# Non-perturbative many-body localization

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# Scope:

- Where it all started...
- Anderson localization - simple statements
- Anderson localization of a BEC – experiments.
- Many-body localization.
- Many-body localization - experiment
- nonperturbative MBL
- Conclusions.

# Prehistory:

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## Atomic Bose and Anderson Glasses in Optical Lattices

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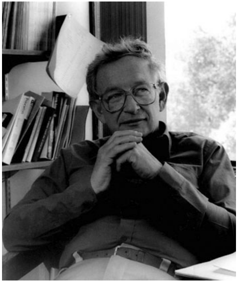
An ultracold atomic Bose gas in an optical lattice is shown to provide an ideal system for the controlled analysis of *disordered* Bose lattice gases. This goal may be easily achieved under the current experimental conditions by introducing a pseudorandom potential created by a second additional lattice or, alternatively, by placing a speckle pattern on the main lattice. We show that, for a noncommensurable

# Anderson localization -simple statements

No interactions $g = 0$ - single particle

$$\left[-\frac{1}{2}\partial_z^2 + V(z)\right]\phi_0 = \mu\phi_0, \qquad \langle\phi|\phi\rangle = 1$$



- tiny random $V(z)$ leads to exponential localization in 1D
- 2D - marginal
- 3D mobility edge extended/localized
- lattice discretization,

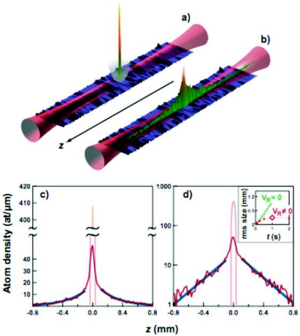
$$\phi_{n-1} + \psi_{n+1} + V_n\psi_n = \epsilon\psi_n$$

$$H = \sum_l \left[-J(a_l^\dagger a_{l+1} + h.c.) + + V_l \; a_l^\dagger a_l\right]$$

# Anderson localization of a BEC

## Experiments

$$\left[-\frac{1}{2}\partial_z^2 + V(z) + g|\phi(z)|^2\right]\phi(z) = \mu\,\phi(z), \qquad \langle\phi|\phi\rangle = N$$




Experiments in the non-interacting limit...

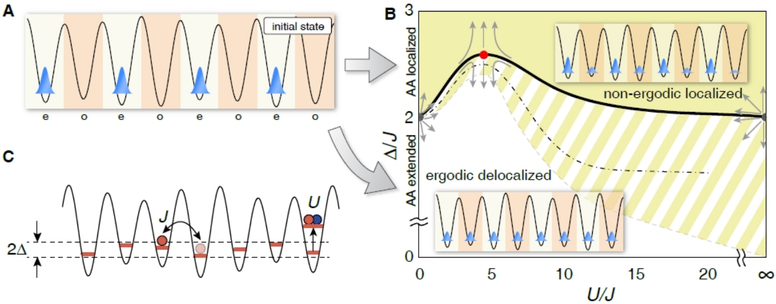
J. Billy et al., Nature **453**, 891 (2008)   G. Roati et al., Nature **453**, 895 (2008)

# Many-body localization

- Localization in the presence of interactions?
- Interference versus mean field
- Thermalization in quantum statistical physics versus many-body localization
- Eigenstate Thermalization Hipothesis (Srednicki)
- Basko, Aleiner, Altschuler - sufficiently strong disorder (perturbative) leads to MBL
- 120 papers over last 12 months

# Many-body localization - experiment

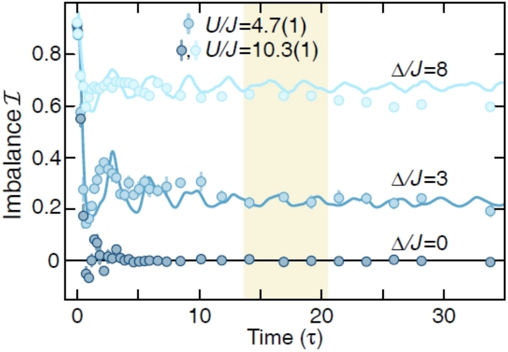
M. Schreiber ... U. Schneider, I. Bloch, Science (2014)




$$\hat{H} = -J\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.}\right) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}.$$

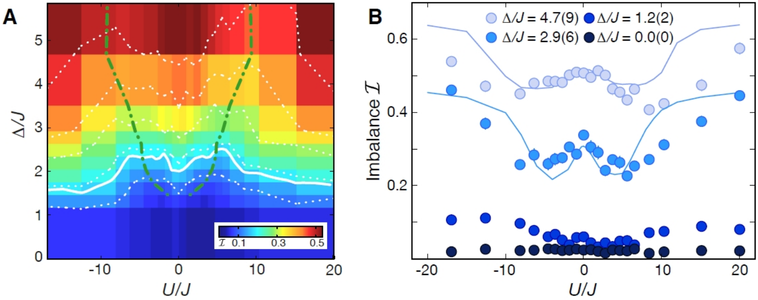
# Many-body localization-experiment

Schreiber ... Bloch, Science (2014)



