Bose-Einstein condensation and classical fields

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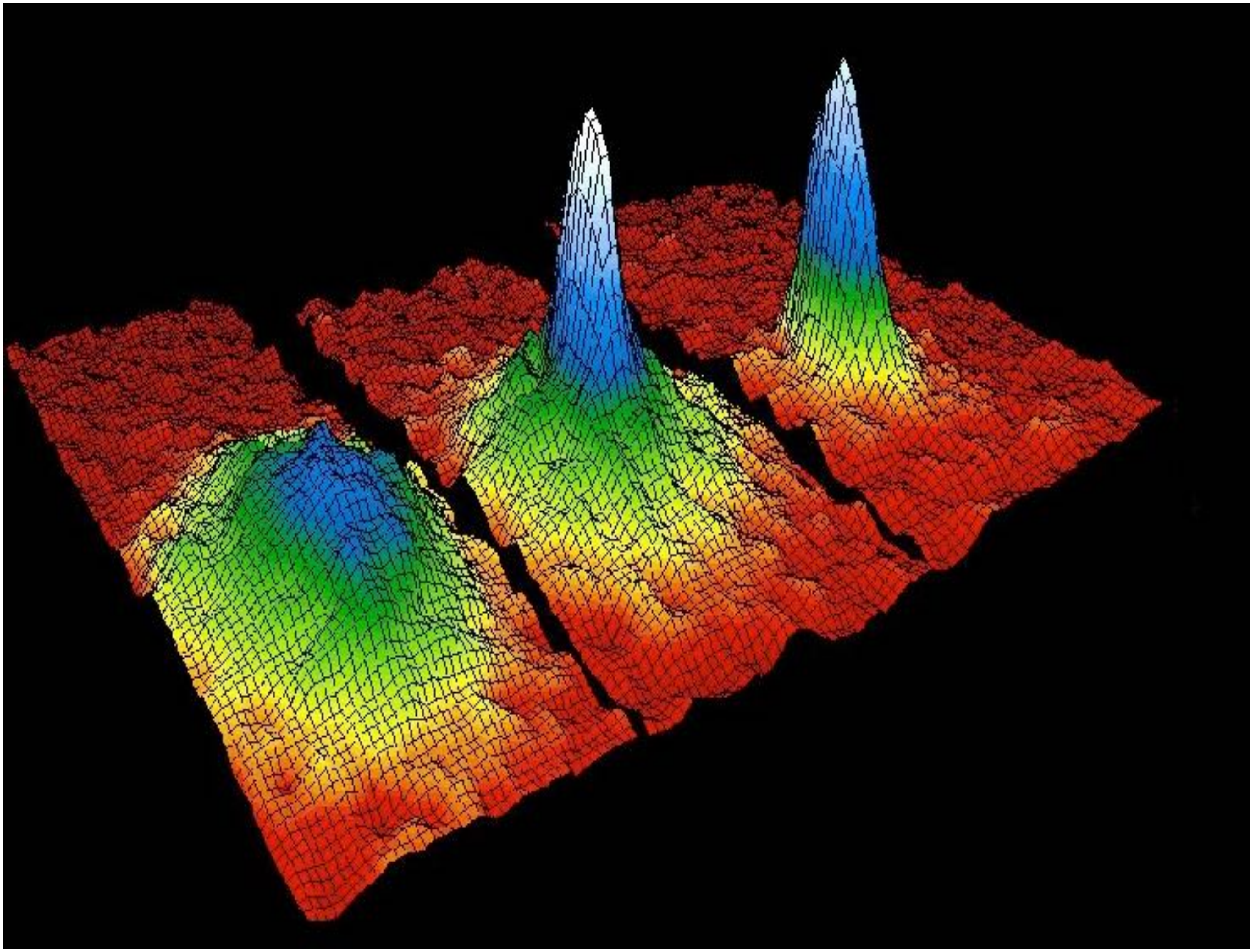
Filip Floegel
Peter Borowski
Demascoth Kadio

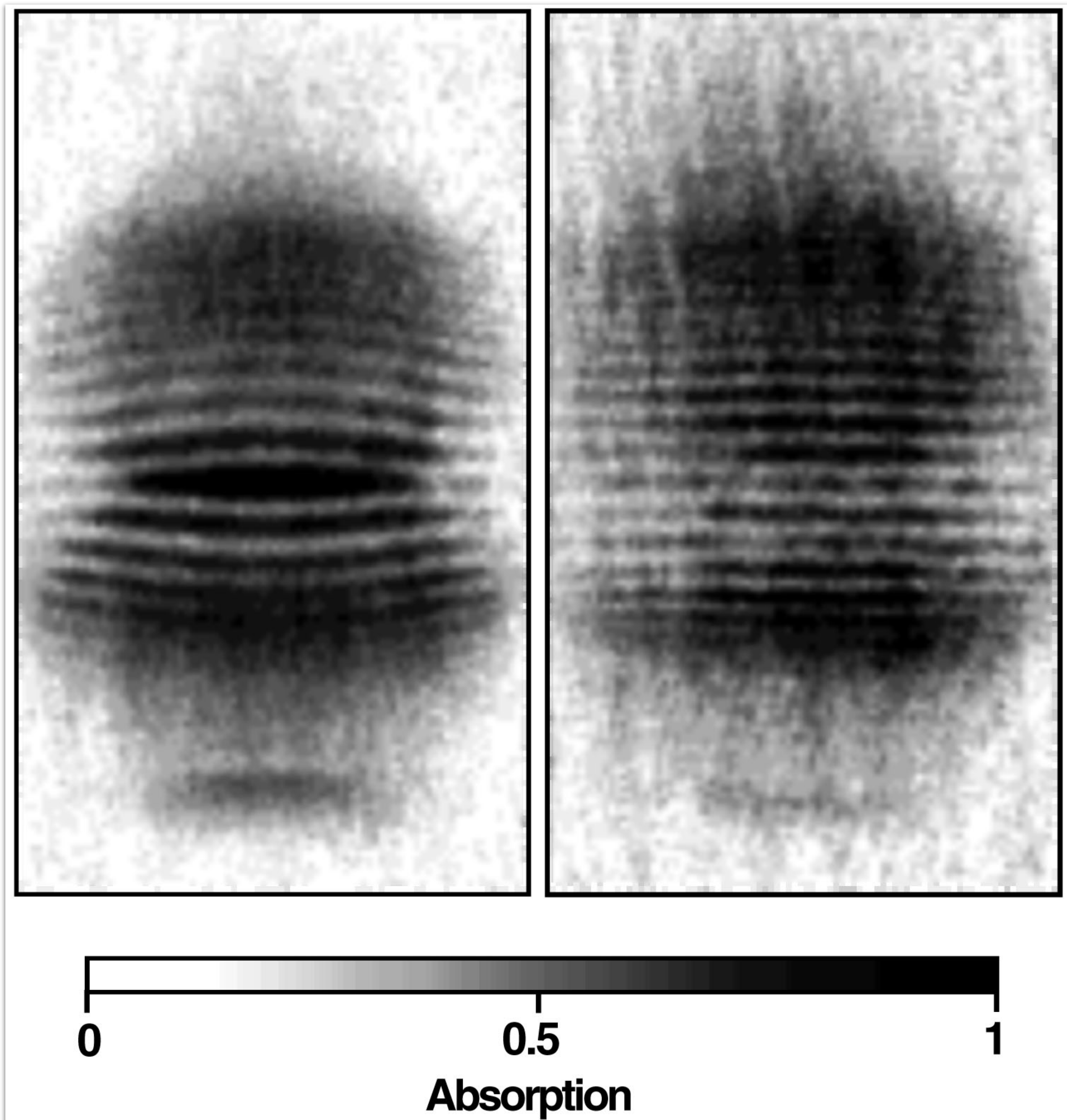
the first (Rubidium) condensate
E. Cornell and C. Wieman, JILA, Boulder, 1995



condensate: macroscopic matter waves

“finger prints” of quantum matter waves

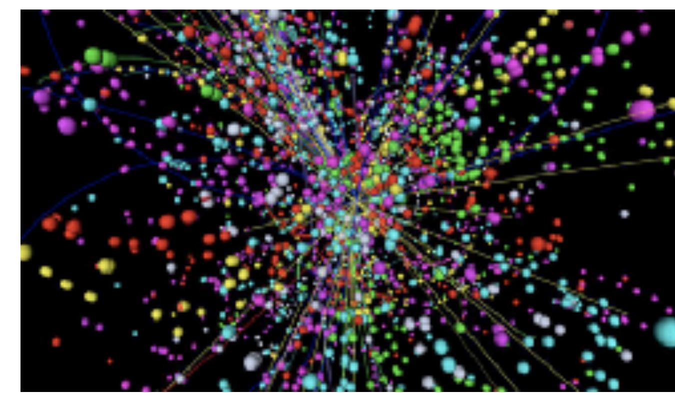
W. Ketterle -MIT, 1996



theory of light:

classical light described by Maxwell equations

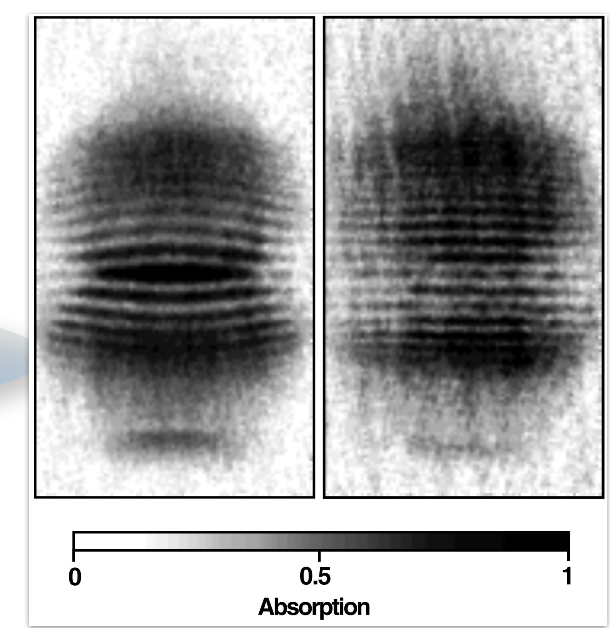
quantum theory of light



theory of cold bosons

atoms are particles

cold bosons are mesoscopic matter waves



Full theory in second quantization:

$$H=\int\hat{\Psi}^{+}\left[\frac{\vec{p}^{2}}{2m}+V_{trap}\right]\hat{\Psi}d^{3}r+\frac{g}{2}\int\hat{\Psi}^{+}\hat{\Psi}^{+}\hat{\Psi}\hat{\Psi}d^{3}r$$

Heisenberg equation for atom field

$$\left[\frac{\vec{p}^{2}}{2m}+V_{trap}(\vec{r})+g\hat{\Psi}^{\dagger}\hat{\Psi}\right]\hat{\Psi}=i\hbar\frac{\partial}{\partial t}\hat{\Psi}$$

Bogoliubov approximation: $\hat{\Psi}=\sqrt{N}\psi+\hat{\delta}$

condensate **thermal**

$$\left[\frac{\vec{p}^{2}}{2m}+V_{trap}(\vec{r})+Ng\,|\psi(\vec{r},t)|^{2}\right]\psi(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

What did we learn from Quantum Optics?

