An instanton-motivated approach to the spontaneous fission of odd nuclei

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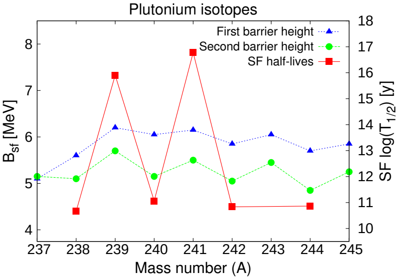
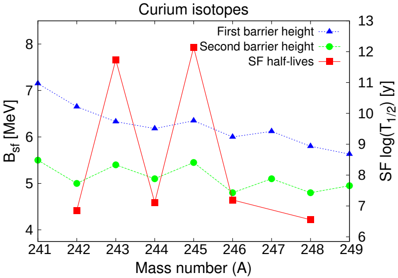
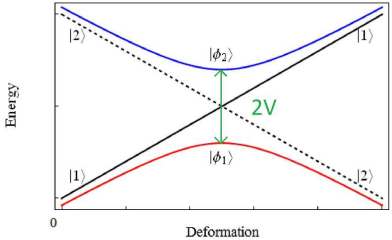
Halflives of nuclei with even/odd number of neutrons





Landau-Zener transition



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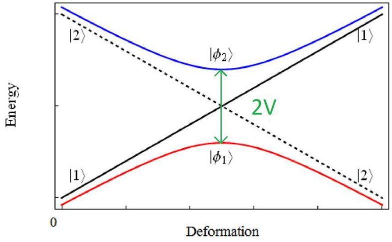
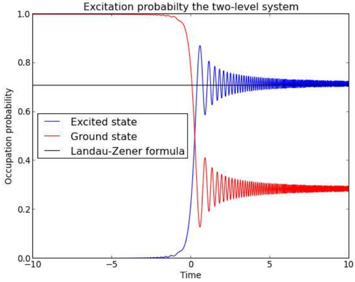


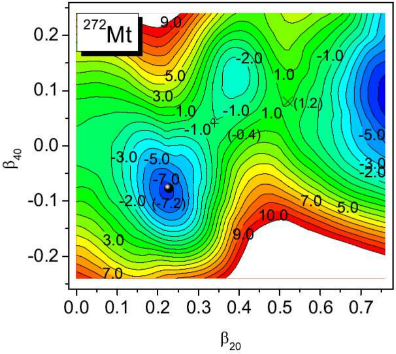


If the system is initially ($t_i = -\infty$) in the state $|\phi_1>$ the probability, that it finds itself in the state $|\phi_2>$ at $t_f = +\infty$ is given by the Landau-Zener formula:

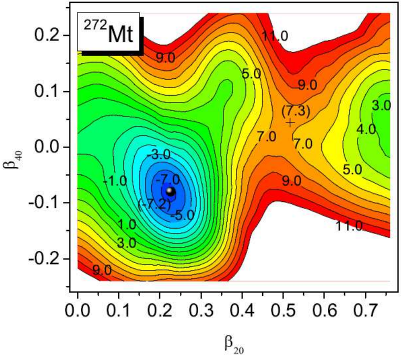
$$P_{|\phi_2>}(t \to +\infty) = exp\left(\frac{-2\pi}{\hbar}\frac{V^2}{\dot{q}\frac{\partial}{\partial q}(E_2 - E_1)}\right)$$

Adiabatic vs. diabatic scenario

Z=109, N=163



Minimization over possible configurations.



Keeping configuration fixed:

$\pi 11/2^{+}\ \nu 13/2^{-}$

Fixing the g.s. configuration rises the barrier by 6 MeV!!!

