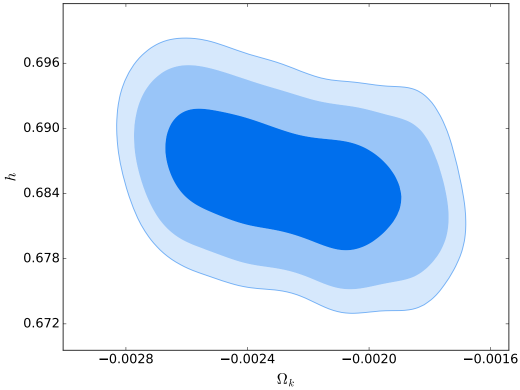


Figure: $1, 2$, and 3σ contours of the curvature parameter Ω_k^0 and the dimensionless Hubble parameter *h* under detailed balance. Solid (blue) colour corresponds to the 1σ limit. Spatial flatness ($\Omega_k^0 = 0$) is excluded at more than 3σ.

Hořava-Lifshitz cosmology

- Our initial simulations revealed that the spatial curvature is actually significantly different from zero.

- The value of $\Lambda \sim 10^{-52}\mathrm{m}^{-2}$ and the cosmological constant in the ΛCDM model is of the same order.

  Details in:

- N. A. Nilsson and E. Czuchry, *Horava-Lifshitz cosmology in light of new data*, Phys. Dark Universe (2018), accepted, in production.

Future prospects

- Recently, LIGO found evidence of a binary black hole merger, which was consistent with the prediction from general relativity for such events. Moreover, LIGO and VIRGO have also observed a binary neutron star which was accompanied by a short γ-ray burst.
- This put strong constraints on the speed of tensorial gravitational waves, as the difference Δt from the speed of light c was found to be $-3 \cdot 10^{-15} < \Delta t/c < 7 \cdot 10^{-16}$.
- As opposed to GR, which permits only tensor metric perturbations, alternative models of gravity have more degrees of freedom, e.g. HL gravity allows vector and tensor modes.
- Recently, the LIGO and VIRGO team reported the first ever direct limits of the strain of scalar and vector modes at $< 1.5 \cdot 10^{-26}$ at 95%.