

Color Instabilities in Quark-Gluon Plasma

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30 years

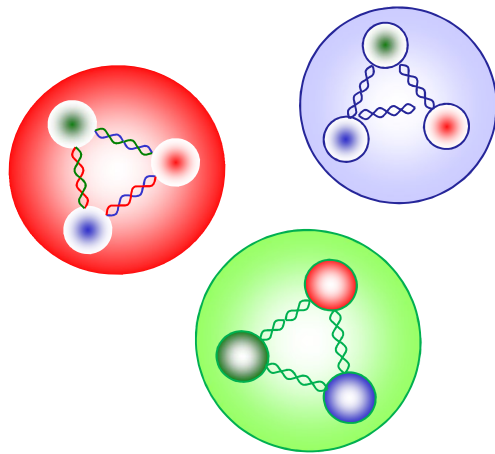
- 1) St. Mrówczyński,
Stream instabilities of the quark-gluon plasma,
Physica Letters B **214**, 587 (1988), Erratum B **656**, 273 (2007)
- 2) St. Mrówczyński,
Plasma Instability at the initial stage of ultrarelativistic heavy-ion collisions,
Physics Letters B **314**, 118 (1993)
- •
•
- 5) St. Mrówczyński and M. Thoma,
Hard loop approach to anisotropic systems,
Physical Review D **62**, 036011 (2000)
- •
•
- 17) St. Mrówczyński, B. Schenke and M. Strickland,
Color instabilities in the quark-gluon plasma,
Physics Reports **682**, 1 (2017)

**Elementary Physics Story
on Color Instabilities
in Quark-Gluon Plasma**

Hadrons, Quarks & Gluons

baryons

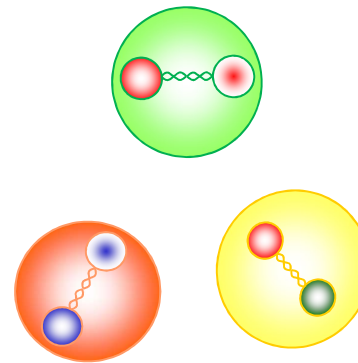
$n, p, \Delta, N^*, \Lambda, \Sigma, \Xi, \Omega, \dots$



(q, q, q)

mesons

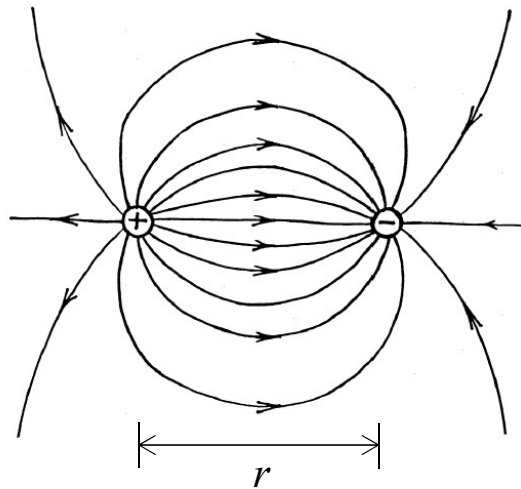
$\pi, K, \rho, \eta, \dots$



(q, \bar{q})

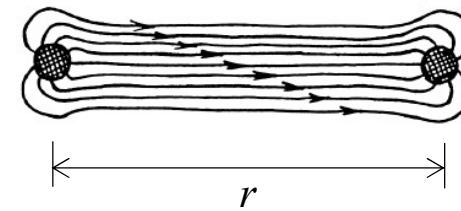
Confinement

Electrodynamics

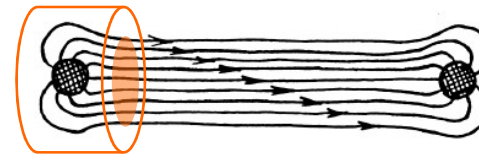


$$E(r) = \frac{e}{r^2} \Rightarrow V(r) = -\frac{e^2}{r}$$

Chromodynamics



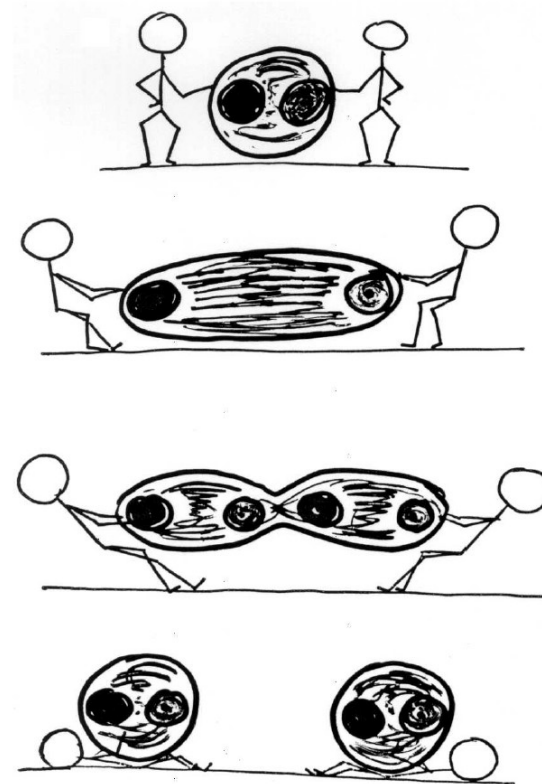
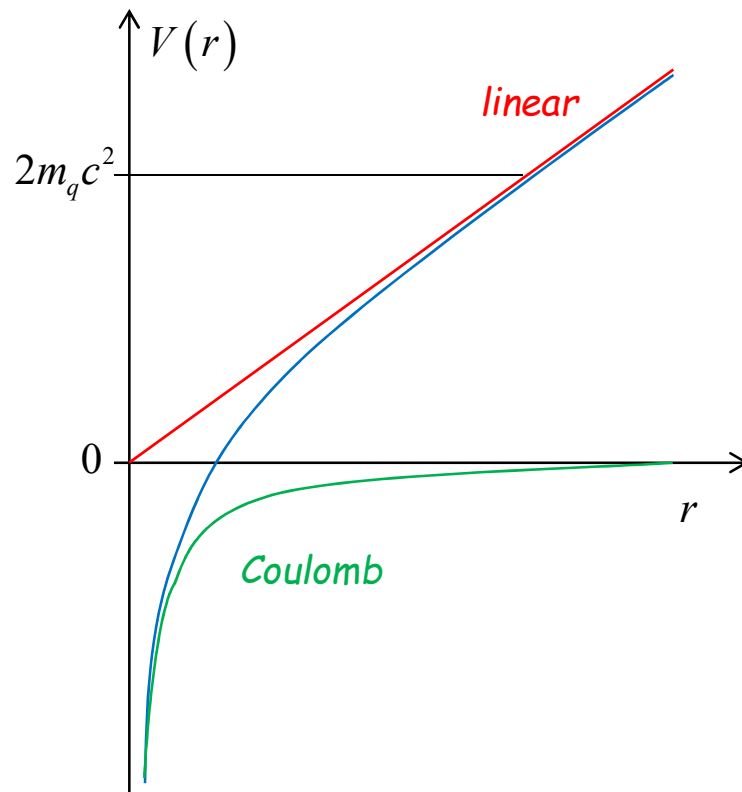
Gauss law



$$\Phi = \sigma E = 4\pi g \quad \sigma = \text{const}$$

$$E(r) = \frac{4\pi g}{\sigma} \Rightarrow V(r) = \frac{4\pi g}{\sigma} r$$

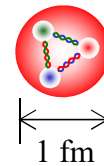
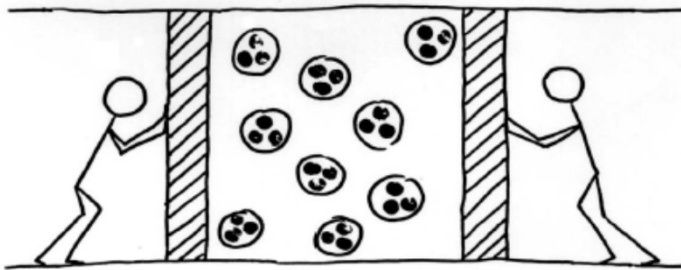
Confinement cont.