- it is useful to know the modes of the field
- many quanta in each mode - classical field

$$\hat{a} \Rightarrow \alpha$$

classical fields approximation

$$\hat{\Psi}(x) = \sum_{n} \varphi_n(x)\hat{a}_n \rightarrow \Psi(x) = \sum_{n}^{n_{\max}} \varphi_n(x)\alpha_n$$

NOTE: this is NOT a common wave function.

close analogy to classical electromagnetic field

to define a meaningful two point correlation function one has to account for the resolution of detectors:

$$< E(x,t)E(x',t') >= \frac{1}{\Delta x \Delta t} \iint_{\Delta x \Delta t} E(x+\xi, t+\tau)E(x'+\xi, t'+\tau)\, d\xi\, d\tau$$

one particle density matrix as determined for a SINGLE realization of BEC experiment - coarse graining:

$$\rho(x,x';t) = \frac{1}{\Delta \tau} \int_{\Delta \tau} \Psi^{*}(x,t+\tau)\Psi(x',t+\tau)\, d\tau$$

The name of the game:

coarse graining

Onsager-Penrose definition of the condensate:

$$\rho(x,x') = \sum_j \frac{N_j}{N} \phi_j^*(x)\phi_j(x')$$

note similarity to
coherence modes for noisy e-m fields
and
natural orbitals for atoms and molecules

trivial example - classical field for an ideal gas

$$\Psi(x,t) = \sum_j \alpha_j \varphi_j(x) \exp\left[-\frac{iE_j t}{\hbar}\right]$$

for sufficiently long observation time shows the content of the state:

$$\rho(x,x') = \sum_j |\alpha_j|^2 \varphi_j^*(x)\varphi_j(x')$$

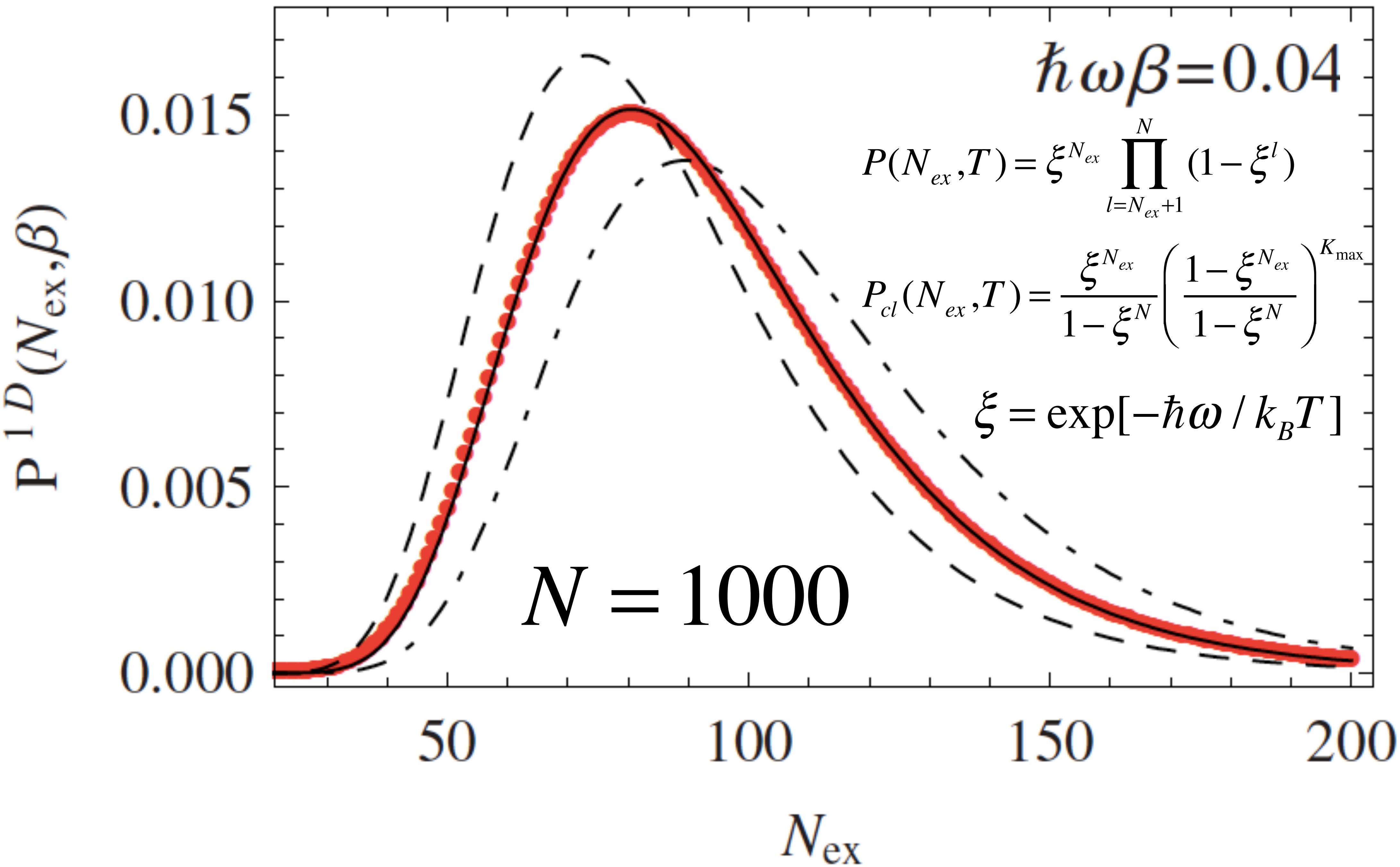
idea of classical field approximation:

long wavelength part of Bose atomic field may be replaced by a classical function.

close analogy to electromagnetic field

the most relevant question:
where to cut?

probability distribution of number of excited atoms



E. Witkowska, M. Gajda, and K. Rzążewski. *Phys. Rev. A*, **79**, 033631 (2009)

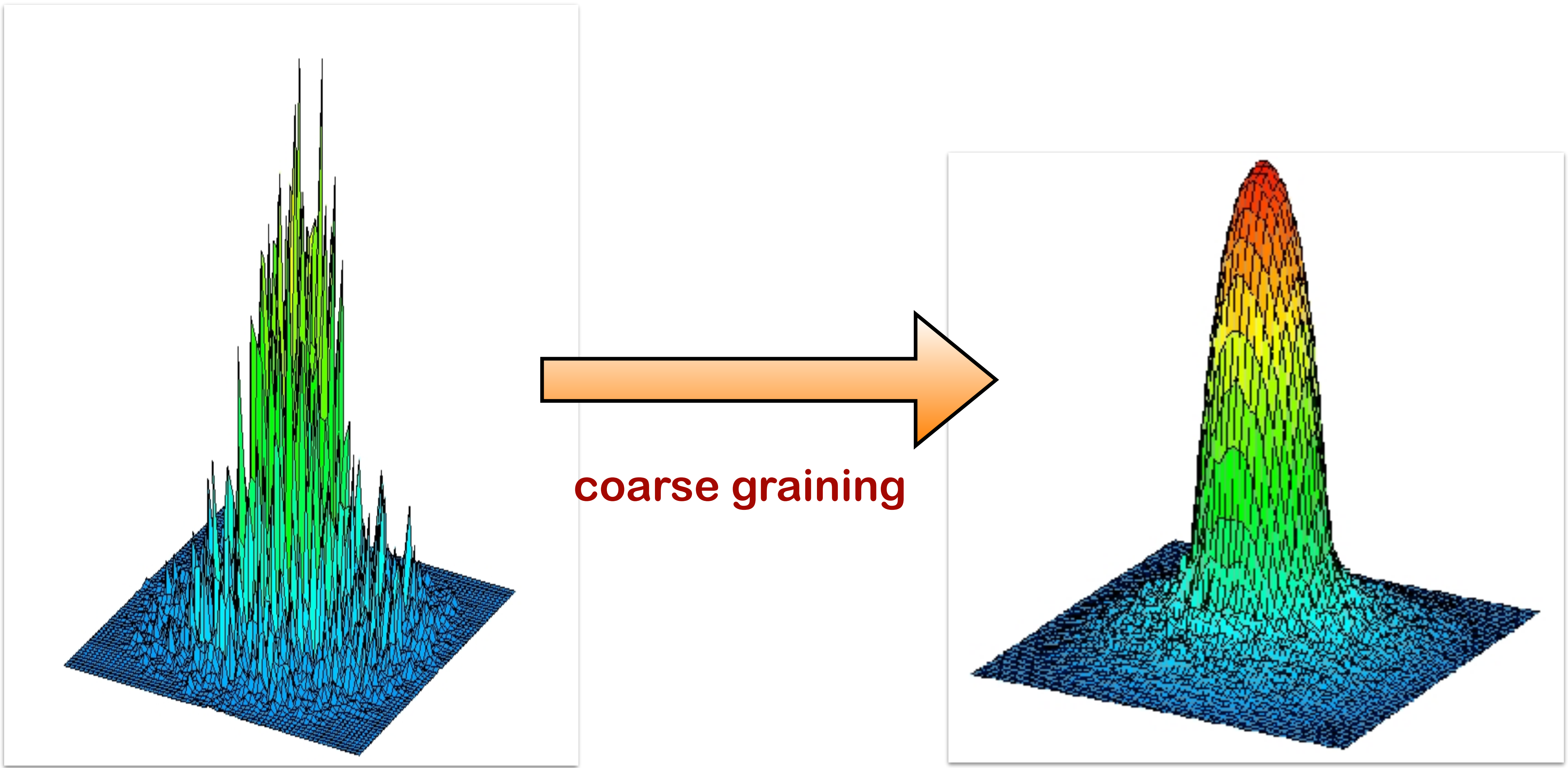
optimal cut-off for D-dimensional harmonic oscillator:

$$\hbar\omega K_{\max}\beta = \begin{cases} 1 & D = 1 \\ \left[\zeta(D)(D-1)(D-1)!\right]^{1/(D-1)} & D \geq 2 \end{cases}$$

E. Witkowska, M. Gajda, and K. Rzążewski. *Phys. Rev. A*, **79**, 033631 (2009)

classical fields approximation -
the only quantum mechanical method that allows to study single realizations of the BEC experiment

application to a steady state in a harmonic trap:



K. Góral, M. Gajda, and K. Rzążewski, Thermodynamics of an isolated Bose gas and a role of observation, Phys. Rev. A **66**, 051602(R), (2002)

some applications of classical fields

classical fields $$\Phi(x) = \sum_{i=0}^{i_{\max}} \alpha_i \varphi_i(x)$$

the energy functional

$$E(\{\alpha_i\}) = \sum_{i=0}^{i_{\max}} \varepsilon_i |\alpha_i|^2 + E_{\mathrm{int}}(\{\alpha_i\}) \qquad N = \sum_{i=0}^{i_{\max}} |\alpha_i|^2$$