Description of spontaneous fission within adiabatic approximation

- Spontaneous fission of a nucleus: collective quantum tunneling process
- Set of collective variables $\{q_i\}$ usually chosen as parameters describing the shape of the fissioning nucleus
- Assumption that variations of the collective degrees of freedom are much slower comparing with the oscillations of individual nucleons $\Rightarrow$ **adiabatic approximation**
- Adiabatic mass parameter:

$$B_{ij} = 2\hbar^2 \sum_k \frac{\langle k|\partial/\partial q_i|0\rangle \langle 0|\partial/\partial q_j|k\rangle}{E_k - E_0}$$

- Calculation of the action corresponding to a trajectory L:

$$S(L, E_0) = \int_L \sqrt{2B_L(\boldsymbol{q}(s))[V(\boldsymbol{q}(s)) - E_0]}\, ds$$

- Minimizing the action in the space of collective coordinates and estimating the SF half-lives as:

$$\Gamma \propto exp\left[-\frac{2}{\hbar} S_{min}\right] \qquad T_{1/2} = \frac{ln2}{\Gamma}$$

Odd nucleus

Ground state of an odd nucleus in the form of BCS state:

$$|0 >= a^+_{\nu_0} \prod_{\mu \neq \nu_0} (u_\mu + v_\mu a^+_\mu a^+_{\bar{\mu}})|vac >$$

Calculating adiabatic mass parameter for this state we obtain the following formula:

$$\begin{aligned} B_{q_i q_j} &= 2\hbar^2 \Bigg[\sum_{\mu,\nu \neq \nu_0} \frac{\langle\mu|\, \partial_{q_i} \hat{H}\, |\nu\rangle \, \langle\nu|\, \partial_{q_j} \hat{H}\, |\mu\rangle}{(E_\mu + E_\nu)^3} (u_\mu v_\nu + u_\nu v_\mu)^2 \\ &+ \frac{1}{8} \sum_{\nu \neq \nu_0} \frac{(\tilde{\varepsilon}_\nu (\partial_{q_i} \Delta) - \Delta(\partial_{q_i} \tilde{\varepsilon}_\nu)) \left(\tilde{\varepsilon}_\nu (\partial_{q_j} \Delta) - \Delta(\partial_{q_j} \tilde{\varepsilon}_\nu)\right)}{E_\nu^5} \Bigg] \\ &+ 2\hbar^2 \sum_{\nu \neq \nu_0} \frac{\langle\nu|\, \partial_{q_i} \hat{H}\, |\nu_0\rangle \, \langle\nu_0|\, \partial_{q_j} \hat{H}\, |\nu\rangle}{(E_\nu - E_{\nu_0})^3} (u_\nu u_{\nu_0} - v_\nu v_{\nu_0})^2 \end{aligned}$$

- if another state comes close to the blocked state ν_0 then mass parameter explodes!
- if the blocked state ν_0 lies higher in energy than other state ν one gets negative values of mass parameter!

Starting point: time dependent Hartree Fock equations (TDHF):

$$i\hbar\partial_t\psi_k = \hat{h}(t)\psi_k(t)$$

where $\hat{h}[\psi^*(t), \psi(t)]\psi_k(t) = \delta\mathcal{H}/\delta\psi_k^*(t) \Rightarrow$ nonlinear dependence of $\hat{h}$ on ψ_k.

Properties:

- $< \psi_l|\psi_k >= const$,
- Energy $\mathcal{H} = const$.

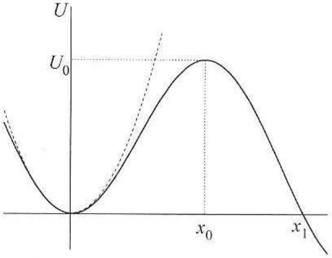
Because of the 2nd property TDHF equations cannot be directly used to describe fission process, one has to transform them to imaginary time i.e. $t \rightarrow -i\tau$. Under this transformation $\psi \rightarrow \psi(x, -i\tau) = \phi(x, \tau)$ and $\psi^* \rightarrow \psi^*(x, -i\tau) = \phi^*(x, -\tau)$.

