Decay law: its non-exponential nature and its connection to time dilatation

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Seminar
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Warsaw

Outline

1. Decay law for a particle at rest: general properties, exponential limit and deviations, experimental evidence.

2. Theory: Lee Hamiltonian.

3. Decay of a moving particle. Is the usual Einstein-formula correct?

Part 1: General discussion and exp. evidence

Exponential decay law

- N_0 : Number of unstable particles at the time t = 0.

  $$\boxed{N(t) = N_0 e^{-\Gamma t}}\,, \quad \tau = 1/\Gamma \text{ mean lifetime}$$

  Confirmend in countless cases!

- For a single unstable particle:

  $$\boxed{p(t) = e^{-\Gamma t}}$$

  is the survival probability for a single unstable particle created at t=0. (Intrinsic probabilty, see Schrödinger´s cat).

For small times: $p(t) = 1 - \Gamma t + \ldots$

Basic definitions

Let $|S\rangle$ be an unstable state prepared at t = 0.

Survival probabilty ammplitude at t > 0:

$$a(t) = \langle S | e^{-iHt} | S \rangle \qquad (\hbar = 1)$$

Survival probability: $p(t) = |a(t)|^2$

Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

Decay theory of unstable quantum systems

L FONDA, G C GHIRARDI and A RIMINI

Deviations from the exp. law at short times

Taylor expansion of the amplitude:

$$a(t)=\langle S|e^{-iHt}|S\rangle = 1 - it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

$$a^*(t)=\langle S|e^{-iHt}|S\rangle = 1 + it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + \ldots$$

It follows:

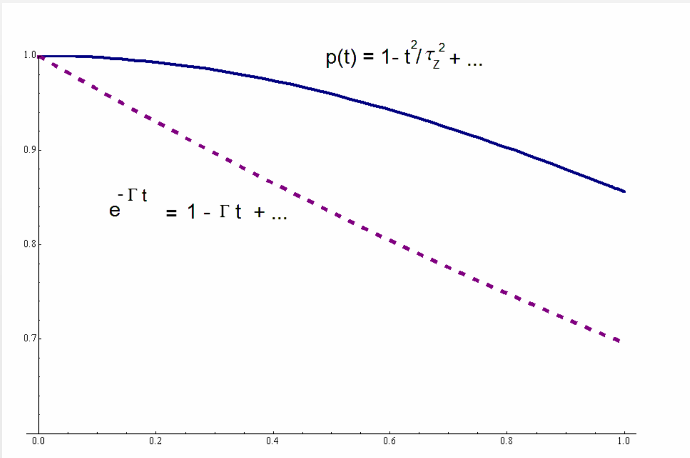
$$\boxed{\mathrm{p(t)} = |\mathrm{a(t)}|^2 = \mathrm{a}^*(\mathrm{t})\mathrm{a(t)} = 1 - \mathrm{t}^2\left(\langle \mathrm{S}|\mathrm{H}^2|\mathrm{S}\rangle - \langle \mathrm{S}|\mathrm{H}|\mathrm{S}\rangle^2\right) + \ldots = 1 - \frac{\mathrm{t}^2}{\tau_Z^2} + \ldots}$$

where $\tau_Z = \dfrac{1}{\sqrt{\langle \mathrm{S}|\mathrm{H}^2|\mathrm{S}\rangle - \langle \mathrm{S}|\mathrm{H}|\mathrm{S}\rangle^2}}$.

p(t) decreases quadratically (<u>not linearly</u>);

no exp. decay for short times.

τ_Z is the `Zeno time´.



Time evoluition and energy distribution (1)

The unstable state $|S\rangle$ is not an eigenstate of the Hamiltonian H.

Let $d_S(E)$ be the energy distribution of the unstable state $|S\rangle$.