canonical probability distribution of the field amplitudes

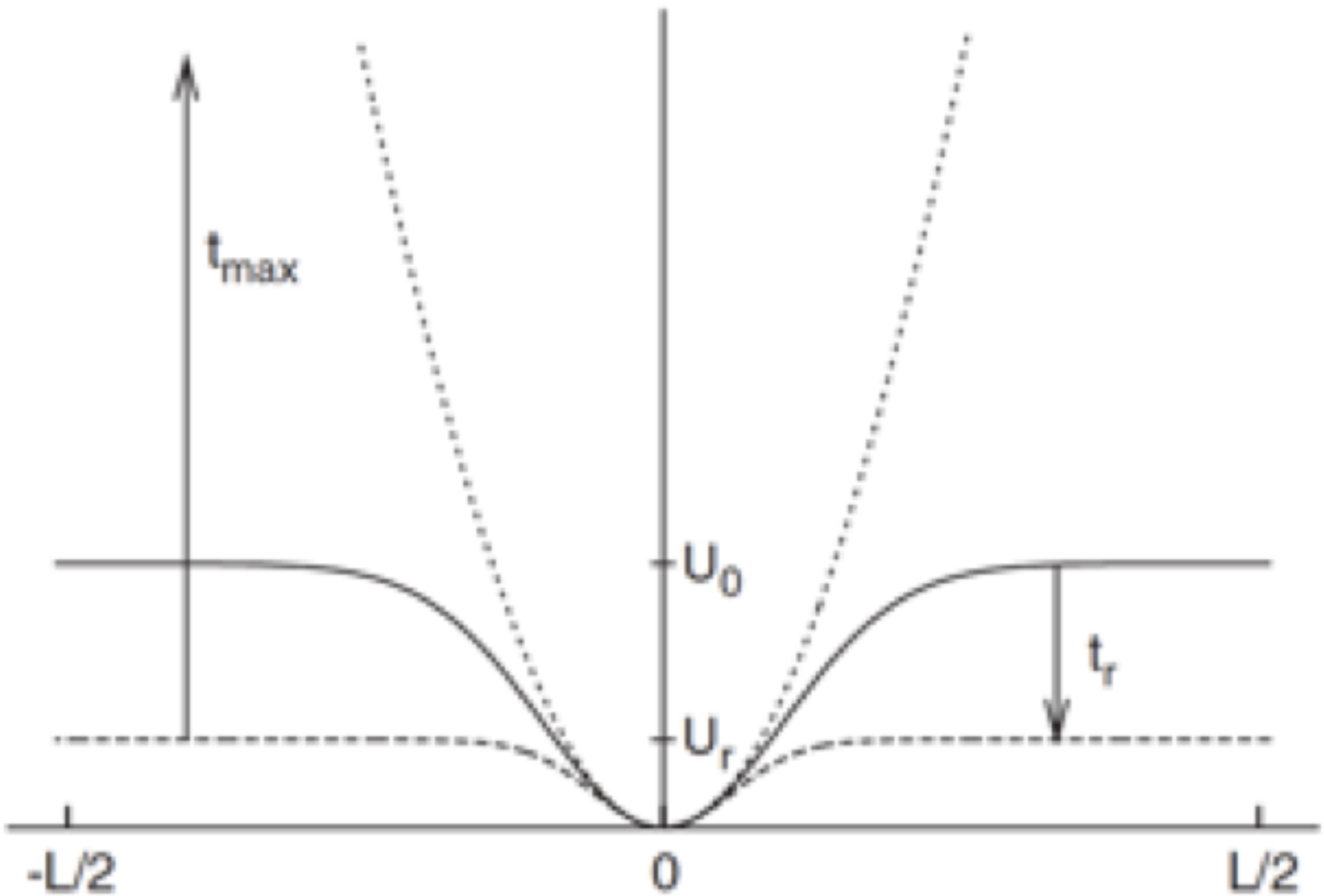
$$P(\{\alpha_i\}, T) = \frac{1}{Z(N,T)} \exp\left[-\frac{E(\{\alpha_i\})}{kT} \right]$$

Monte Carlo methods!

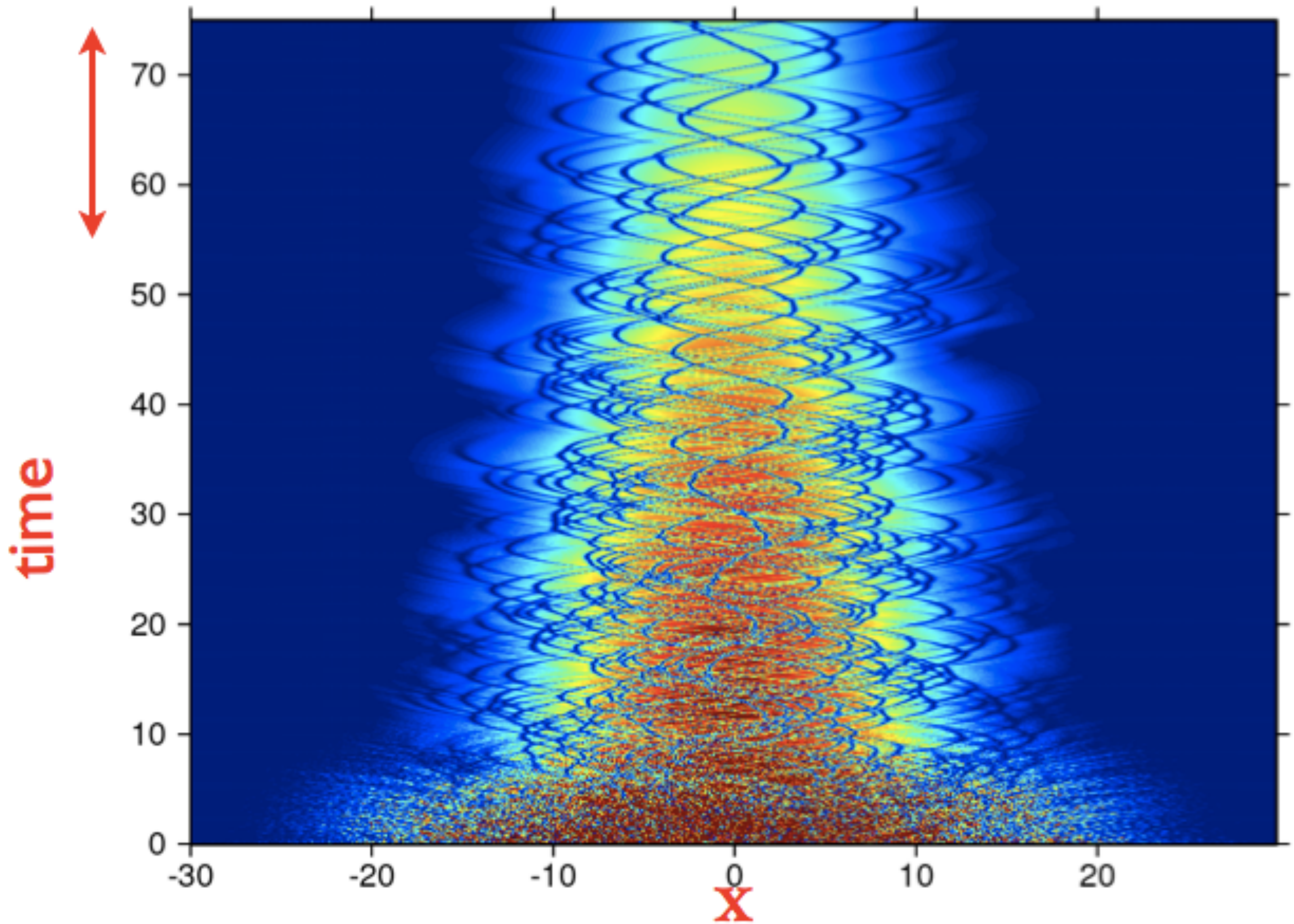
Note: we have access not only to mean values but also to the properties of a single copy!

spontaneous soliton formation in rapid cooling of quasi one dimensional Bose gas (W. Żurek - 2009)

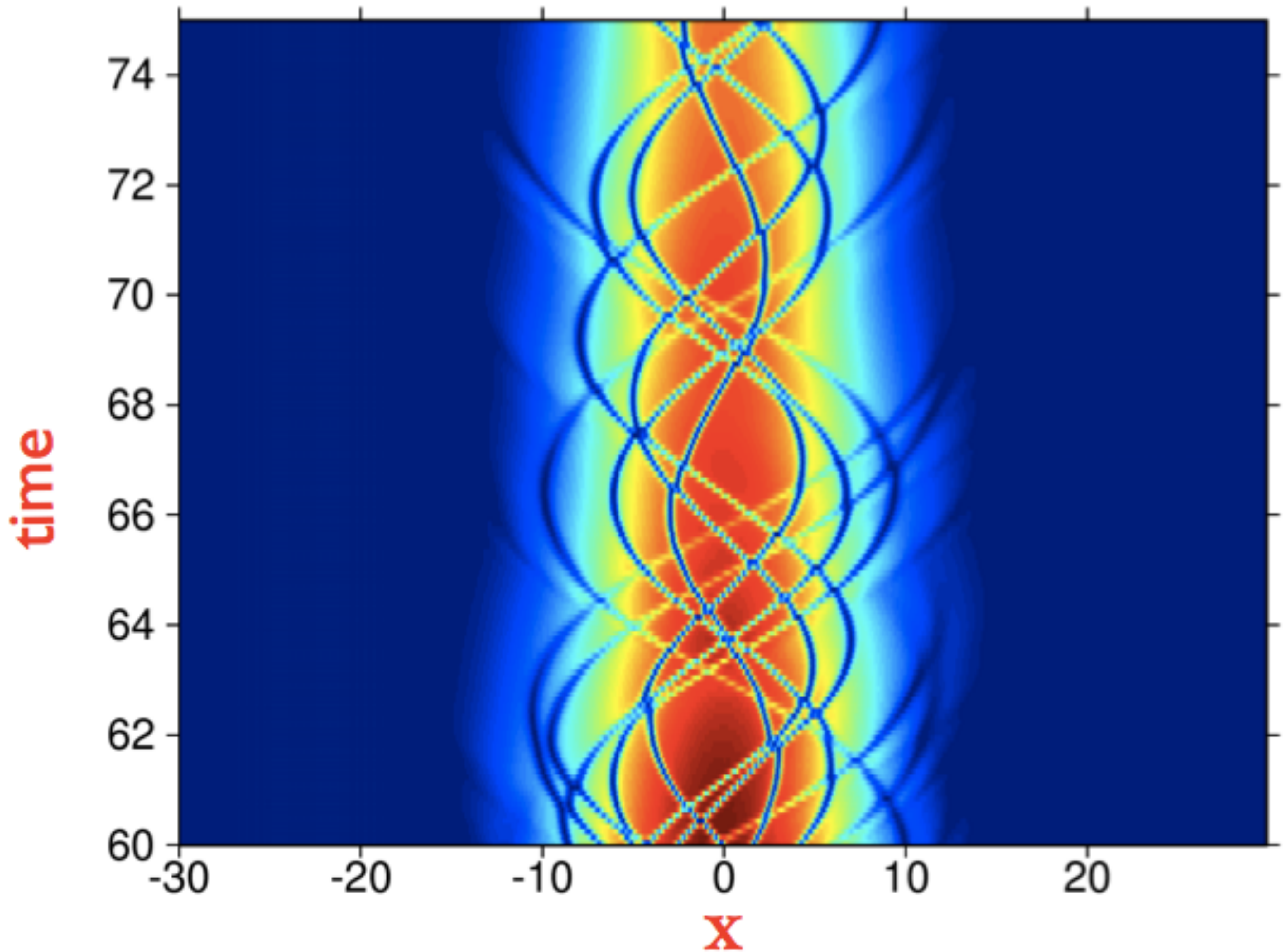
model of rapid cooling



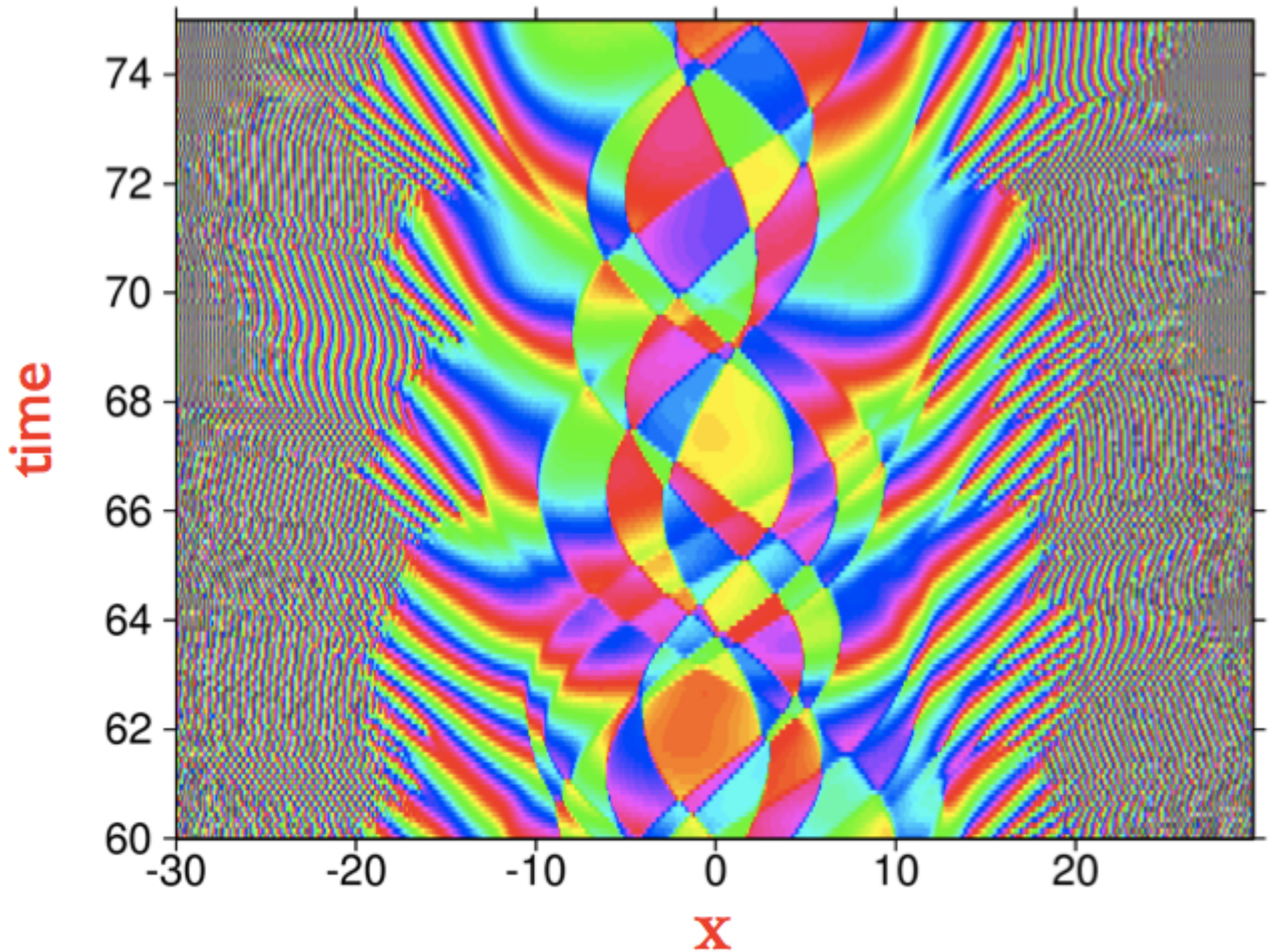
emergence of gray solitons during cooling