# Many-body localization - experiment

Schreiber ... Bloch, Science (2014)

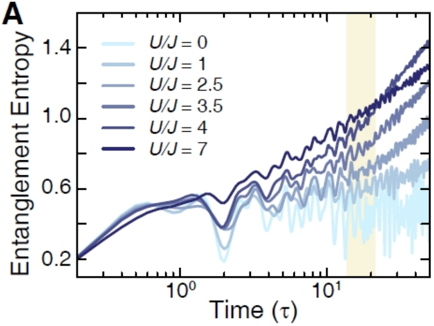



dominated by single particle processes

# Many-body localization - simulation

Entropy of entanglement between $A$ and $B$

$$S_{ent} = -\mathrm{Tr}\rho_A \ln \rho_A \qquad \rho_A = \mathrm{Tr}_B \rho$$




reveals a log growth as postulated for MBL.

# Nonperturbative Many-body localization

- Need to find a system not dominated by a single particle physics

# Nonperturbative Many-body localization

- Need to find a system not dominated by a single particle physics
- Single particle physics - not localized, localization due to random interactions.
- standard Bose-Hubbard model:

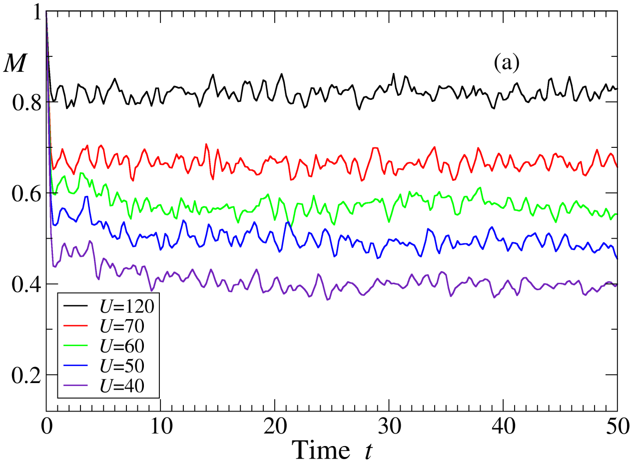
$$H = \sum_l \left[ -J(a_l^\dagger a_{l+1} + h.c.) + \frac{U}{2} a_l^\dagger a_l^\dagger a_l a_l + V_l \; a_l^\dagger a_l \right]$$

- Let us take instead

$$H = \sum_l \left[ -J(a_l^\dagger a_{l+1} + h.c.) + \frac{U_l}{2} a_l^\dagger a_l^\dagger a_l a_l \right]$$

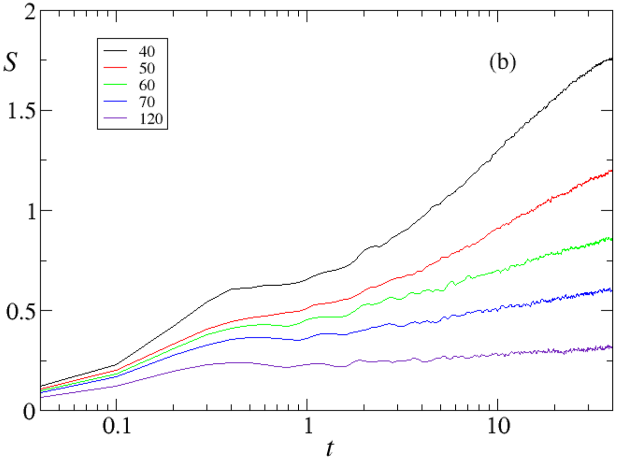
# Nonperturbative Many-body localization

We follow Schreiber, Bloch route: initial state $2, 1, 2, 1, 2, 1, 2, ..$
Propagation using MPS (tDMRG home made, parallelized)
$M \equiv$ (Popul. in odd sites) – (Popul. in even sites)



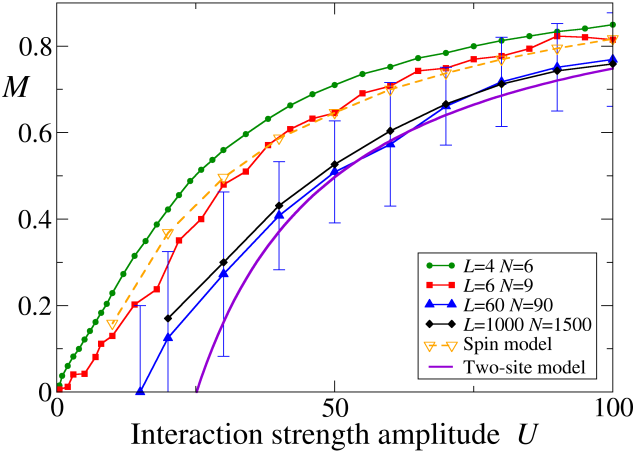
# Nonperturbative Many-body localization