Normalization holds: $\int_{-\infty}^{+\infty} d_S(E)dE = 1$

$$\boxed{a(t) = \int_{-\infty}^{+\infty} d_S(E)e^{-iEt}dE}$$

In stable limit : $d_S(E) = \delta(E-M) \rightarrow a(t) = e^{-iMt} \rightarrow p(t) = 1$

Time evoluition and energy distribution (2)

Breit-Wigner distribution:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \Gamma^2/4} \rightarrow a(t) = e^{-iM_0 t - \Gamma t/2} \rightarrow p(t) = e^{-\Gamma t}.$$

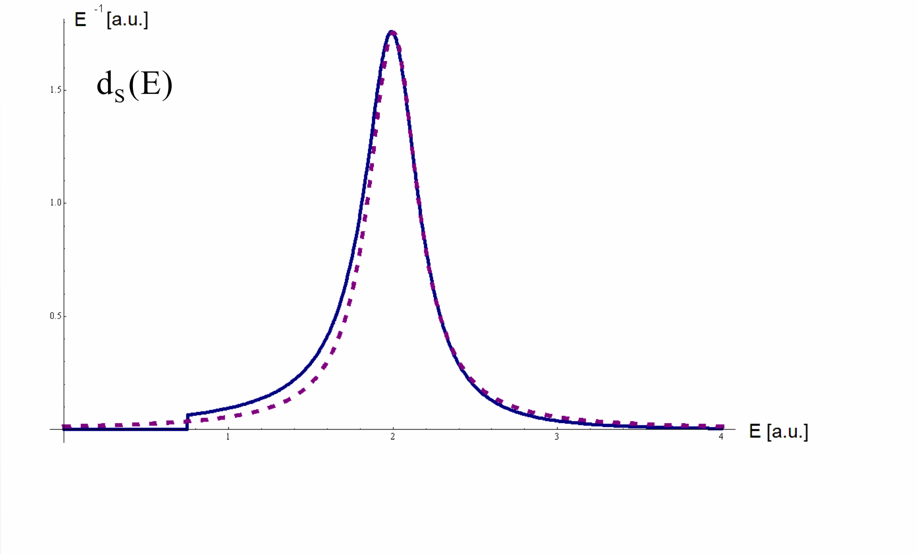
The Breit-Wigner energy distribution cannot be exact.

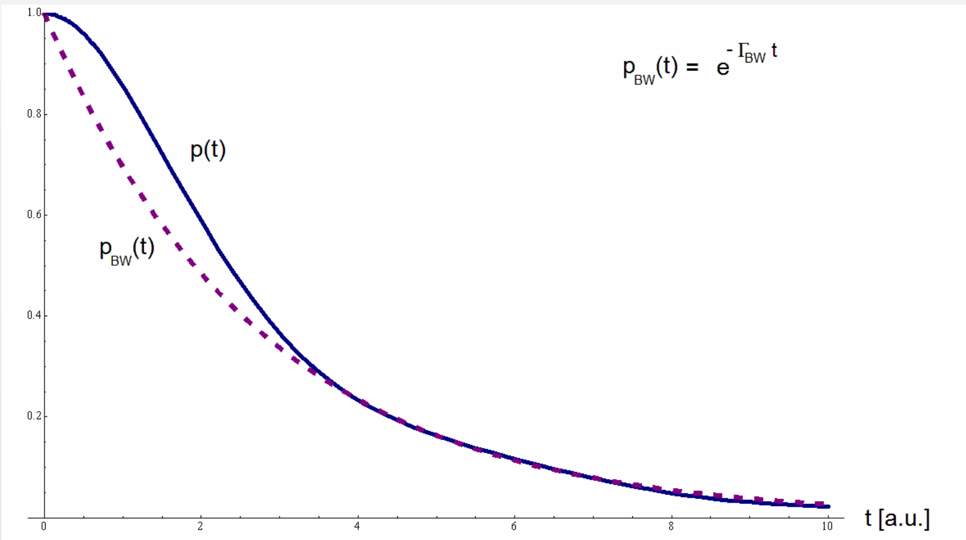
Two physical conditions for a realistic $d_S(E)$ are:

1) Minimal energy: $d_S(E) = 0 \text{ for } E < E_{\min}$

2) Mean energy finite: $\langle \mathrm{E} \rangle = \int_{-\infty}^{+\infty} d_S(E) E dE = \int_{E_{\min}}^{+\infty} d_S(E) E dE < \infty$