*The potential is studied
in spectroscopy of quarkonia.*

Creation of Quark-Gluon Plasma

compression of nuclear matter



$\rho_0 = 0.12 \text{ fm}^{-3}$
normal nuclear density

heating up hadron gas

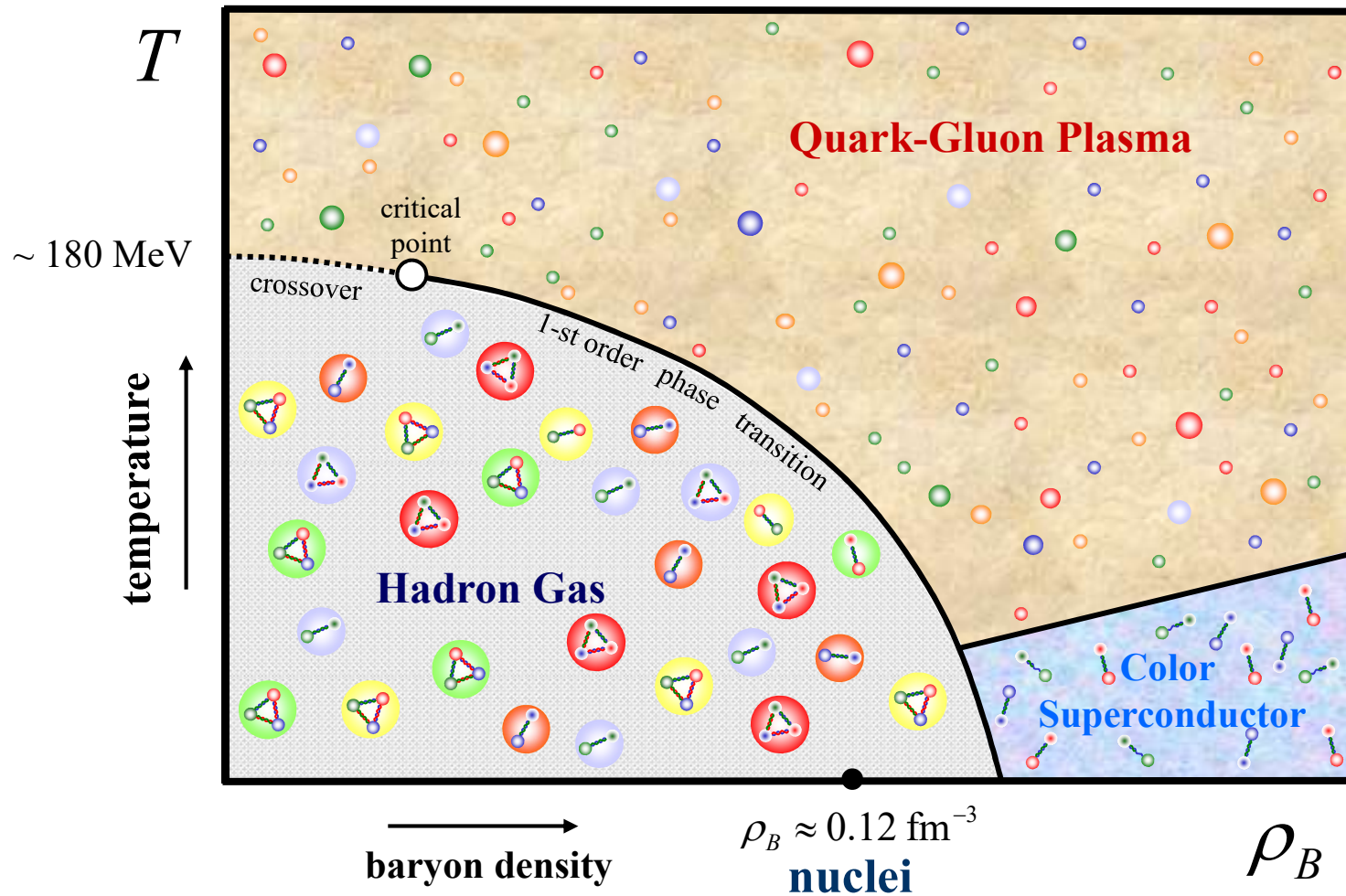


hadron
density

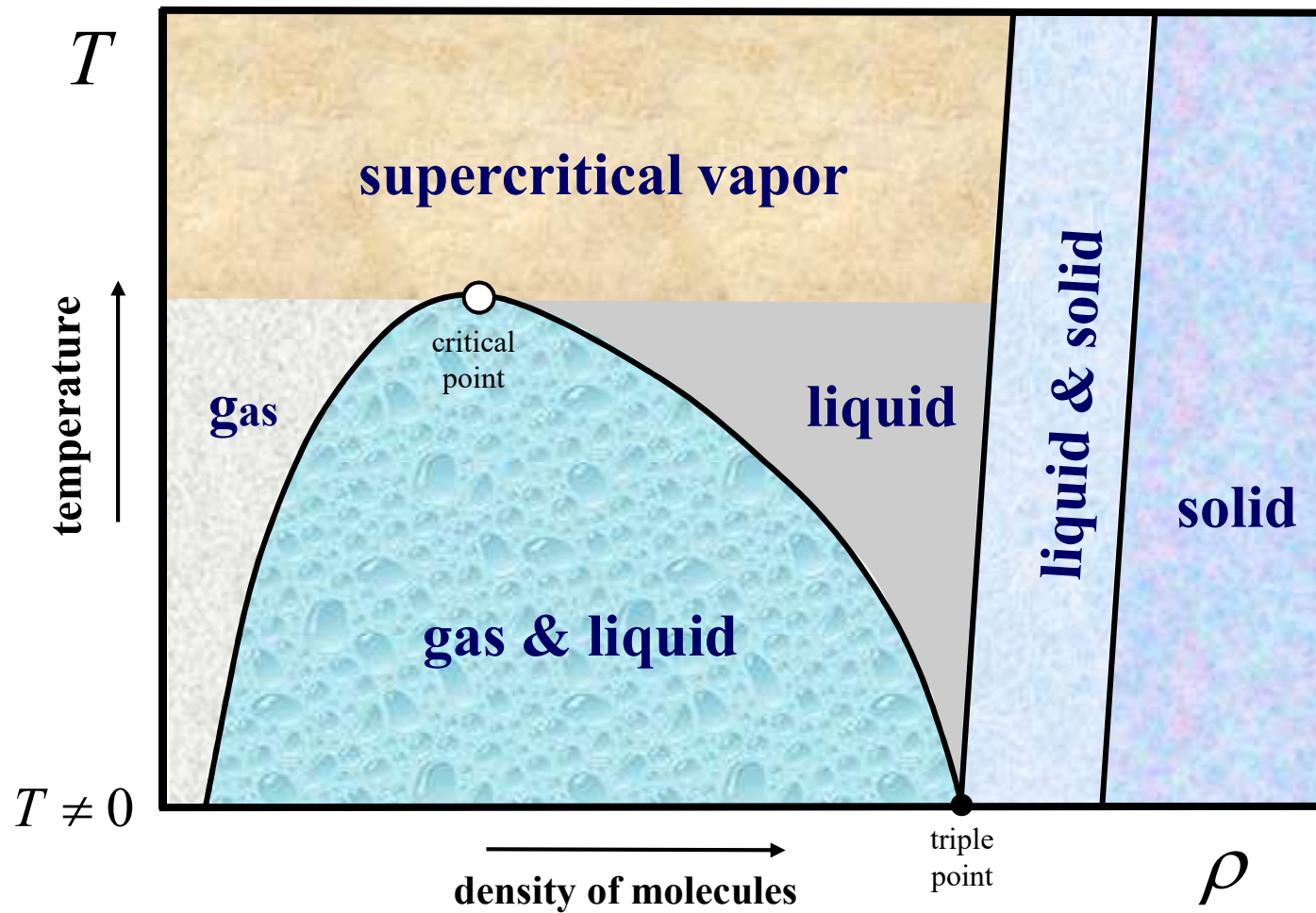
$$\rho \sim T^3$$
$$m_\pi \ll T$$

Natural system of units: $\hbar = c = k_B$

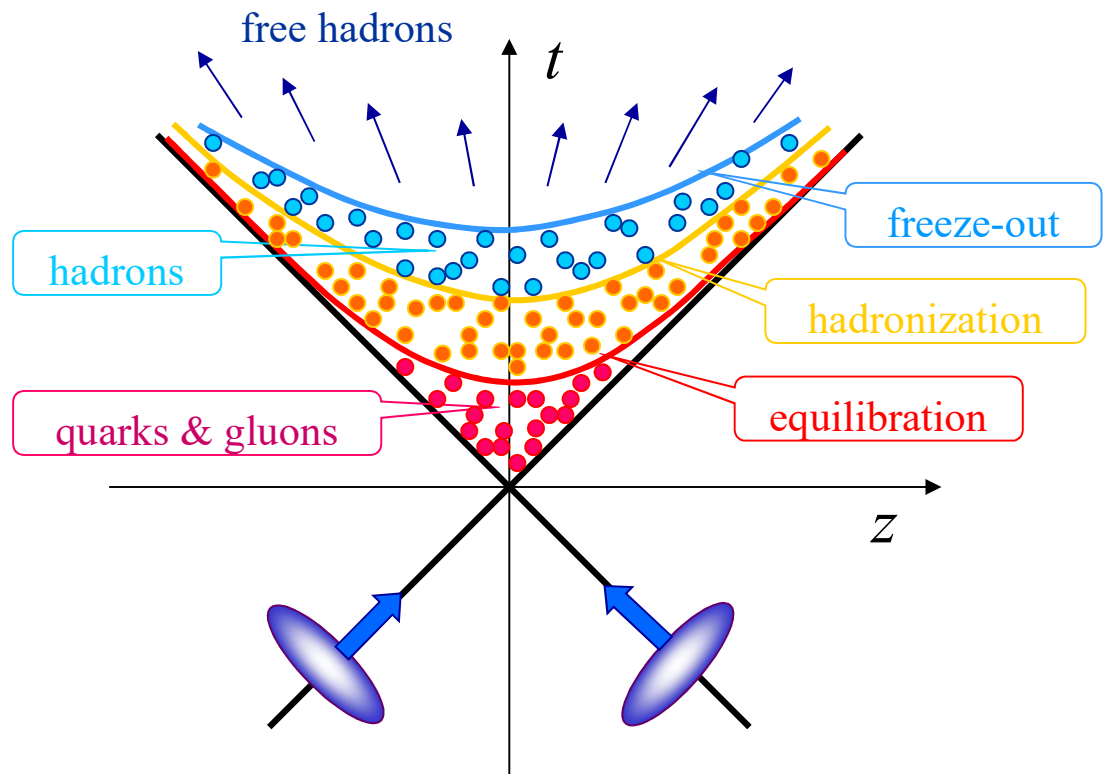
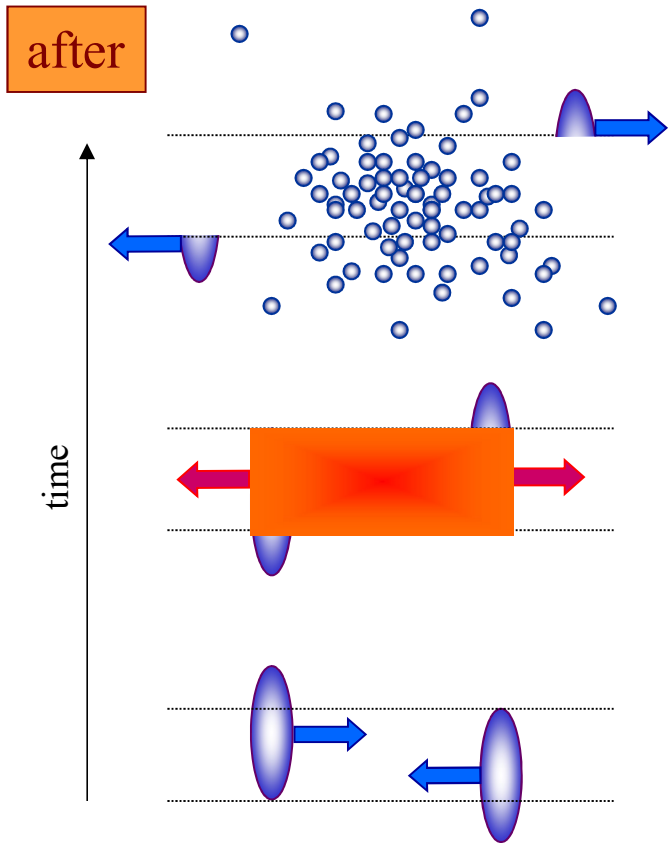
Phase diagram of strongly interacting matter



Schematic phase diagram of a simple fluid

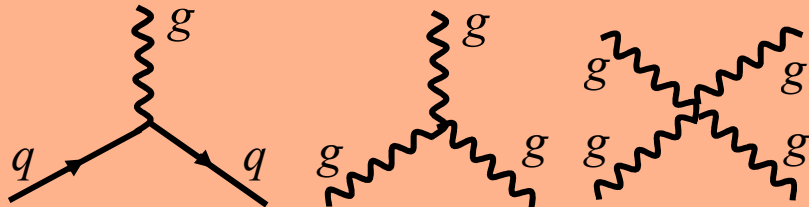
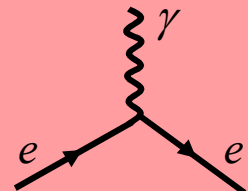


Relativistic heavy-ion collisions



An important role of boost invariance $\tau = \sqrt{t^2 - z^2}$

Quark-Gluon Plasma vs. EM Plasma