In field theory:
S. Coleman, Phys. Rev. D 15 (1977) 2929
In nuclear mean-field theory:

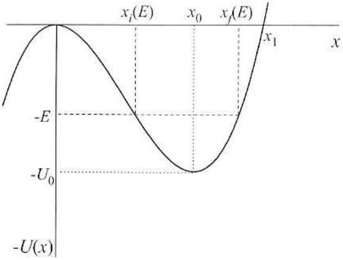
S. Levit, J.W. Negele and Z. Paltiel, Phys. Rev. C22 (1980) 1979

Dynamics in real and imaginary time

$$L = T - V$$



$$L = -(T + V)$$



After transformation of the TDHF equations to the imaginary time we obtain:

$$\hbar\frac{\partial \phi_k(\tau)}{\partial \tau} = -\hat{h}(\tau)\phi_k(\tau)$$

where $\hat{h}(\tau) = \hat{h}[\phi^*(-\tau), \phi(\tau)]$.
Since we require our solution to be periodic, i.e. $\phi_k(-T/2) = \phi_k(T/2)$, we add the periodicity fixing term $\epsilon_k\phi_k$ obtaining the instanton equations:

$$\boxed{\hbar\frac{\partial \phi_k(\tau)}{\partial \tau} = -(\hat{h}(\tau) - \epsilon_k)\phi_k(\tau)}$$

The action of an instanton can be calculated in the following way:

$$\boxed{S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \phi_k(-\tau) \middle| \partial_\tau \phi_k(\tau) \right\rangle}$$

Approximation: We replace the selfconsistent potential in the hamiltonian $\hat{h}(\tau)$ by the phenomenological Woods-Saxon potential.

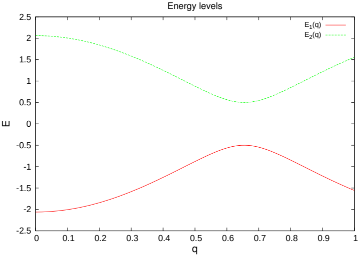
Action for the instanton:

$$S_{inst} = \hbar \int_{-T/2}^{T/2} d\tau \, \left\langle \psi(-\tau) \middle| \partial_\tau \psi(\tau) \right\rangle =$$

$$= \hbar \int_{-T/2}^{T/2} d\tau \sum_{i=1}^{2} c_i(-\tau) c_i(\tau)(\epsilon - E_i(q(\tau)))$$

Adiabatic action:

$$S_{adiab} = 2\hbar^2 \int_{-T/2}^{T/2} d\tau \, \dot{q}^2 \frac{\left|\langle \phi_2 | \partial_q \phi_1 \rangle\right|^2}{E_2 - E_1}$$



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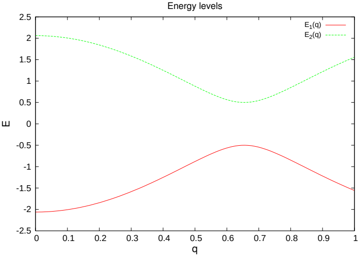
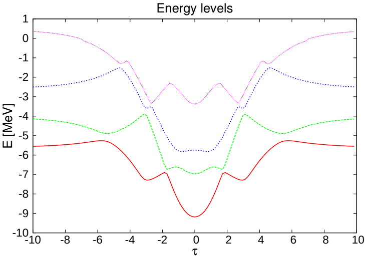
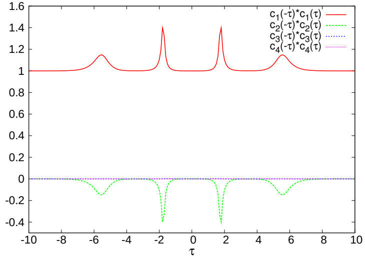