A very simple numerical example





$$M_0 = 2;\ E_{\min} = 0.75;\ \ \Gamma = 0.4;\ \Lambda = 3$$

$$\mathrm{d_S(E) = N_0 \frac{\Gamma}{2\pi} \frac{e^{-(E^2 - E_0^2)/\Lambda^2}\theta(E - E_{min})}{(E - M_0)^2 + \Gamma^2/4}}$$

$$\mathrm{d_{BW}(E) = \frac{\Gamma_{BW}}{2\pi} \frac{1}{(E - M_0)^2 + \Gamma_{BW}{}^2/4}}$$

Γ_{BW}, such that $\mathrm{d_{BW}(M_0) = d_S(M_0)}$

$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt} dE\ ;\quad p(t) = |a(t)|^2$$

$$p_{BW}(t) = e^{-\Gamma_{BW} t}$$

The quantum Zeno effect

We perform N inst. measurements:

the first one at time $t = t_0$, the second at time $t = 2t_0$, ..., the N-th at time $T = Nt_0$.

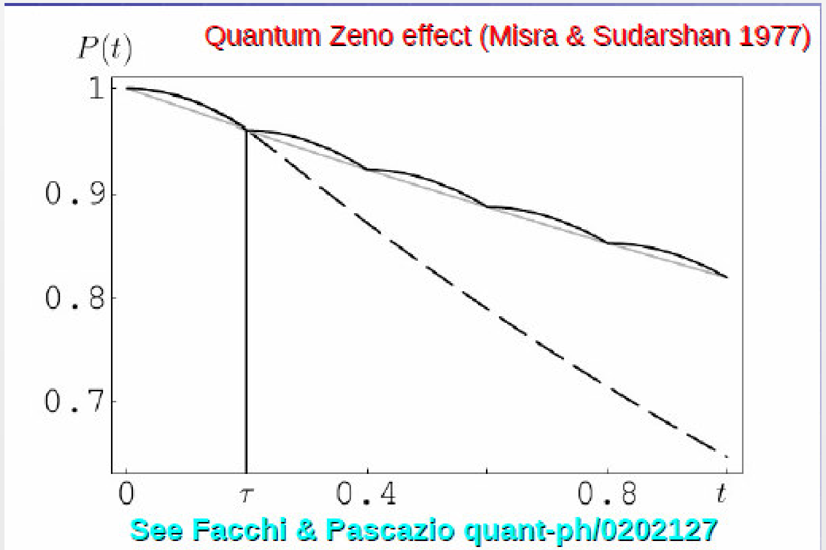
$$p_{\text{after N measurements}} = p(t_0)^N \approx \left(1 - \frac{t_0^2}{\tau_Z^2}\right)^N = \left(1 - \frac{T^2}{N^2 \tau_Z^2}\right)^N$$

under the assumption that t_0 is small enough.

If $N >> 1$ (at fixed T): $p_{\text{after N measurements}} \approx e^{-\frac{T^2}{N\tau_Z^2}} \approx 1.$

For large but finite N :

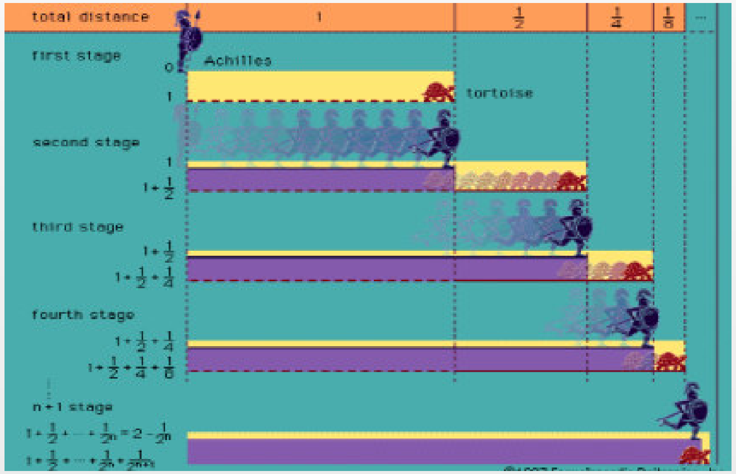
$\rightarrow$ slowing down of the decay.