E. Witkowska, P. Deuar, M. Gajda, and K. Rzążewski, Solitons as the Early Stage of Quasicondensate Formation during Evaporative Cooling, *Phys. Rev. Lett.* **106**, 135301 (2011) enjoy the solitons...



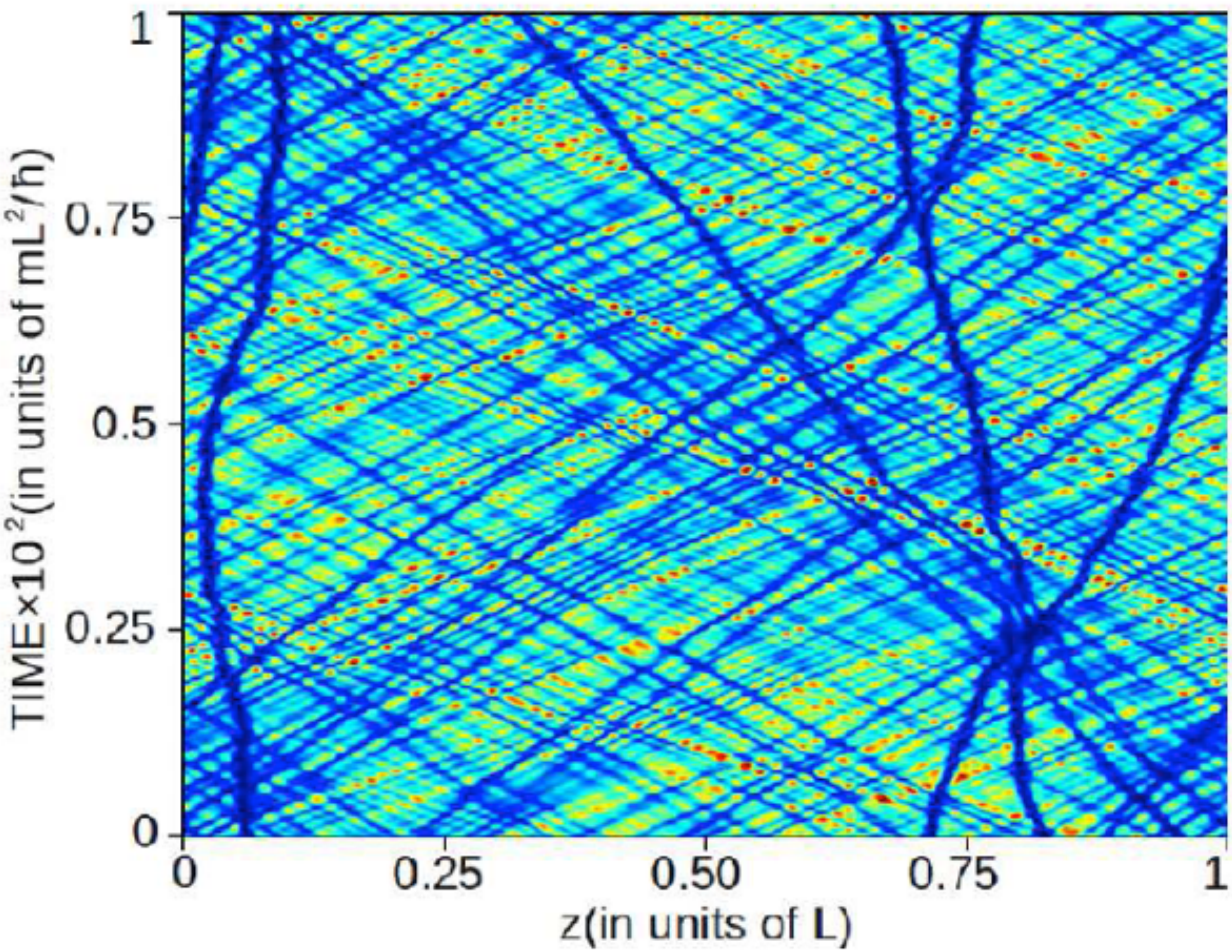
domains of (nearly) constant phase



Action of external agents is not necessary.

Solitons exist also in the thermal equilibrium!

T. Karpiuk, P. Deuar, P. Bienias, E. Witkowska, K. Pawłowski, M. Gajda, K. Rzążewski and M. Brewczyk, Spontaneous Solitons in the Thermal Equilibrium of a Quasi-1D Bose Gas, *Phys. Rev. Lett.*, **109**, 205302 (2012)



TIME×10^2(in units of mL^2/ħ)
1
0.75
0.5
0.25
0
0
0.25
0.50
0.75
1
z(in units of L)

Two types of spectra in 1D soluble model of Lieb and Liniger

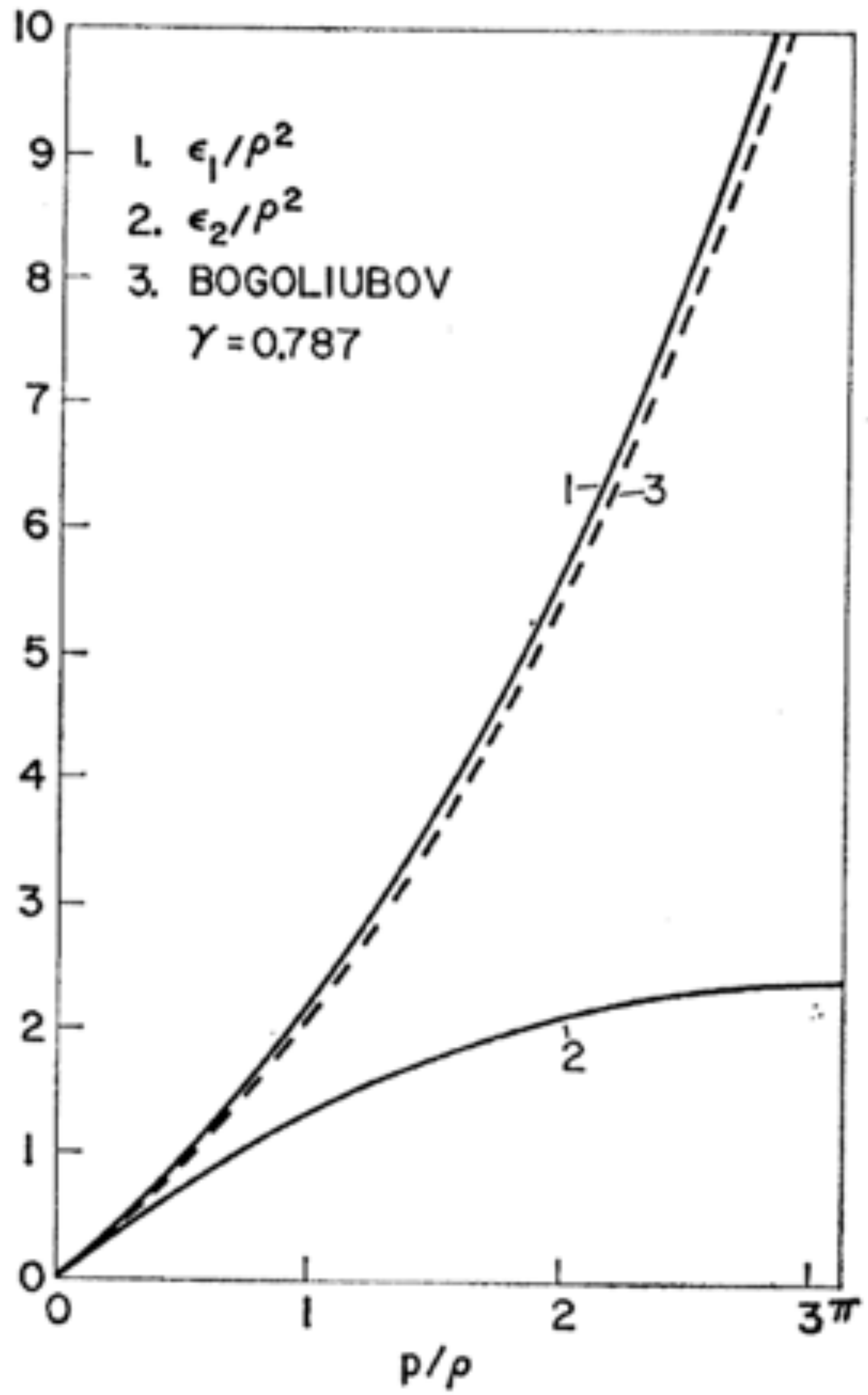


FIG. 4. A comparison plot of the two types of excitations, ϵ_1 and ϵ_2, for $\gamma = 0.787$. The dashed curve is Bogoliubov's spectrum which is quite close to the type I spectrum. The type II spectrum does not exist in Bogoliubov's theory.