Entropy of entanglement (averaged over realizations of disorder)



# Nonperturbative Many-body localization

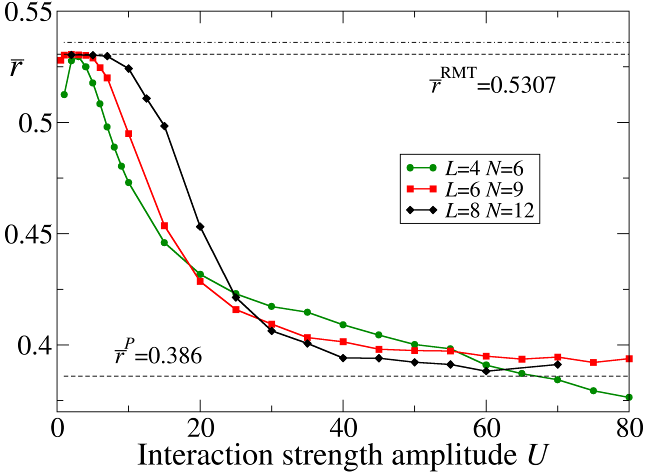
small versus large systems



# Nonperturbative Many-body localization

Random matrix type approach

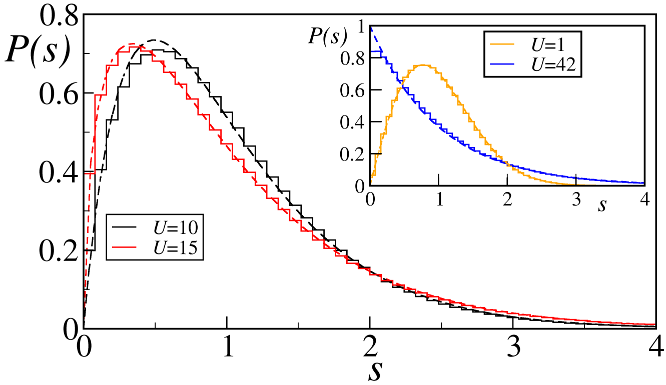
$r_n = \min[\delta_n^E, \delta_{n-1}^E]/\max[\delta_n^E, \delta_{n-1}^E]$, with $\delta_n^E = E_n - E_{n-1}$,



# Nonperturbative Many-body localization

Random matrix type approach

SemiPoisson distribution $P(s) = As^{\beta} \exp[-(\beta+1)s]$



# Approximate microscopic models

- Spin-1/2 approximation
  — Relevant subspace: permutations of 1 and 2 e.g. $|121212>$, $|122121>$, etc.
  — This is XX spin model with random magnetic field

$$\mathcal{H} = -2J\sum_{i}^{L-1}(S_{i+1}^{+}S_{i}^{-} + S_{i+1}^{-}S_{i}^{+}) + \sum_{i} U_i(S_i^z + 1/2)$$

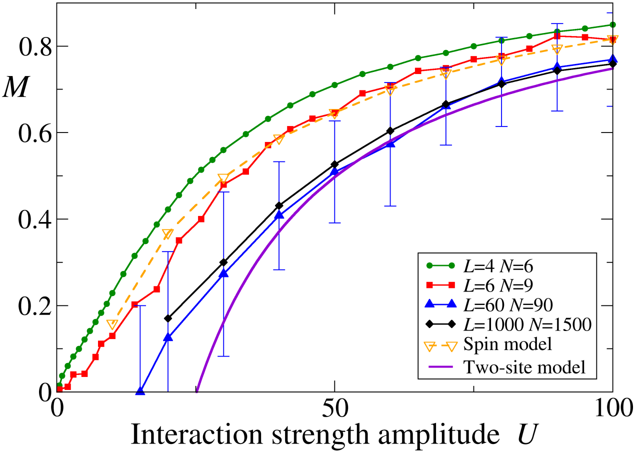
  — It maps on noninteracting fermions - Anderson localization
- For large $U$ – two sites approximation

$$\begin{pmatrix} U_1 & -2J \\ -2J & U_2 \end{pmatrix}$$

  — Diagonalize, average over disorder and time oscillations...
  $M \approx 1 - 8\pi J/U$

# Nonperturbative Many-body localization

Approximations work:



# Conclusions

- Many-body localization possible without its single particle counterpart
- Experimentally feasible model
- MBL explained microscopically via a simplified (abstract) system
- Piotr Sierant, Dominique Delande, and Jakub Zakrzewski, *Many-body localization due to random interactions* arXiv:1607.00227, PRL submitted.

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http://chaos.if.uj.edu.pl/∼kuba/Opus10/

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