		Quark-Gluon Plasma	Electromagnetic Plasma
Underlying Microscopic Theory		QCD	QED
Elementary Interactions			
Constituents	Fermions	quarks, antiquarks	electrons, positrons
	Massless Gauge Bosons	gluons	photons
		-	massive ions
Coupling		$\alpha(Q^2) = \frac{g^2}{4\pi} \approx 0.1-1$	$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$

Ultrarelativistic Quark-Gluon Plasma

Plasma constituents – quarks & gluons – are massless!

$$m_q \ll T$$

Temperature T is often the only dimensional parameter.

density: $\rho \sim T^3$

inter-particle spacing: $l \sim T^{-1}$

energy density: $\varepsilon \sim T^4$

pressure: $p \sim T^4$

Weakly Coupled Quark-Gluon Plasma

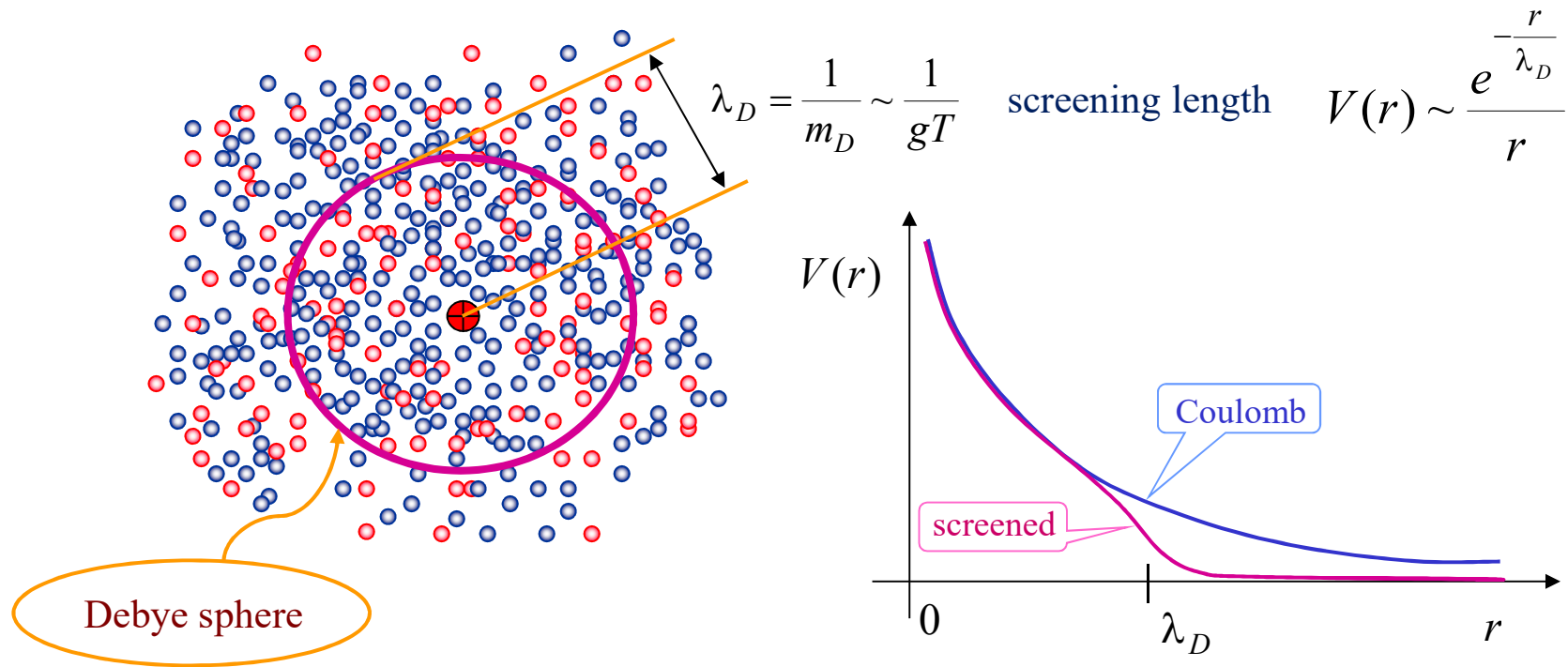
Plasma from the earliest stage of relativistic heavy-ion collisions is assumed to be weakly coupled.