E. H.Lieb, Exact analysis of an interacting Bose gas, Excitations spectrum, Phys. Rev. **130**, 1616 (1963)

Comparison of thermal distributions of dark solitons and Yang-Yang collective excitations

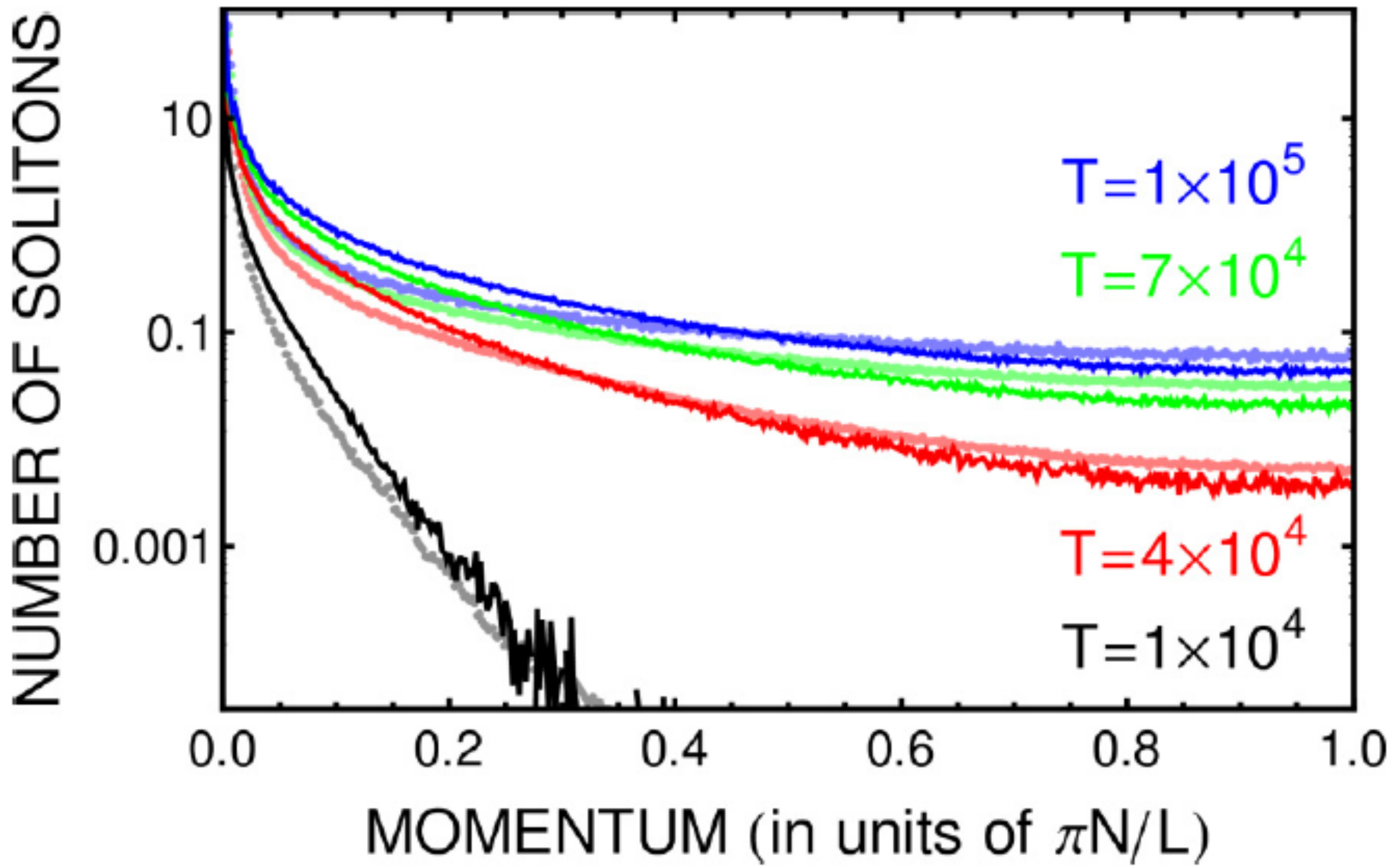


FIG. 7. (Color online) Comparison between the Yang-Yang model and the classical field approximation for the temperatures (from the top to bottom) 10^5, 7×10^4, 4×10^4, and 10^4. Frame

T. Karpiuk, T. Sowiński, M. Gajda, K. Rzążewski, and M. Brewczyk, Correspondence between dark solitons and the type II excitations of the Lieb-Liniger model. *Phys. Rev. A*, **91**, 013621 (2015)