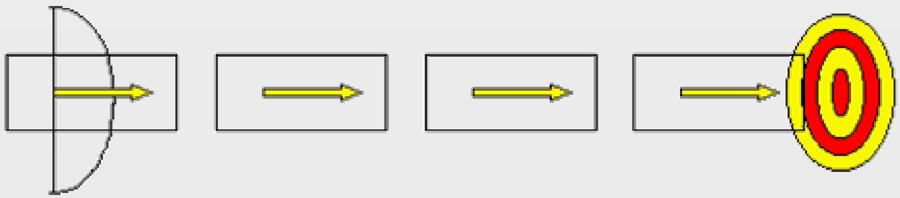


Zeno of Elea

489/431 a.c., Elea

Paradoxes:





Nowadays:
Zeno-effect, Zeno dynamics, Zeno subspaces,…

Experimental confirmation of non-exponential decays (1)

Uniwersytet
Jana Kochanowskiego w Kielcach

NATURE | VOL 387 | 5 JUNE 1997

Experimental evidence for non-exponential decay in quantum tunnelling

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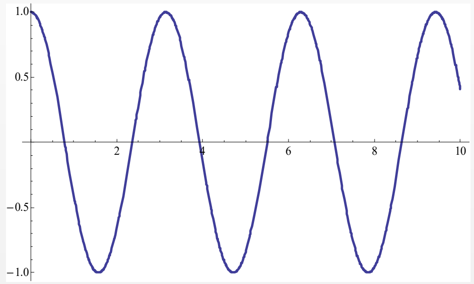
An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.

Cold Na atoms in a optical potential

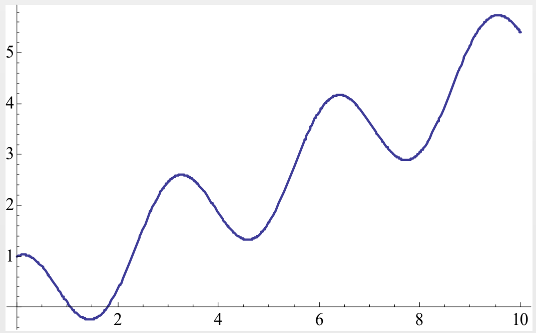
$V(x,0)$



$$V(x,t) = V_0 \cos\left(2k_L x - k_L a t^2\right)$$

x [a.u.]

$U(x')$



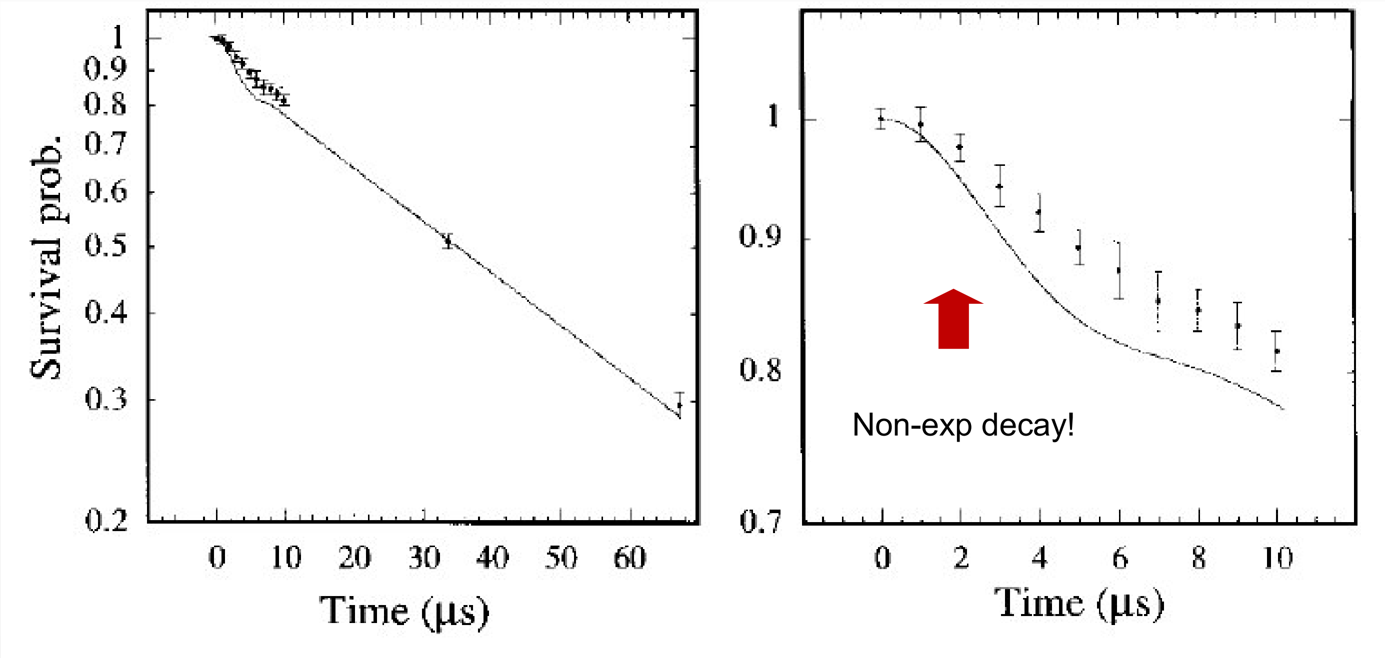
x' [a.u.]