$\hbar\dot{q}/(E_2 - E_1)$	0.16	0.08	0.05	0.08	0.04	0.025
$V_{int}[MeV]$		0.5			1.0	
$S_{inst}/\hbar$	1.183	0.770	0.569	0.398	0.218	0.149
$S_{adiab}/\hbar$	2.015	1.007	0.672	0.459	0.229	0.152

More realistic case: four $1/2^+$ states taken from the deformed Woods-Saxon potential for Z=109, N=163





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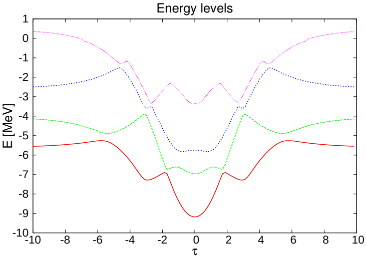

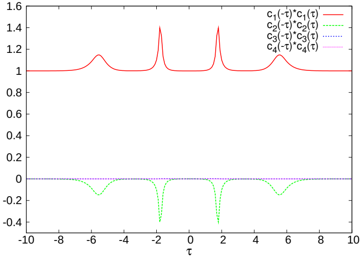

Instanton vs adiabatic action (1st state):

$\hbar\dot{q}_{max}$ [MeV]	$S_{inst}/\hbar$	$S_{adiab}/\hbar$
0.14	2.6818	55.048
0.09	2.4892	36.699
0.06	2.3492	25.689

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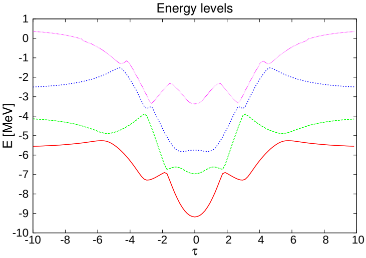

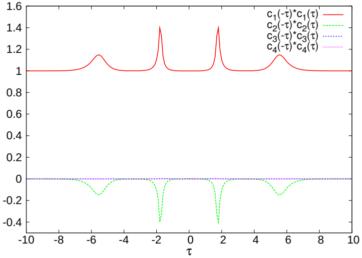

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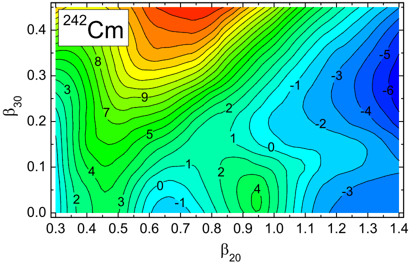
By the comparison of both action values we can see, how far from the adiabaticity condition we actually are!

...not only odd nuclei, but also (high) K-isomers!

Case of ^{242}Cm



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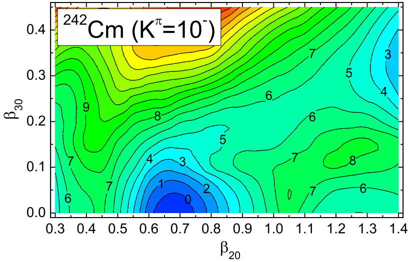

Keeping configuration fixed:

$\nu 11/2^{+}\ \nu 9/2^{-}$

Conclusions:

- Calculations of the SF halflives will provide predictions of where we could expect particularly long-living superheavy systems
- The usual treatment of spontaneous fission employing the adiabatic approximation breaks down in case of odd nuclei due to the presence of level crossings $\Rightarrow$ possible nonadiabatic transitions;
- **An approach based on the instanton formalism may provide more suitable way of describing fission process. The method does not break around level crossing points and gives correct value in the adiabatic limit.**

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The preliminary results of our approach were presented at:

- XXXV Mazurian Lakes Conference on Physics, Piaski, Poland, September 3 – 9, 2017,
- 3rd International Symposium on Super-Heavy Elements in Kazimierz Dolny, Poland, September 10 – 14, 2017,

and are subject of the publication in the Proceedings of XXXV Mazurian Lakes Conference on Nuclear Physics (Acta Phys. Pol. B)

Thank you for your attention!