solitons in quasi-one-dimensional Bose gas at thermal equilibrium

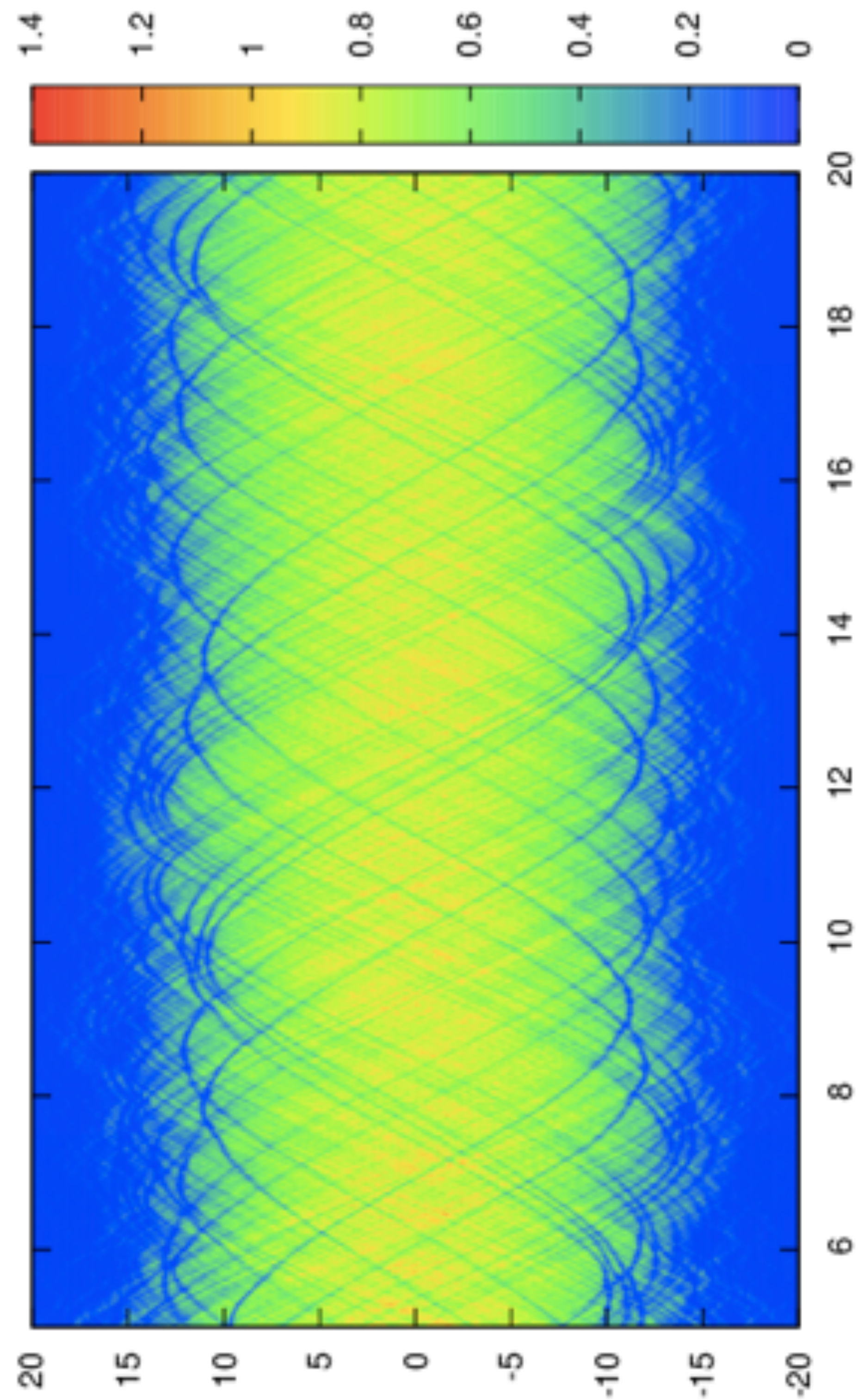
N=1000

$$\omega_{x,y} = 2\pi \cdot 1000\text{Hz}$$

$$\omega_z = 2\pi \cdot 10\text{Hz}$$

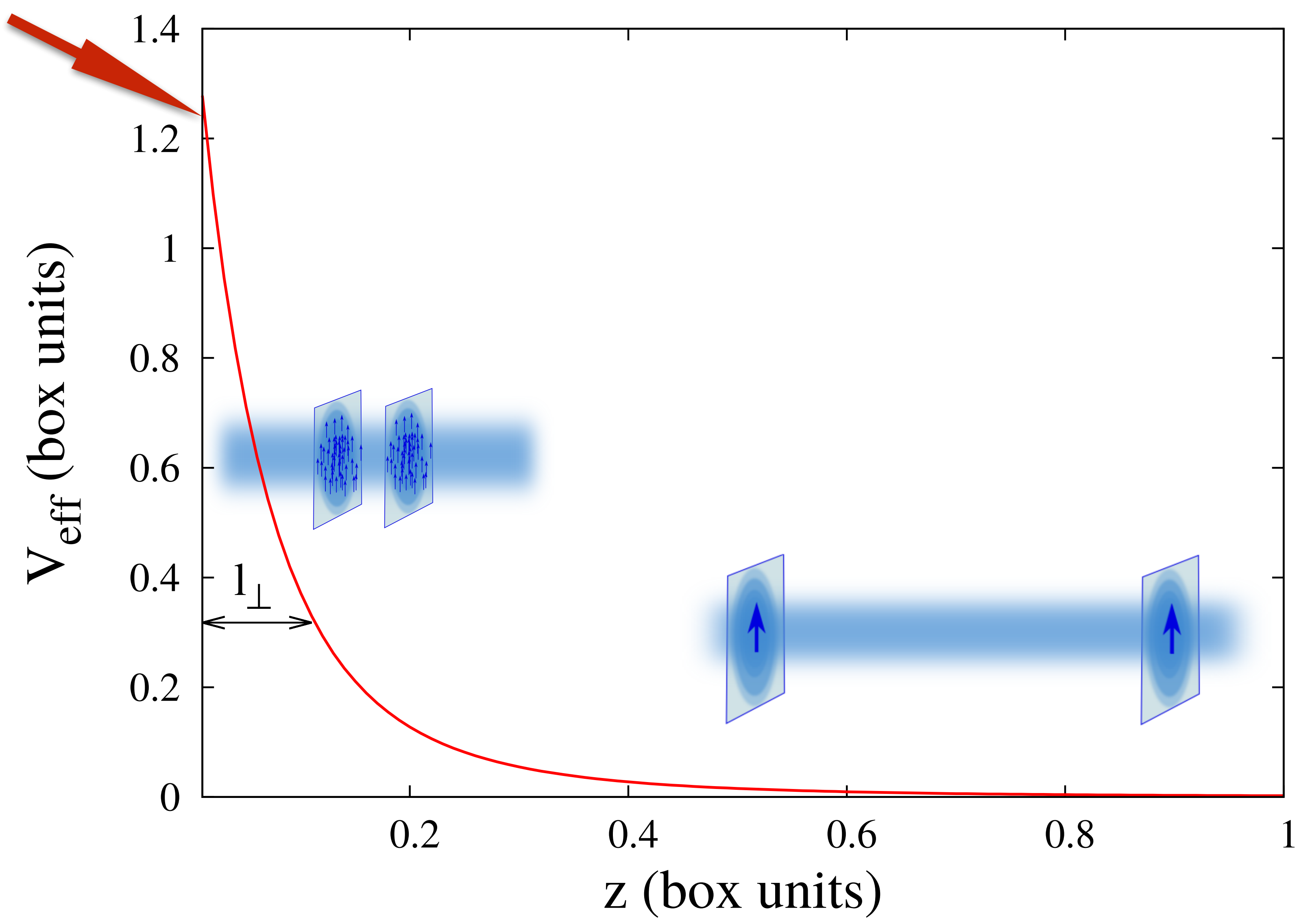
$$N_0 / N = 0.2$$

$$k_B T \approx \hbar \omega_{x,y}$$

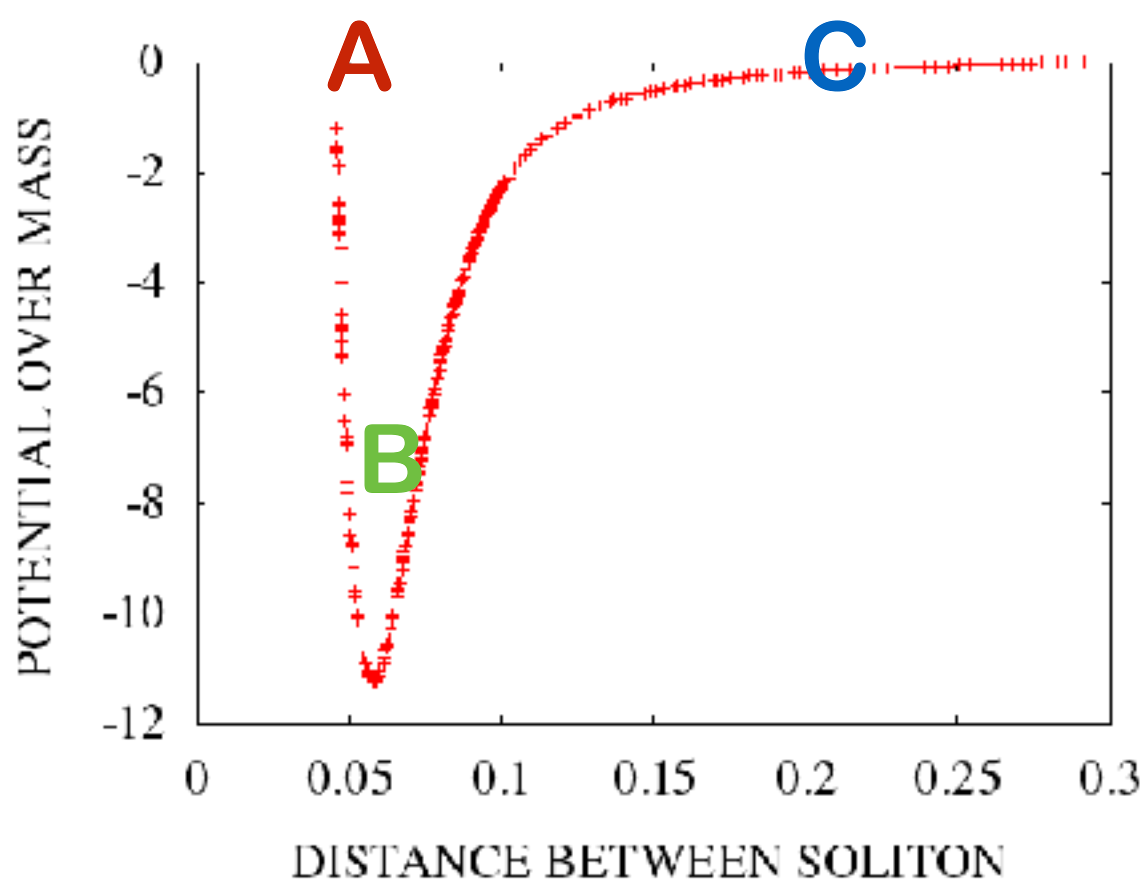


solitons in dipolar condensate

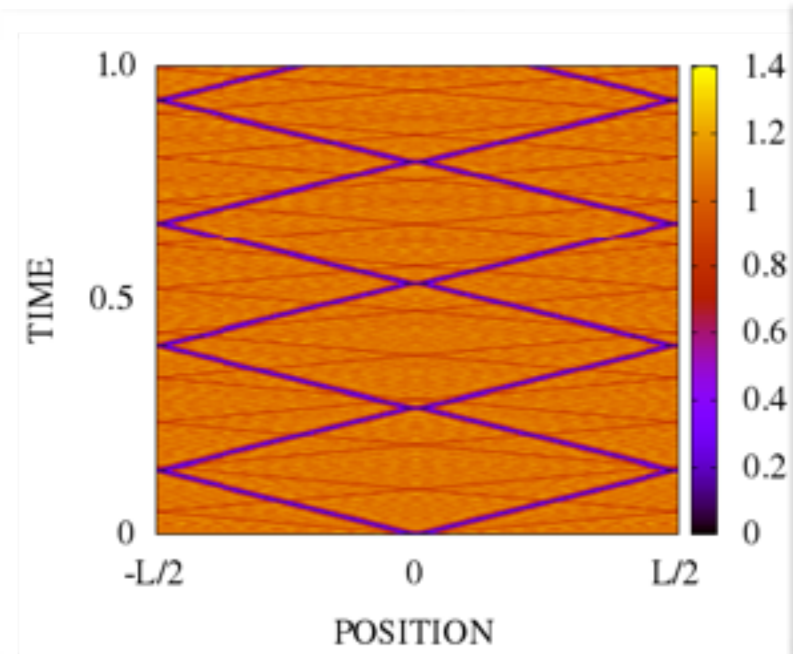
finite value



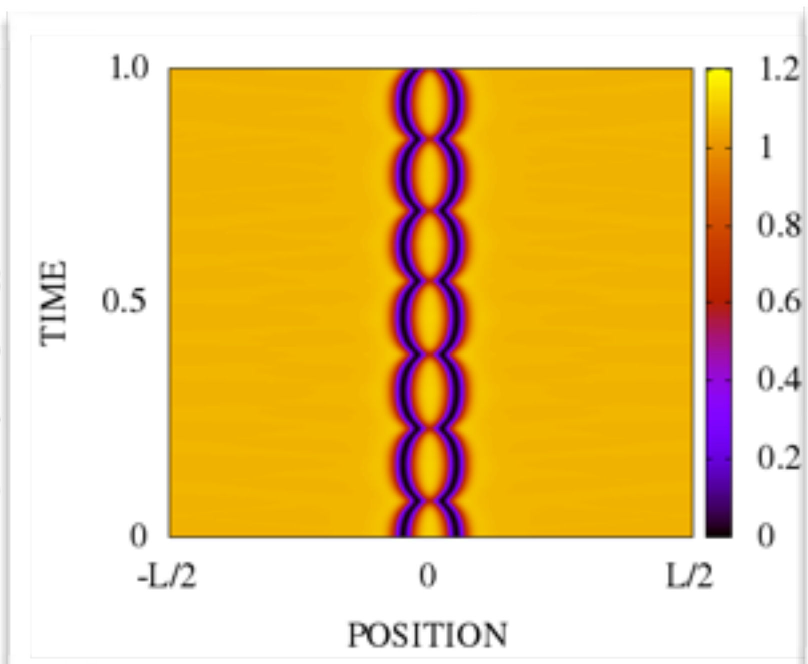
inter-soliton potential



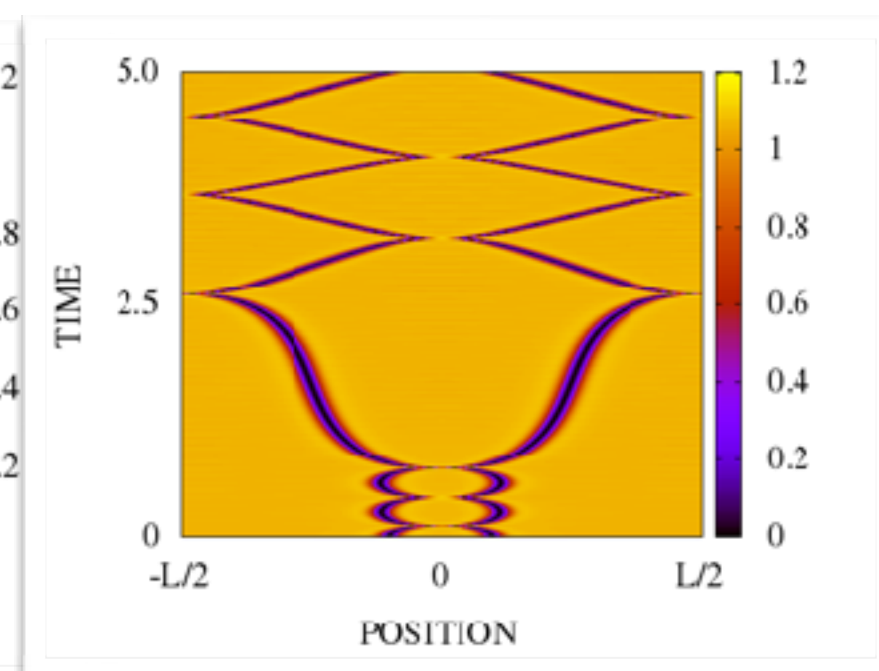
collisions
elastic?