Asymptotic freedom formula:
$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f) \ln\left(\frac{Q^2}{\Lambda_{\text{QCD}}^2}\right)}$$

Dimensional argument:
$$Q \rightarrow \mathcal{E}^{1/4}$$

\mathcal{E} - energy density

Plasma manifests collective behavior



$$V_D = \frac{4}{3} \pi \lambda_D^3 \sim \frac{1}{g^3 T^3}, \quad n \sim T^3, \quad n V_D \sim \frac{1}{g^3} \gg 1 \text{ if } g \ll 1$$

In a weakly coupled plasma, there are many particles in a Debye sphere!

Screening length

Poisson equation

$$\Delta V(\mathbf{r}) = -e\rho(\mathbf{r})$$

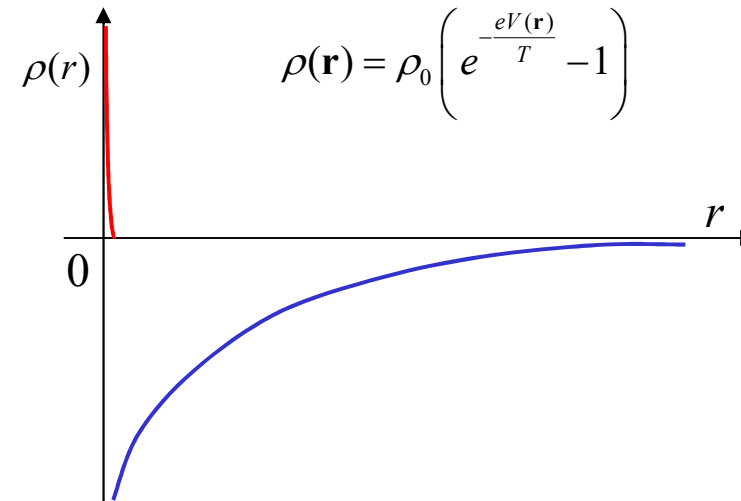
$$e|V(\mathbf{r})| \ll T$$

$$e^{\frac{eV(\mathbf{r})}{T}} - 1 \approx 1 - \frac{eV(\mathbf{r})}{T} \dots - 1 = -\frac{eV(\mathbf{r})}{T}$$

$$\Delta V(\mathbf{r}) = -e\rho(\mathbf{r}) \approx \frac{e^2 \rho_0}{T} V(\mathbf{r})$$

$$\frac{d^2}{dx^2} V(x) = m_D^2 V(x) \quad \Rightarrow \quad V(x) \sim e^{-m_D|x|}$$

charge density

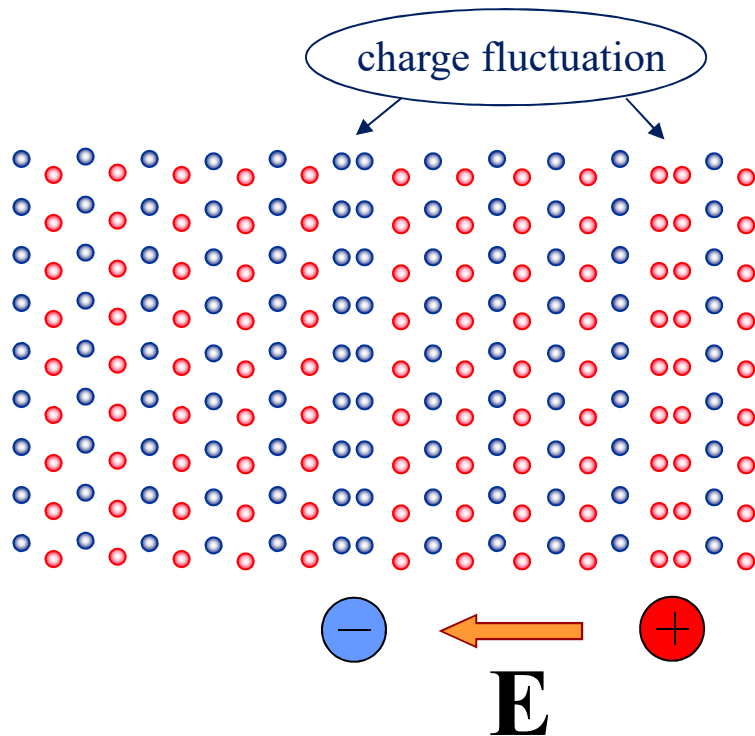


Debye mass

$$m_D \equiv e \sqrt{\frac{\rho_0}{T}} = \frac{1}{\lambda} \sim eT$$

$$\rho_0 \sim T^3$$

Plasma oscillations

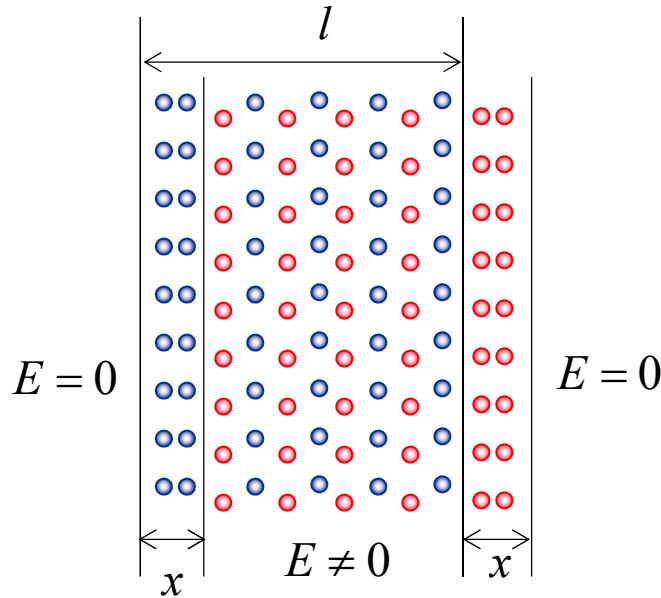


$$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}_0 \cos(\omega(\mathbf{k})t - \mathbf{k} \cdot \mathbf{r} + \varphi)$$

$$\omega(\mathbf{k}) \underset{\mathbf{k} \rightarrow 0}{\approx} \omega_p \sim gT$$