$$x' = x - \frac{1}{2} a t^2$$

$$U(x') = V_0 \cos\left(2k_L x'\right) + Max'$$

Experimental confirmation of non-exponential decays (2)

Measured survival probabilty p(t)



Experimental confirmation of non-exponential decays and Zeno /Anti-Zeno effects

VOLUME 87, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 2001

Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System

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(Received 30 March 2001; published 10 July 2001)

We report the first observation of the quantum Zeno and anti-Zeno effects in an unstable system. Cold sodium atoms are trapped in a far-detuned standing wave of light that is accelerated for a controlled duration. For a large acceleration the atoms can escape the trapping potential via tunneling. Initially the number of trapped atoms shows strong nonexponential decay features, evolving into the characteristic exponential decay behavior. We repeatedly measure the number of atoms remaining trapped during the initial period of nonexponential decay. Depending on the frequency of measurements we observe a decay that is suppressed or enhanced as compared to the unperturbed system.

Same exp. setup,
but with measurements in between

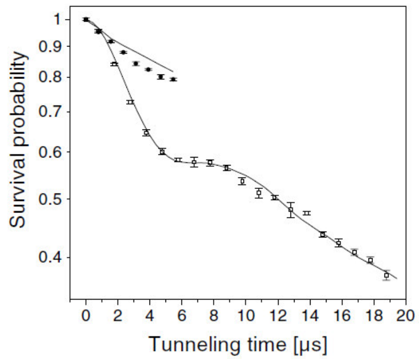


FIG. 3. Probability of survival in the accelerated potential as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 50 μs duration every 1 μs. The error bars denote the error of the mean. The data have been normalized to unity at $t_{\mathrm{tunnel}} = 0$ in order to compare with the simulations. The solid lines are quantum mechanical simulations of the experimental sequence with no adjustable parameters. For these data the parameters were $a_{\mathrm{tunnel}} = 15\,000\ \mathrm{m/s^2}$, $a_{\mathrm{interr}} = 2000\ \mathrm{m/s^2}$, $t_{\mathrm{interr}} = 50\ \mu\mathrm{s}$, and $V_0/h = 91$ kHz, where h is Planck's constant.