A

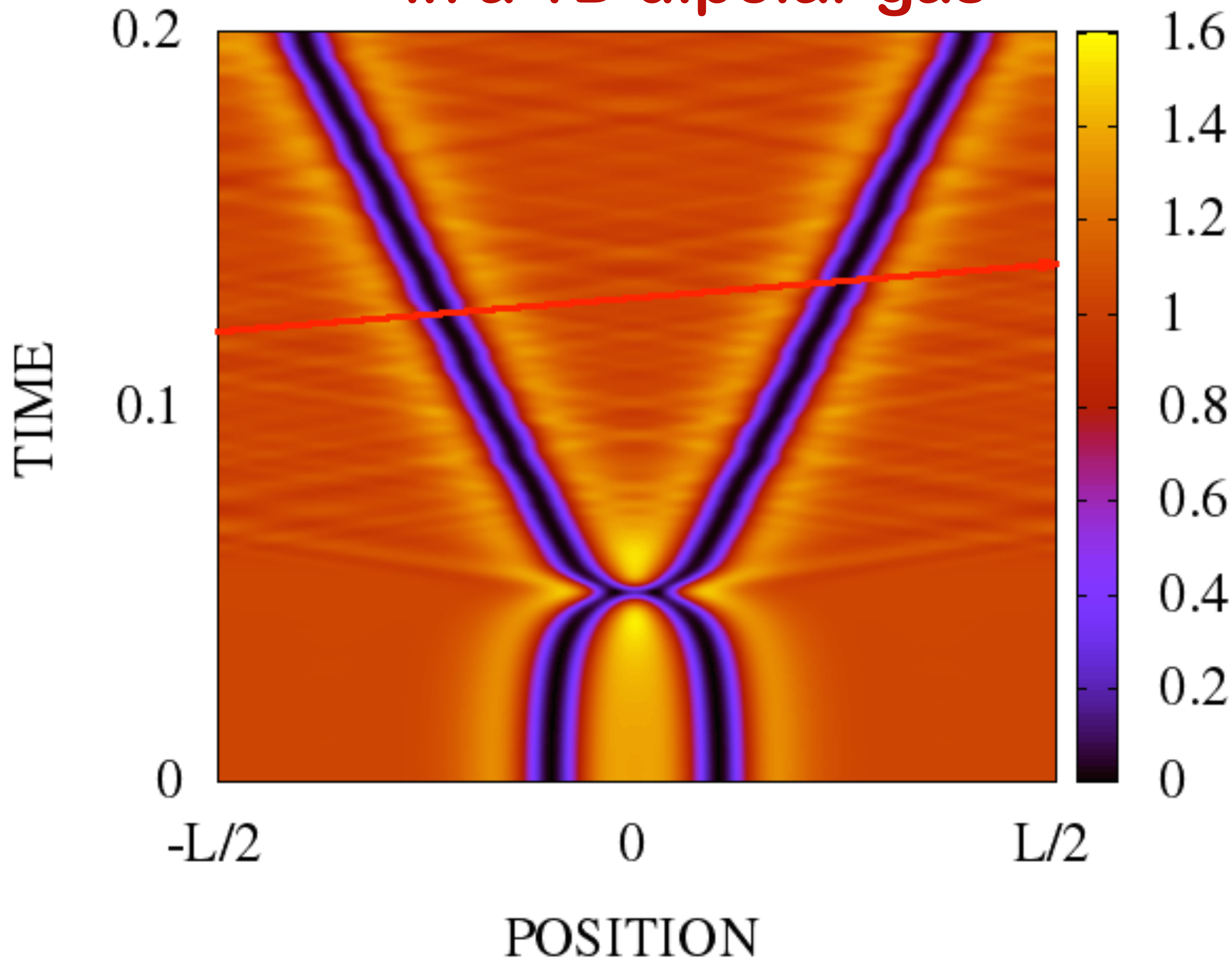


B



C

collision of two solitons in a 1D dipolar gas



K. Pawłowski and K. Rz. Dipolar dark solitons. *New J. Phys*. 17 (2015) 105006

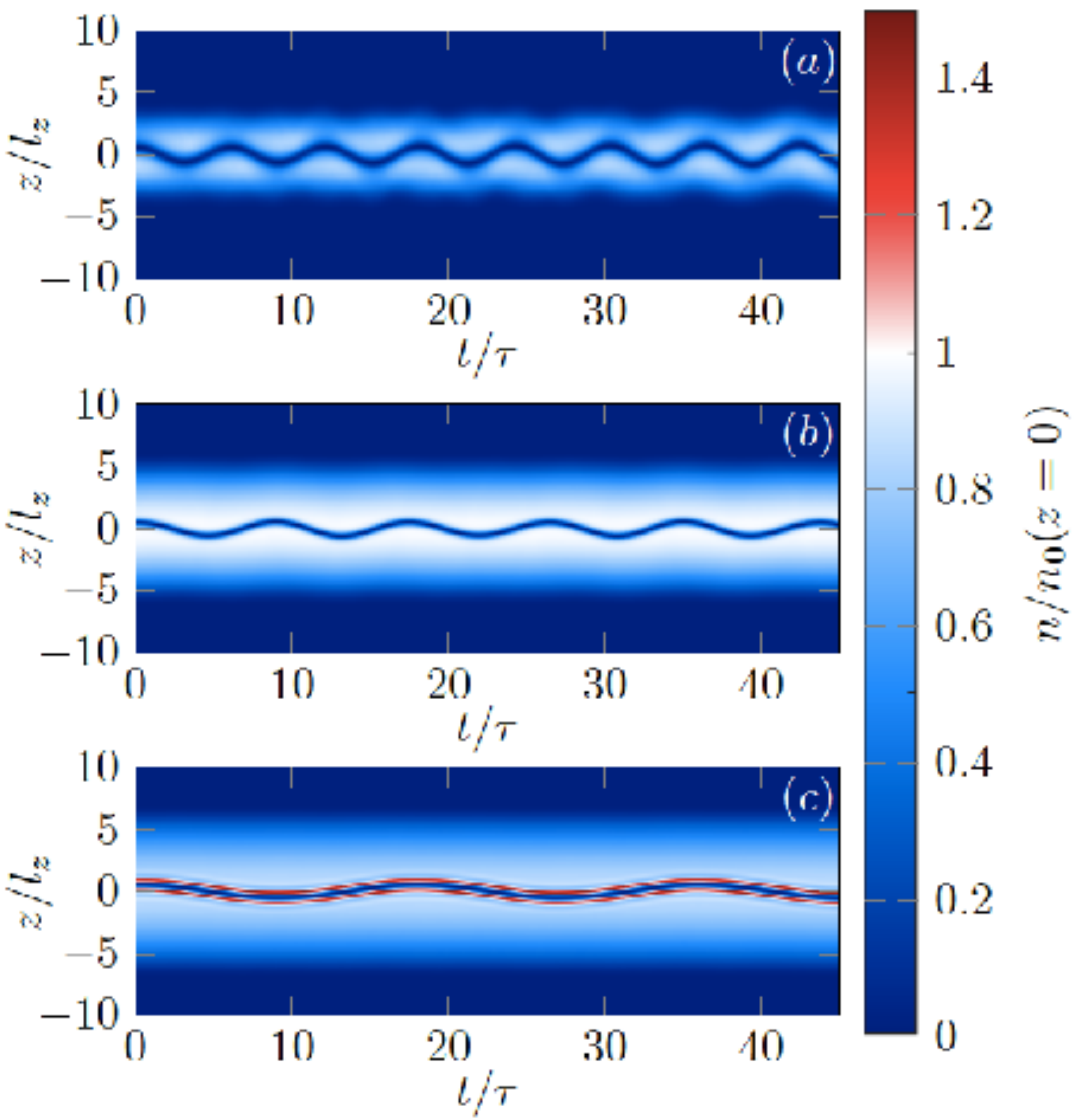
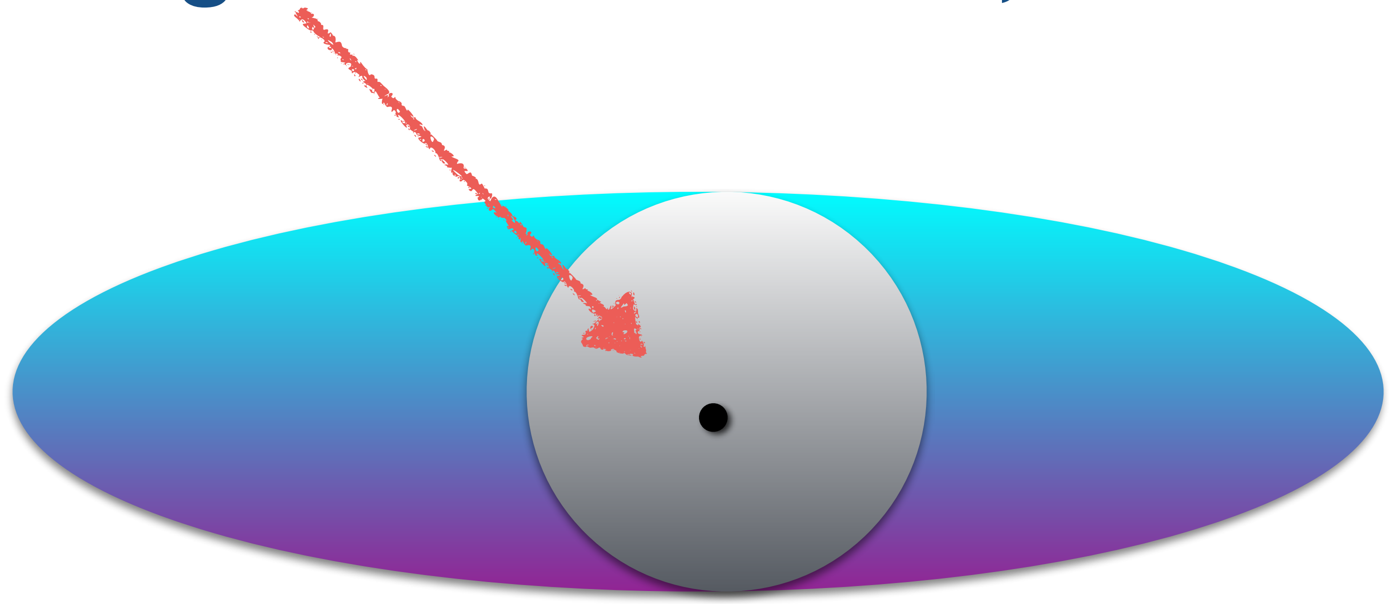


FIG. 2. (Color online) Density dynamics of a dark soliton in the quasi-1D dipolar condensate for (a) $\varepsilon_{dd} = -1.7$, (b) $\varepsilon_{dd} = 0$, and (c) $\varepsilon_{dd} = -5.5$. These values correspond to close to $a_{eff} = 0$, the non-dipolar case, and close to the roton instability, respectively. Remaining parameters as in Fig. 1.

T. Bland, K. Pawłowski, M. J. Edmonds, K. Rzążewski, and N. G. Parker, Anomalous oscillations of dark solitons in trapped dipolar condensates, arXiv:1610.02002

single Rydberg atom in a condensate

a single atom in N=202, l=0 state



80 000 ^{87}Rb atoms

J. Balewski, A. T. Krupp, A. Gaj, D. Peter, H. P. Büchler, R. Löw, S. Hofferberth & Tilman Pfau, Nature **502**, 664, (2013)

New Journal of Physics

Detecting and imaging single Rydberg electrons in a Bose-Einstein condensate

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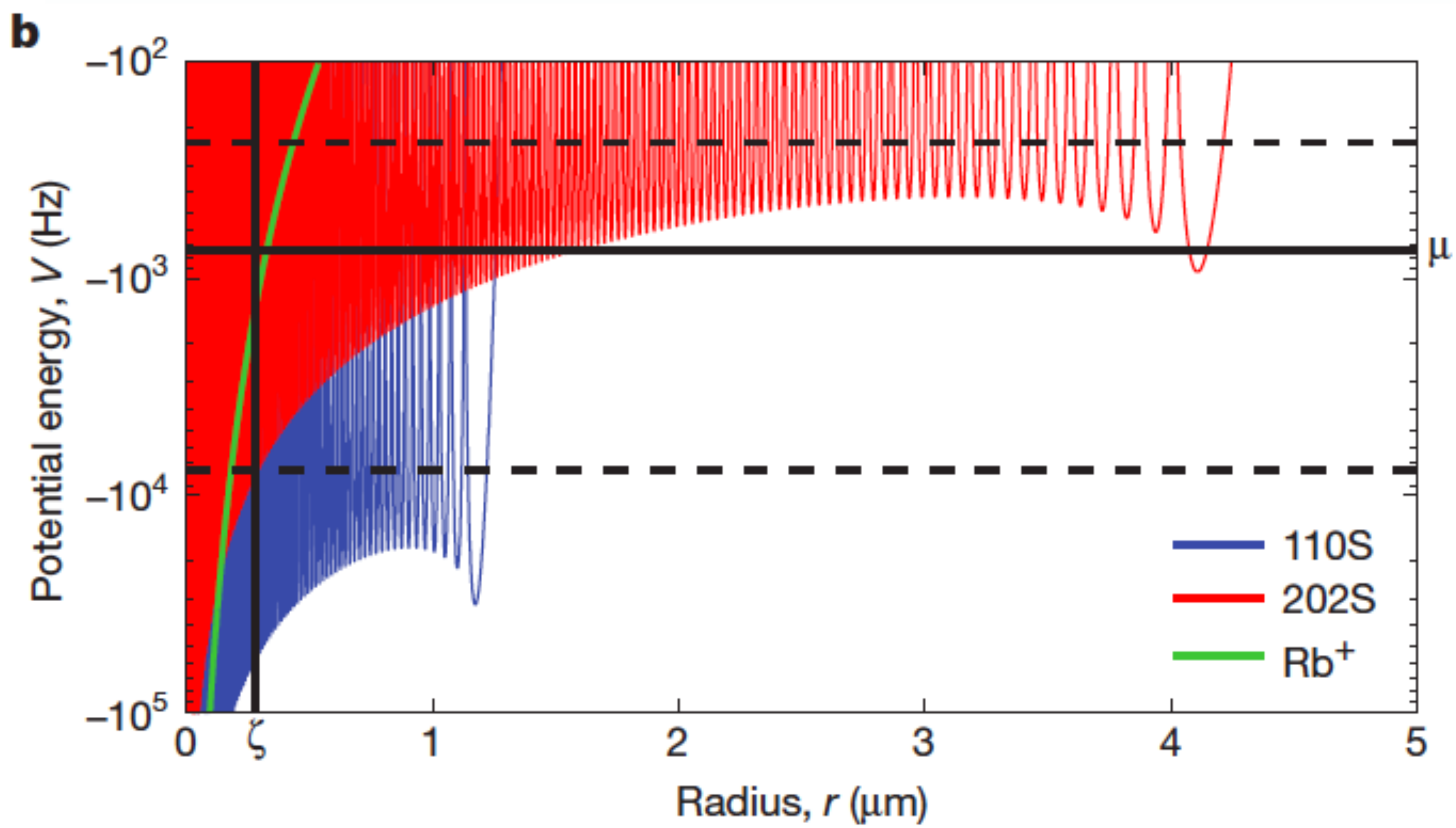
[3] *Center for Theoretical Physics PAN, Al. Lotników 32/46, 02-668 Warsaw, Poland*