plasma or Langmuir frequency

Plasma frequency



Gauss theorem

$$\Phi = Q_s$$

Flux $\Phi = ES$

Charge $Q_s = e\rho Sx$

Electric field $E = e\rho x$

Equation of motion

$$M \ddot{x} = F$$

Mass $M = \rho mSl$

Force $F = QE$

Charge $Q = e\rho Sl$

Harmonic oscillator

$$\ddot{x} = -\omega_p^2 x$$

plasma frequency

$$\omega_p \equiv e \sqrt{\frac{\rho}{m}}$$

$$\lambda \rightarrow \infty$$

Quark-gluon plasma

$$e \rightarrow g$$

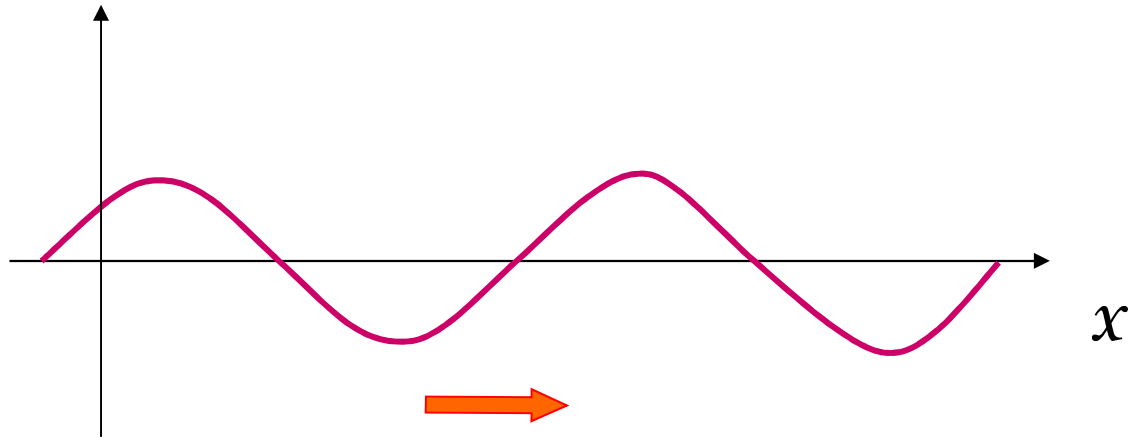
$$\rho \sim T^3$$

$$m \sim T$$

$$\omega_p \sim gT$$

Landau damping

$$E^x(t, x) = E_0 \cos(\omega_0 t - kx)$$



$$v_\phi = \frac{\omega_0}{k}$$

Resonance energy transfer from electric field to particles with $v = v_\phi$

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

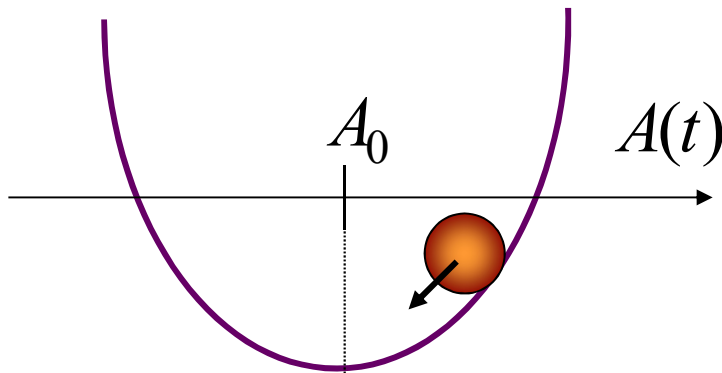
fluctuation

Instability

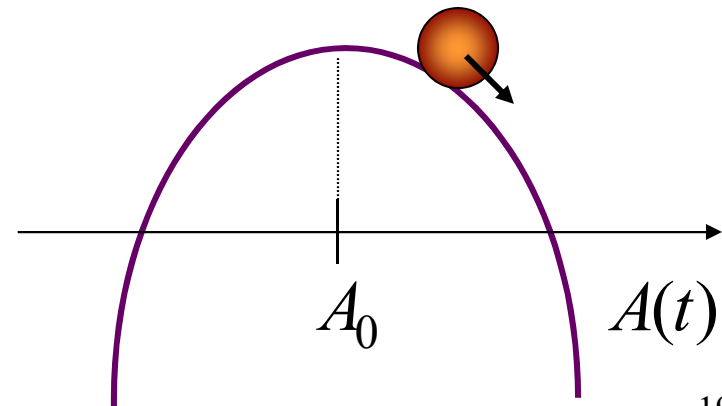
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Plasma instabilities

▶ instabilities in configuration space – **hydrodynamic instabilities**

▶ instabilities in momentum space – **kinetic instabilities**

instabilities due to non-equilibrium
momentum distribution

$$f(\mathbf{p}) \text{ is not } \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

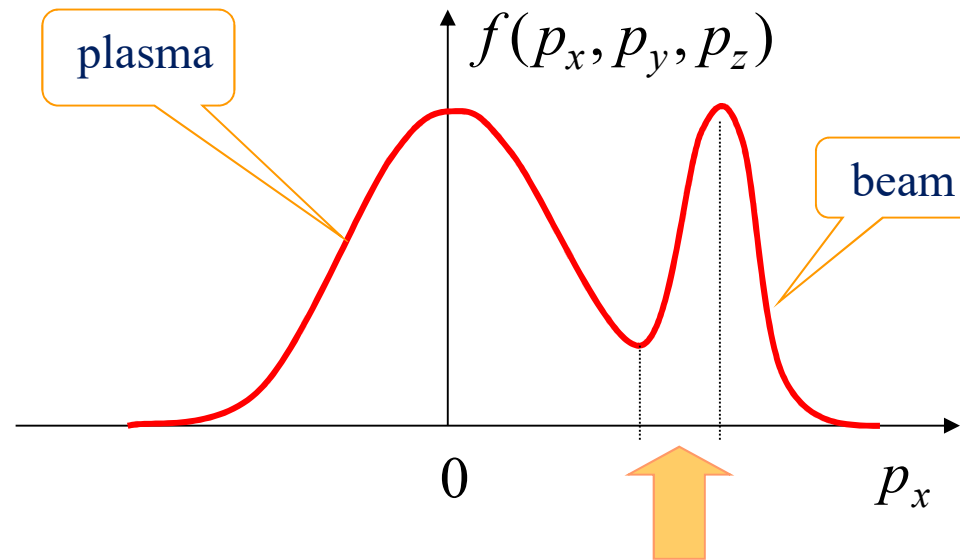
- ▶ longitudinal modes – $\mathbf{k} \parallel \mathbf{E}$, $\delta\rho \sim e^{-i(\omega t - \mathbf{k}\mathbf{r})}$
- ▶ transverse modes – $\mathbf{k} \perp \mathbf{E}$, $\delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{k}\mathbf{r})}$

\mathbf{E} – electric field, \mathbf{k} – wave vector, ρ – charge density, \mathbf{j} - current

Which modes are relevant for QGP
from relativistic heavy-ion collisions?

Logitudinal modes

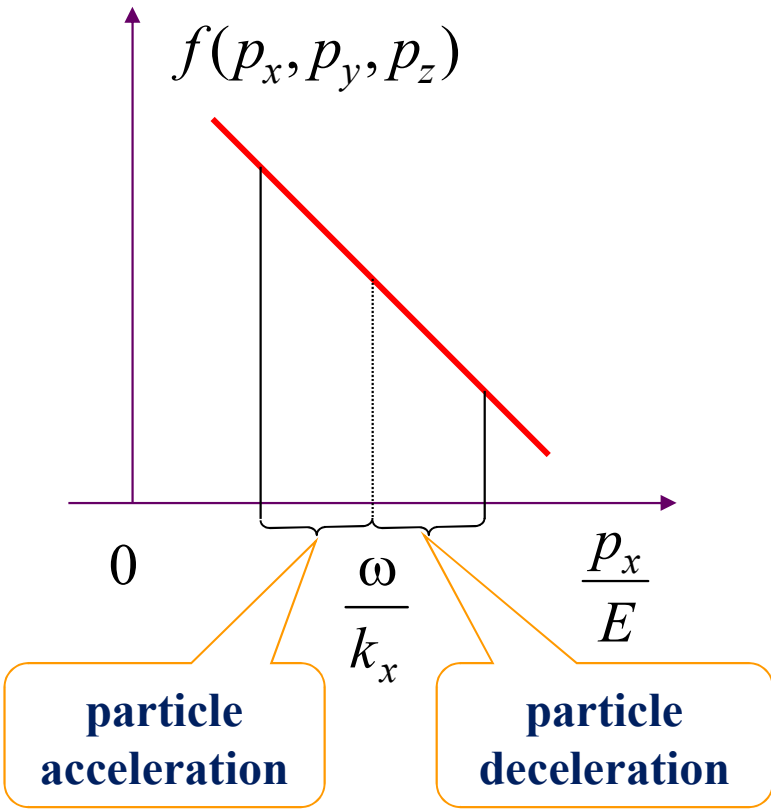
unstable configuration



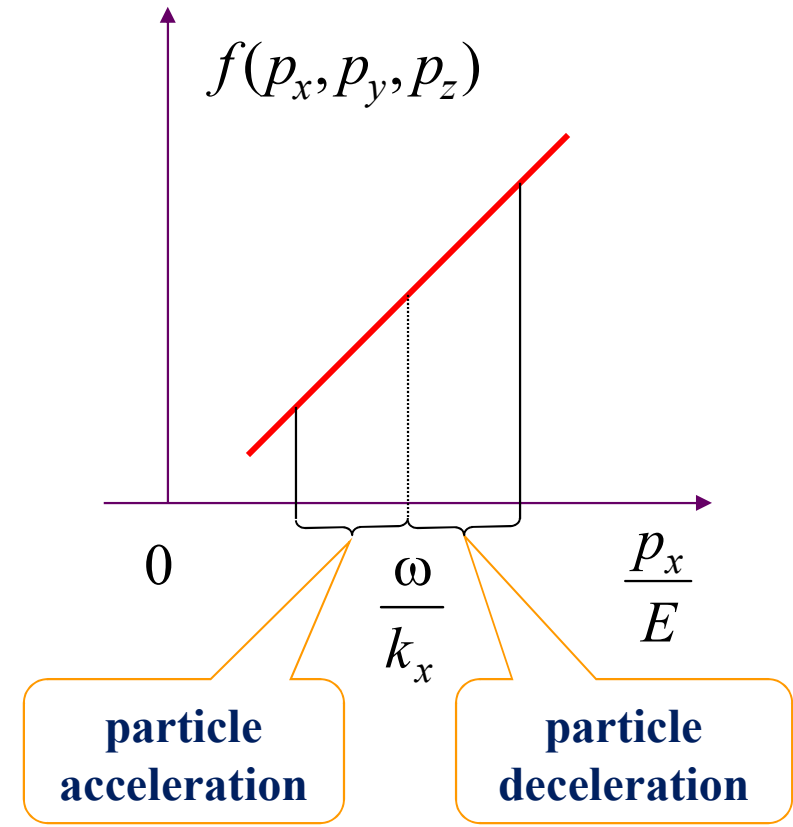
Energy is transferred from particles to fields.