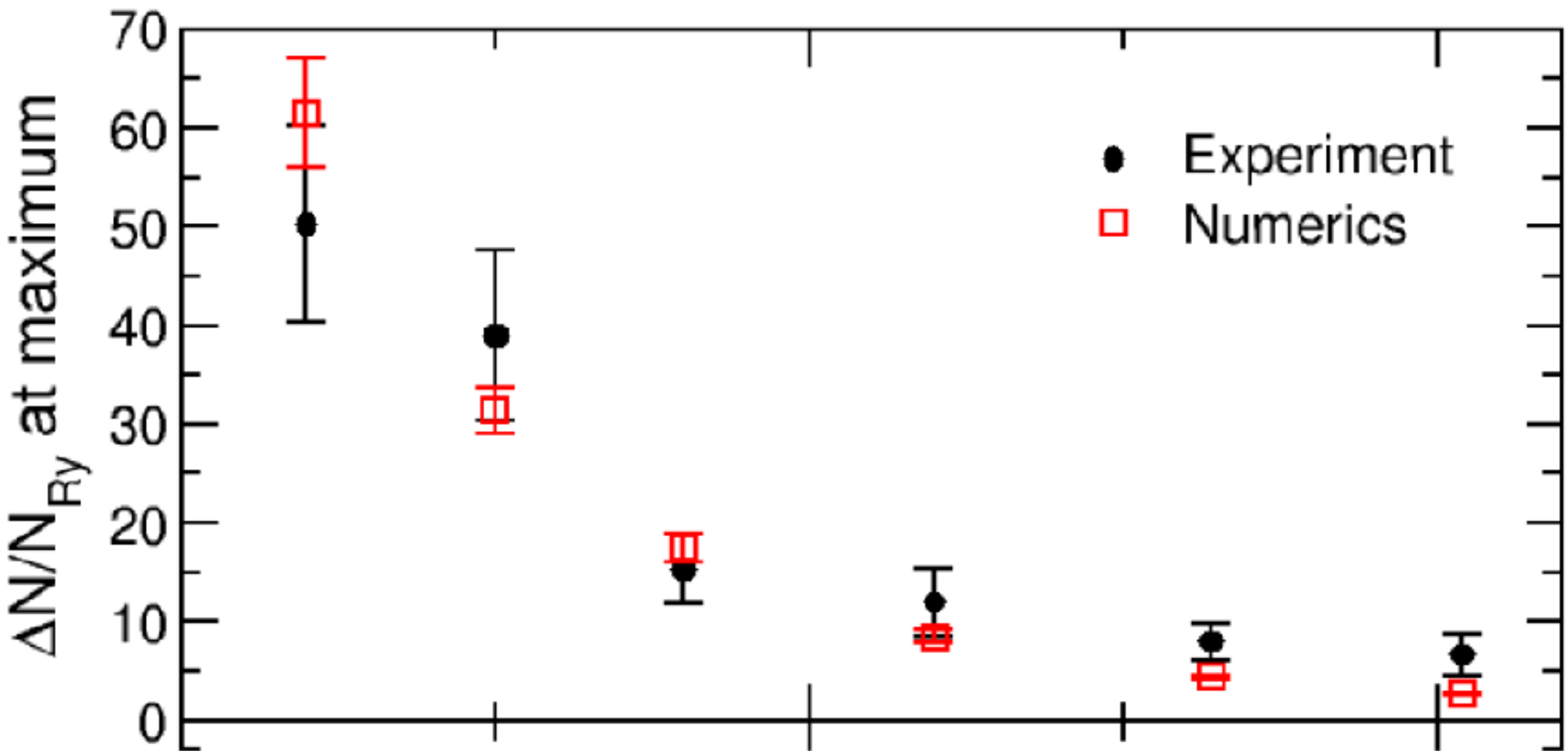
Jonathan B. Balewski[4], Alexander T. Krupp[4], Anita Gaj[4], Robert Löw[4], Sebastian Hofferberth[4] and Tilman Pfau[4]
[4] *5. Physikalisches Institut, Universität Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany*

“potential well” formed by Rydberg electron



$$V_{Ryd}(\vec{r}) = \frac{2\pi\hbar^2 a}{m_e}|\Psi_{Ryd}(\vec{r})|^2$$

fraction of atoms evaporating from a condensate as a result of intermittent action of a Rydberg electron



T. Karpiuk, M. Brewczyk, K. Rzążewski, A. Gaj, J. B. Balewski, A. T. Krupp, M. Schlagmüller, R. Löw, S. Hofferberth and T. Pfau, Imaging single Rydberg electrons in a Bose–Einstein condensate, *New J. Phys.* **17**, 053046 (2015)

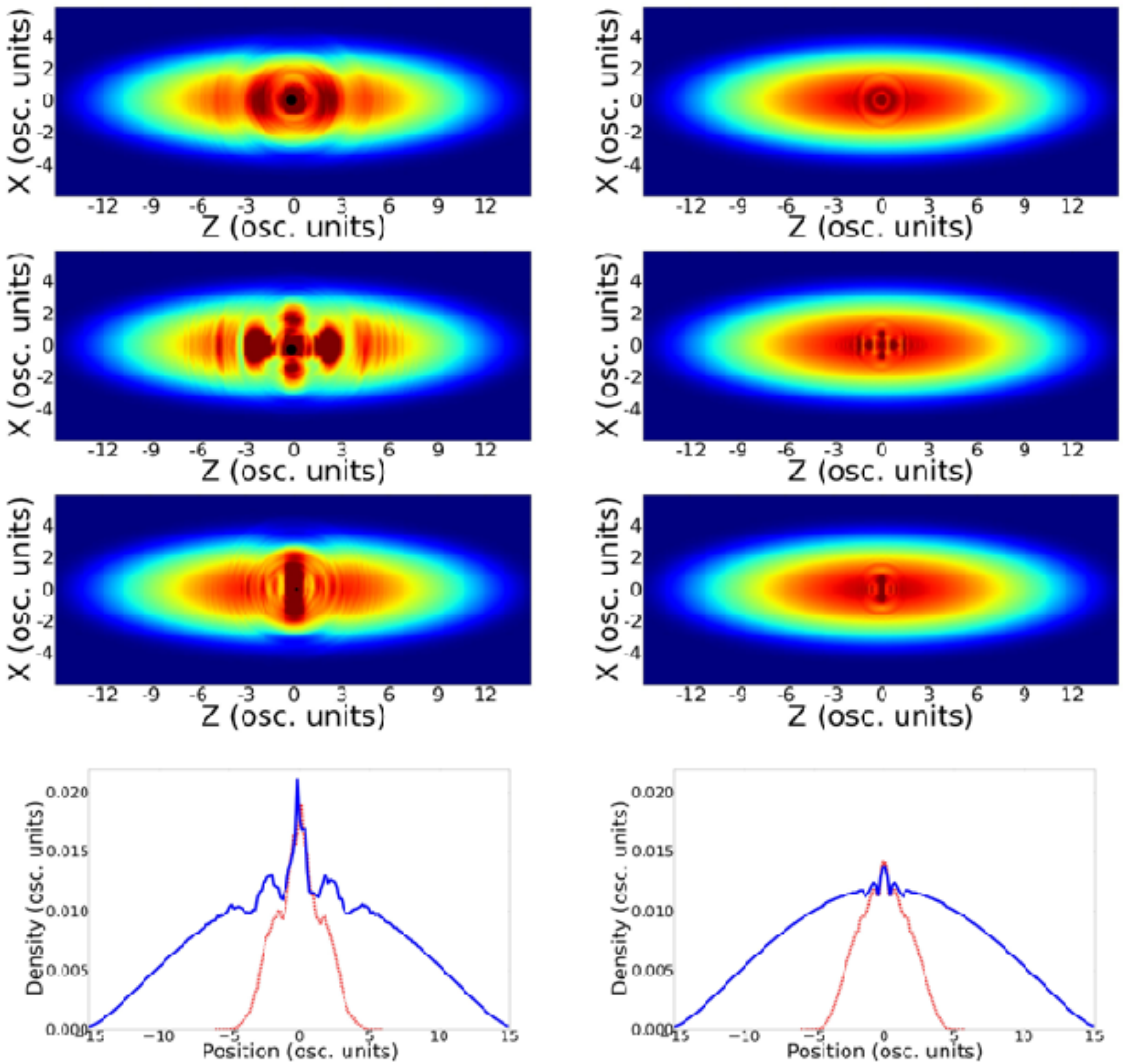
“imaging” orbital

sequence of 50 atoms — **single atom**

***N*=182, *l*=0, *m*=0**

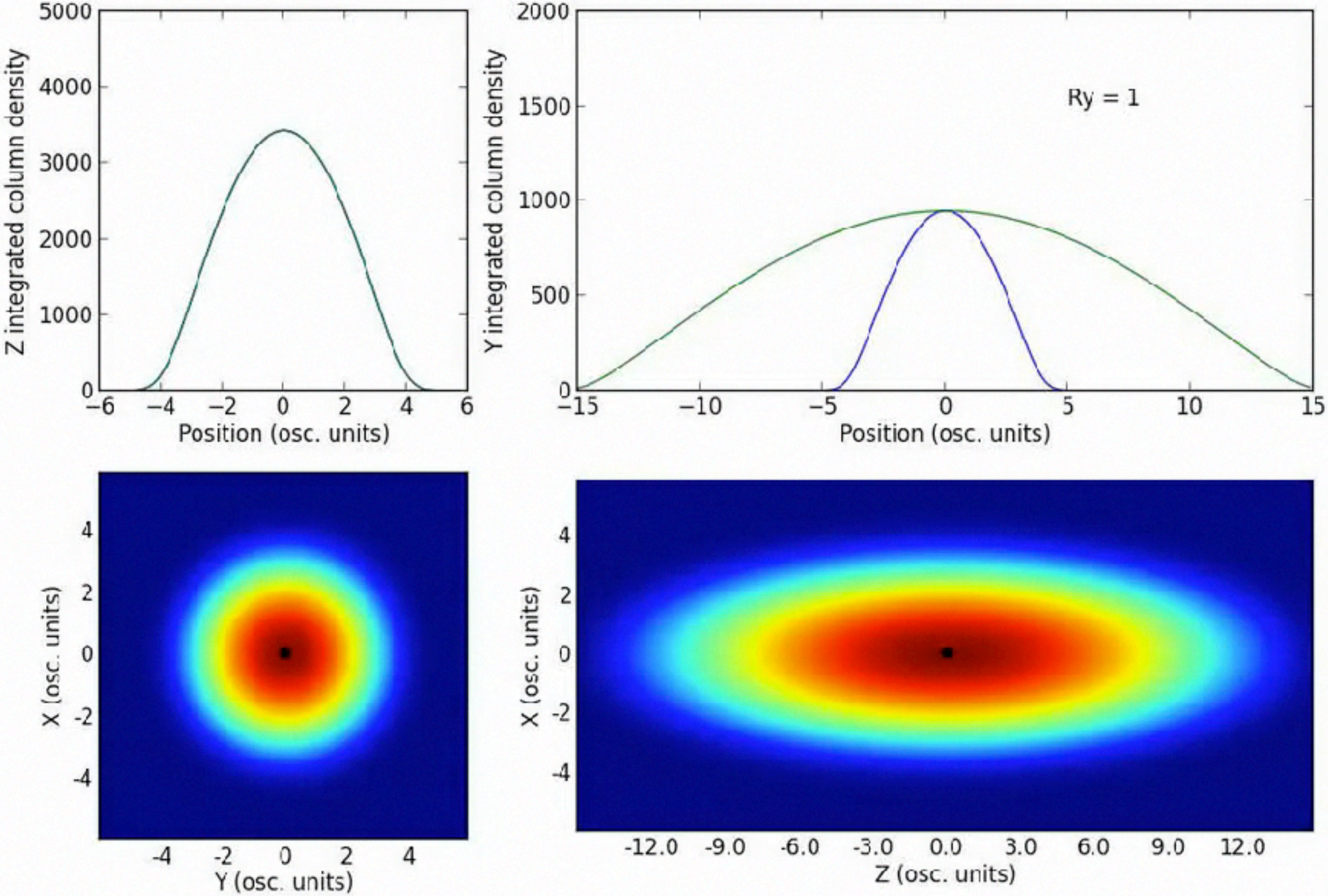
***N*=180, *l*=2, *m*=0**

***N*=180, *l*=2, *m*=2**



N=110

N=202, l=2, m=0



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J. B. Balewski, A. T. Krupp, A. Gaj, M. Schlagmüller, R. Löw, S. Hofferberth, and T. Pfau,
Condensate losses and oscillations induced by Rydberg atoms, *Journal of Physics* B, (in press)

summary

- classical optics is a source of inspiration for classical fields approach to cold bosons
- link to experiment requires account for the finite resolution of the measurement
- unified description of a nonzero temperature gas - analogy to partially coherent light