Logitudinal modes

Electric field decays - damping



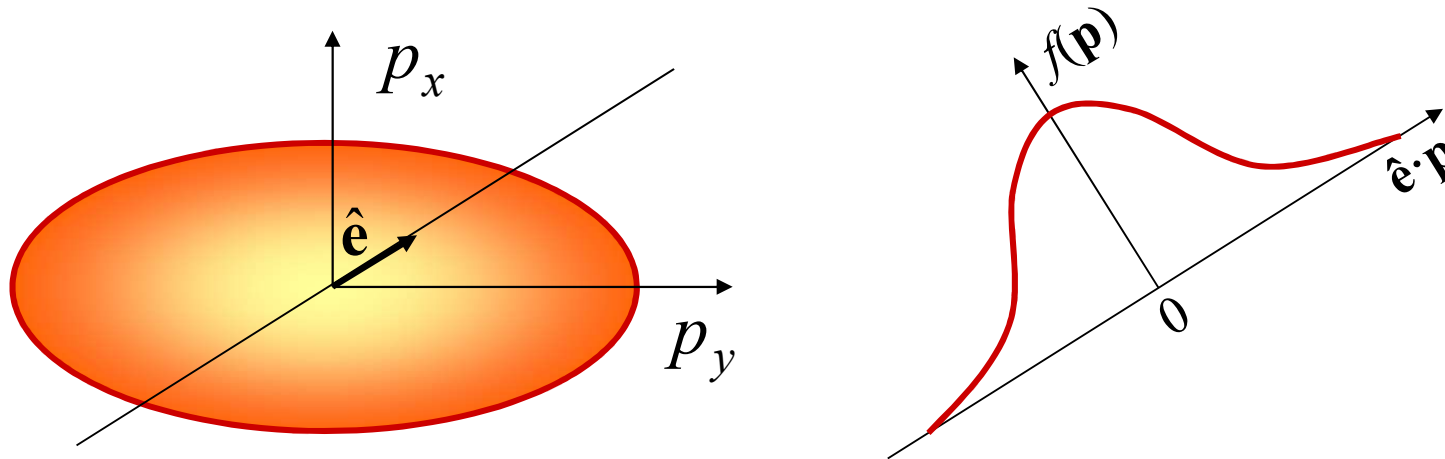
Electric field grows - instability



$\frac{\omega}{k_x}$ - phase velocity of the electric field wave,

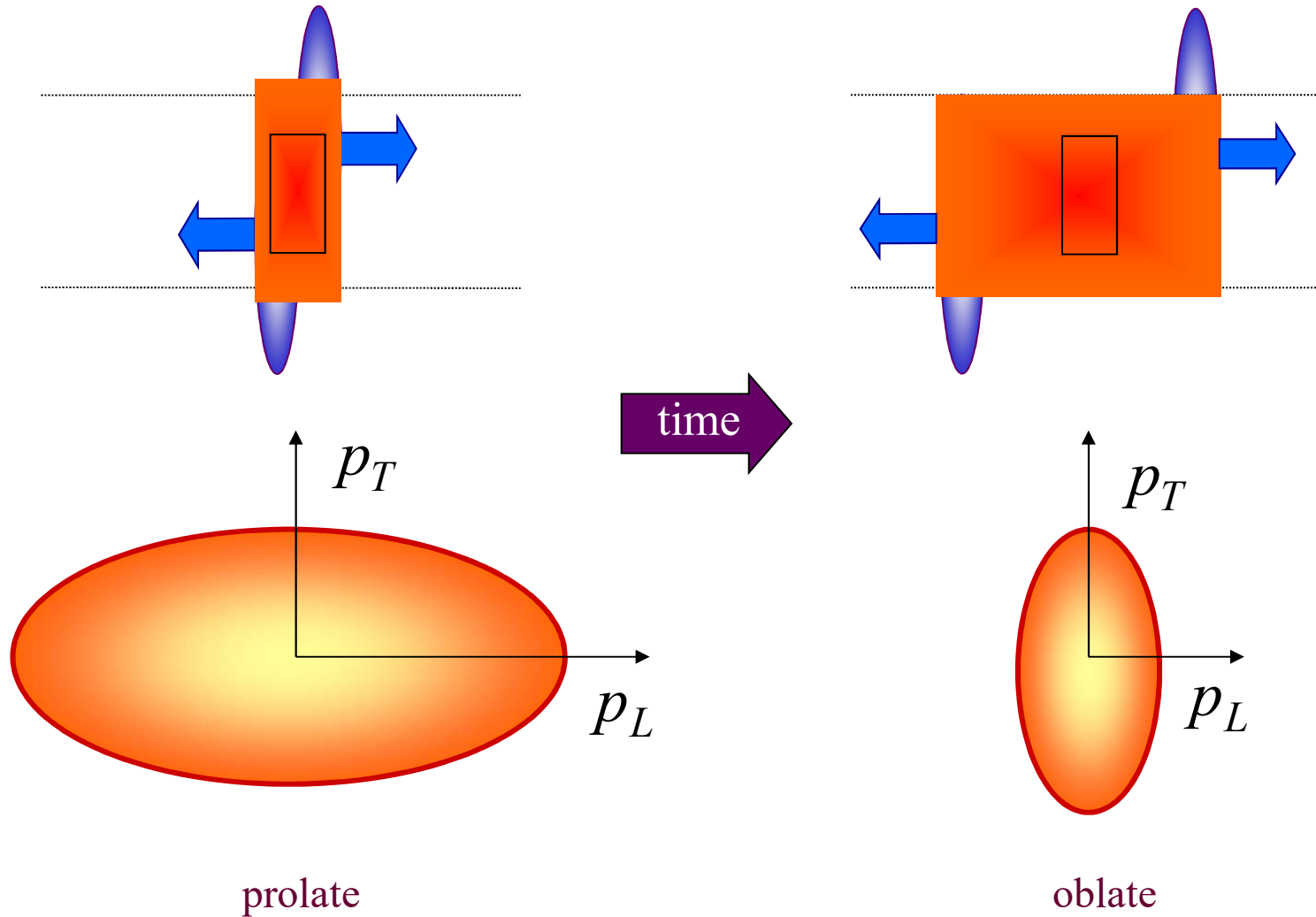
$\frac{p_x}{E}$ - particle's velocity

Parton momentum distribution in AA collisions



- ▶ Momentum distribution has a single maximum and monotonously decreases in every direction.
- ▶ Longitudinal unstable modes are irrelevant for relativistic heavy-ion collisions.
- ▶ There are unstable transverse modes.

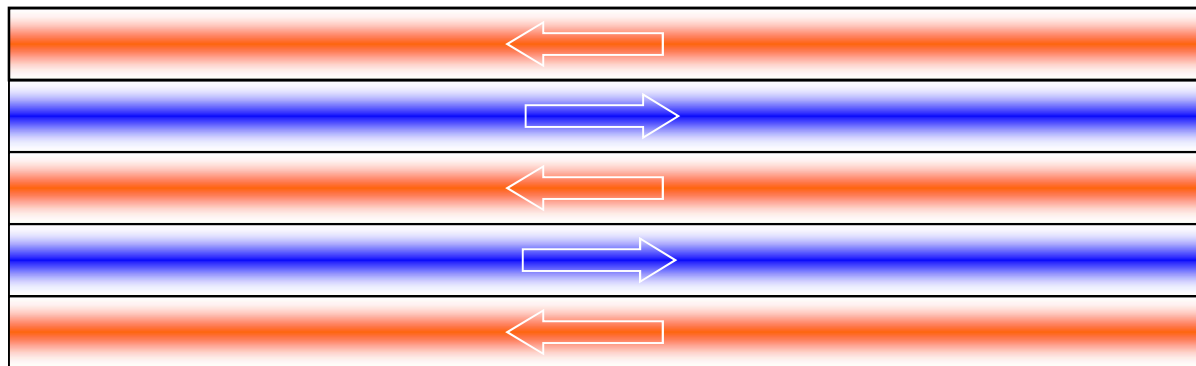
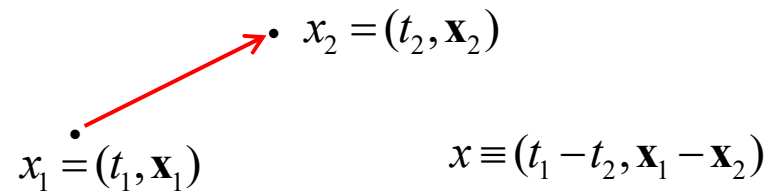
Evolution of Parton Momentum Distribution



Seeds of instability

$\langle j_a^\mu(x) \rangle = 0$ but current fluctuations are finite

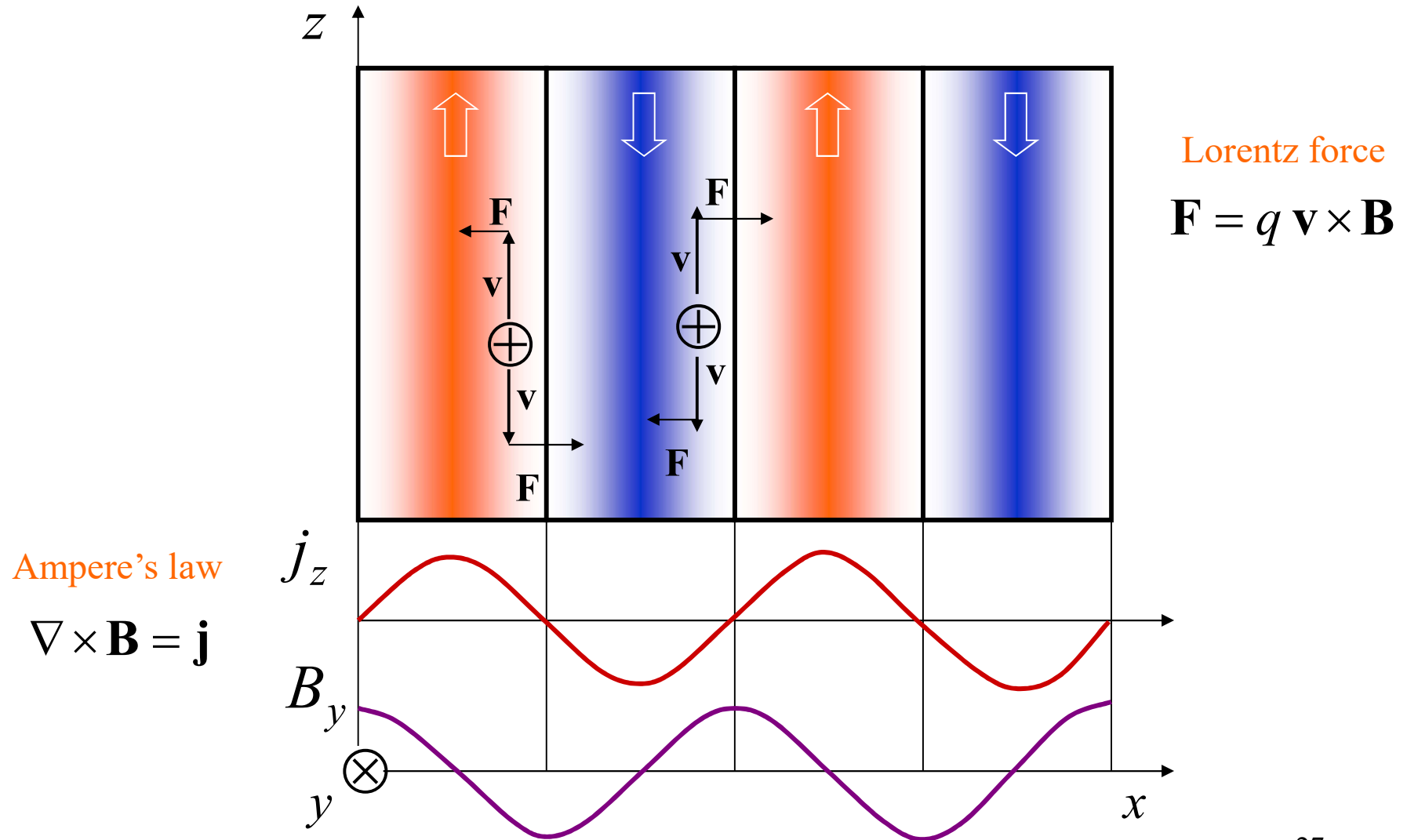
$$\langle j_a^\mu(x_1) j_b^\nu(x_2) \rangle = \frac{1}{2} \delta^{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0$$



Direction of the momentum surplus



Mechanism of filamentation

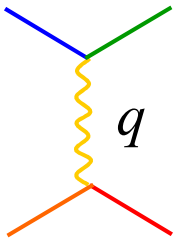


Time scale & collisional damping

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{gT} \Rightarrow \nu_{\text{collec}} \sim \frac{1}{t_{\text{collec}}} \sim gT$$

Parton-parton scattering



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Frequency of collisions

$$\nu_{\text{hard}} \sim g^4 \ln(1/g) T$$

$$\nu_{\text{soft}} \sim g^2 \ln(1/g) T$$

$$g^2 \ll 1 \Rightarrow \nu_{\text{hard}} \ll \nu_{\text{soft}} \ll \nu_{\text{collec}}$$

The instabilities are fast!

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

$$A_a^\mu = A_a^\mu(t, z)$$

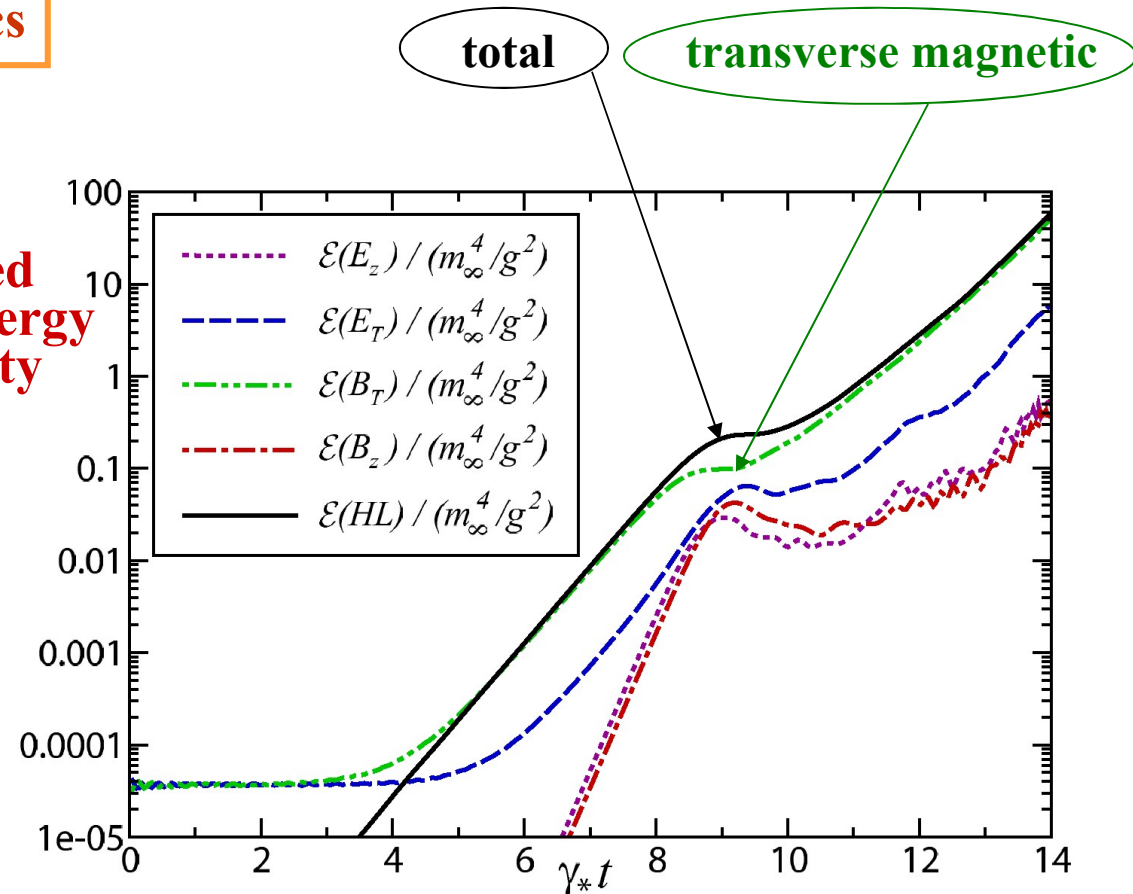
Scaled
field energy
density

Anisotropic particle's
momentum distribution

$$f(\mathbf{p}) = f_{\text{iso}}(|\mathbf{p}| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi} \int_0^\infty dp p^2 \frac{df_{\text{iso}}(p)}{dp}$$

(m_D, ζ)



Strong anisotropy $\zeta = 10$

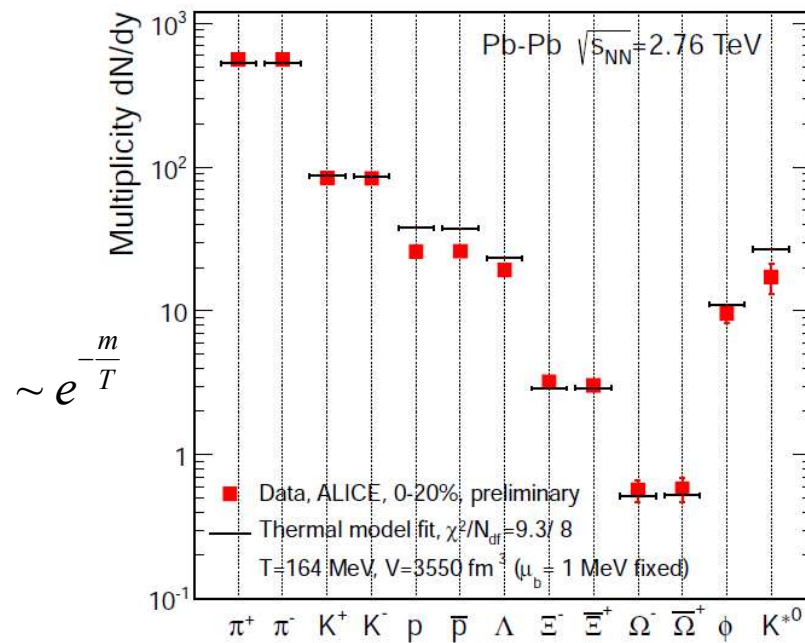
γ_* - maximal growth rate

What is the role of instabilities in nuclear collisions?

Instabilities speed up equilibration of quark-gluon plasma

Thermodynamic equilibrium in nuclear collisions

Chemical equilibrium

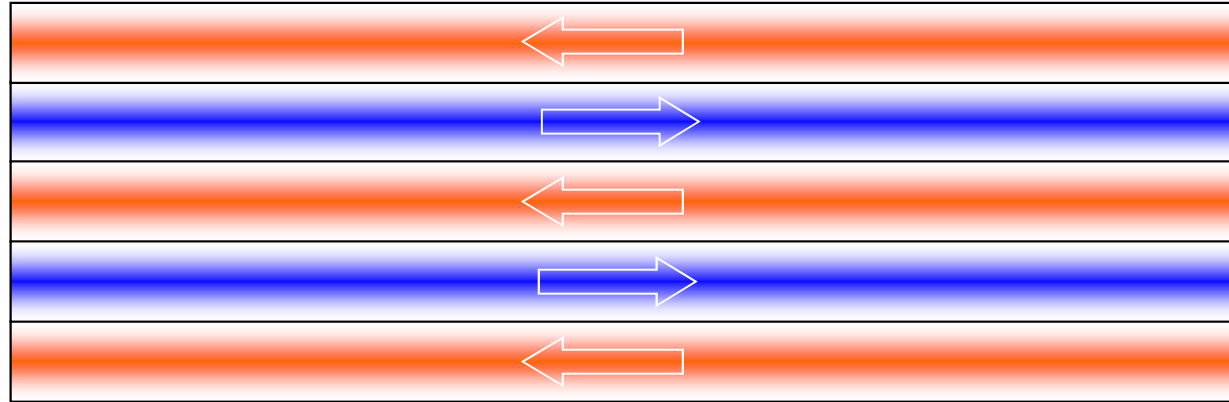


Kinetic equilibrium

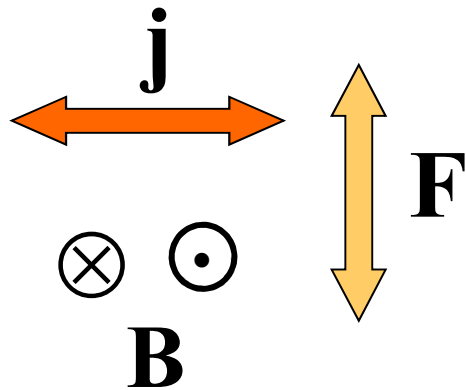
- particle spectra
- collective flow

A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel,
 Nuclear Physics A **904-905**, 535c (2013)

Isotropization - particles



Direction of the momentum surplus



$$\Delta \mathbf{p} = \int dt \mathbf{F}$$

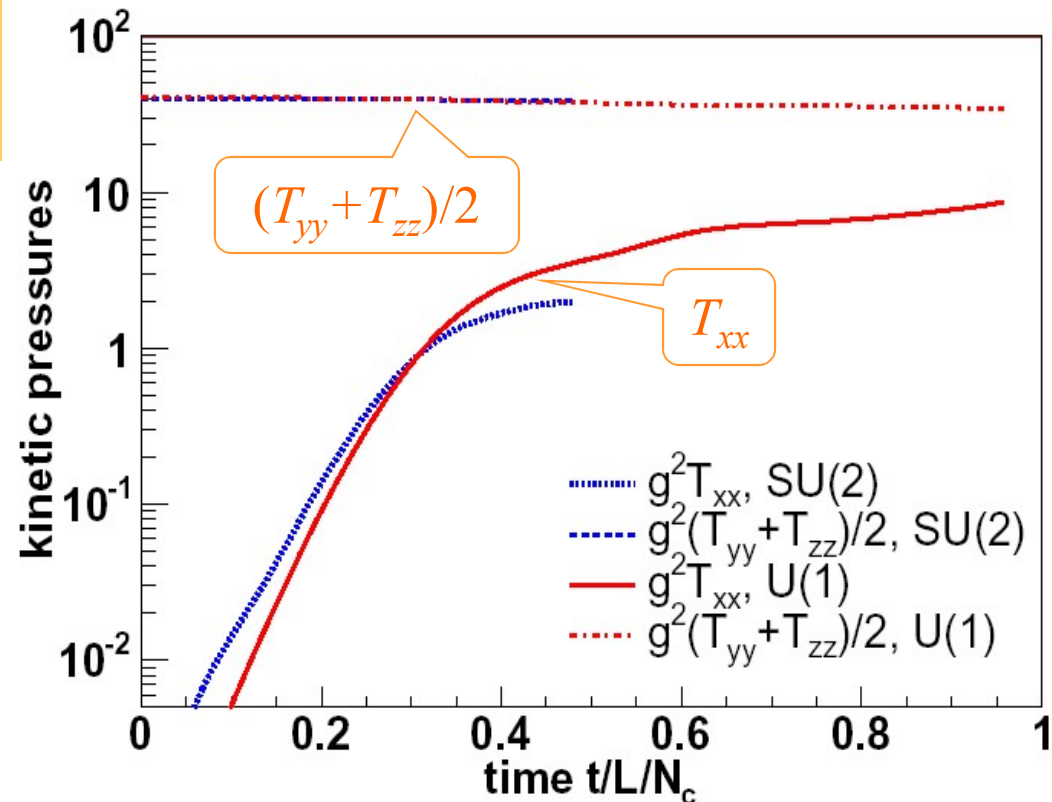
Isotropization – numerical simulation

Classical system of colored particles & fields

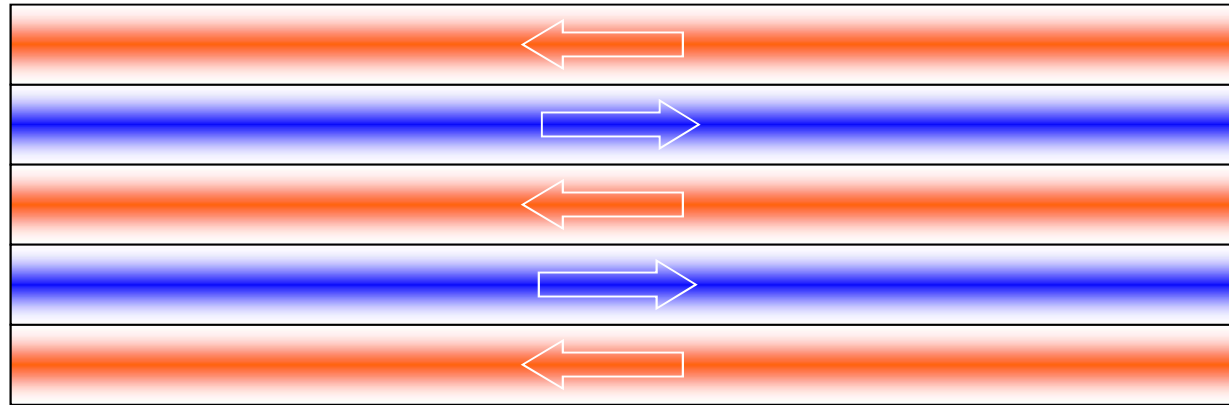
$$T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

Isotropy:

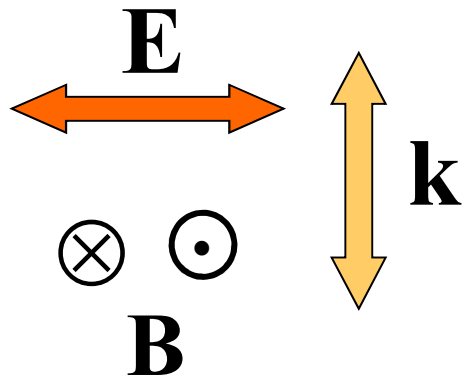
$$T_{xx} = (T_{yy} + T_{zz}) / 2$$



Isotropization - fields



Direction of the momentum surplus



$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^a \times \mathbf{E}^a \sim \mathbf{k}$$

Conclusions

- **Non-equilibrium QGP can be unstable**
- **Unstable transverse modes are relevant for AA collisions**
- **Instabilities drive equilibration**