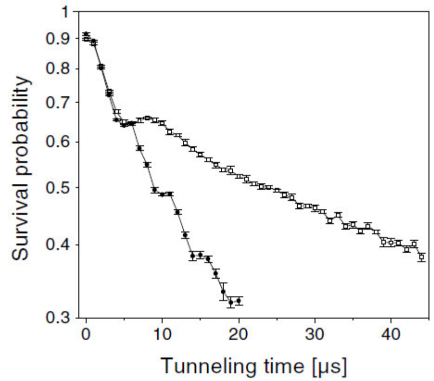


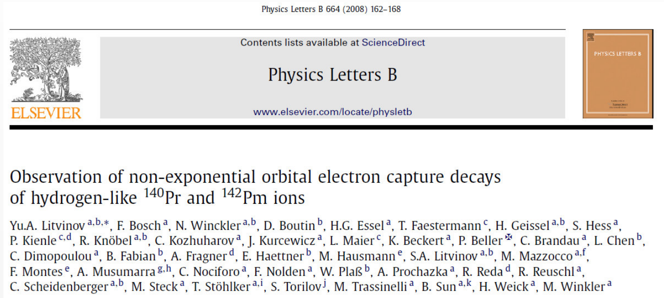
FIG. 4. Survival probability as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 40 μs duration every 5 μs. The error bars denote the error of the mean. The experimental data points have been connected by solid lines for clarity. For these data the parameters were: $a_{\mathrm{tunnel}} = 15\,000\ \mathrm{m/s^2}$, $a_{\mathrm{interr}} = 2800\ \mathrm{m/s^2}$, $t_{\mathrm{interr}} = 40\ \mu\mathrm{s}$, and $V_0/h = 116$ kHz.

Zeno effekt

Anti-Zeno effect

GSI-oscillations

Measurement of weak decays of ions.


Physics Letters B 664 (2008) 162–168

Contents lists available at ScienceDirect

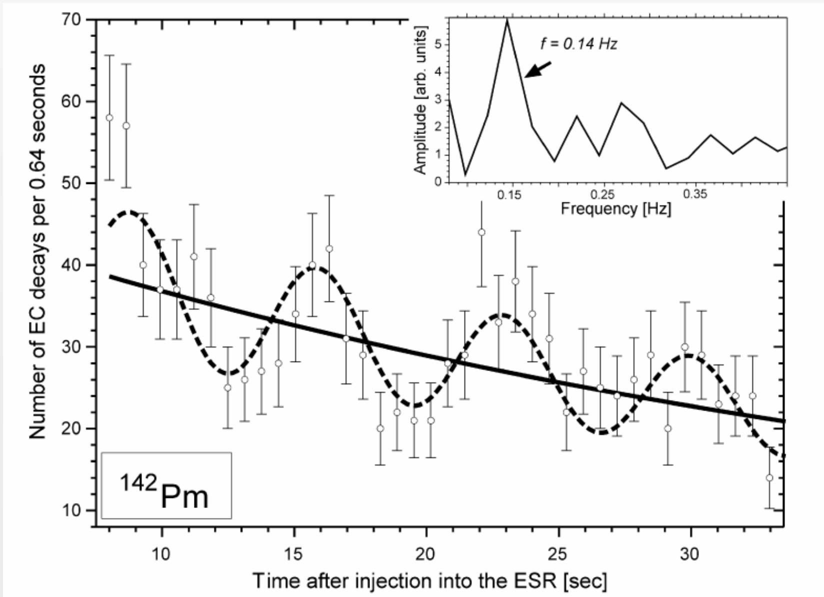
Physics Letters B

www.elsevier.com/locate/physletb

Observation of non-exponential orbital electron capture decays of hydrogen-like ^{140}Pr and ^{142}Pm ions

Yu.A. Litvinov [a,b,*], F. Bosch [a], N. Winckler [a,b], D. Boutin [b], H.G. Essel [a], T. Faestermann [c], H. Geissel [a,b], S. Hess [a], P. Kienle [c,d], R. Knöbel [a,b], C. Kozhuharov [a], J. Kurcewicz [a], L. Maier [c], K. Beckert [a], P. Beller [✠], C. Brandau [a], L. Chen [b], C. Dimopoulou [a], B. Fabian [b], A. Fragner [d], E. Haettner [b], M. Hausmann [e], S.A. Litvinov [a,b], M. Mazzocco [a,f], F. Montes [e], A. Musumarra [g,h], C. Nociforo [a], F. Nolden [a], W. Plaß [b], A. Prochazka [a], R. Reda [d], R. Reuschl [a], C. Scheidenberger [a,b], M. Steck [a], T. Stöhlker [a,i], S. Torilov [j], M. Trassinelli [a], B. Sun [a,k], H. Weick [a], M. Winkler [a]

Decay of H-like Pm into:
neutrino + Nd



Measurement was:

$$\frac{dN_{decays}}{dt} \propto -\frac{dp(t)}{dt}$$

Oscillations later confirmed.

arXiv:1309.7294 [nucl-ex]. Explanation still missing!

Late-time deviations

PRL **96,** 163601 (2006)   PHYSICAL REVIEW LETTERS   week ending 28 APRIL 2006

Violation of the Exponential-Decay Law at Long Times

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(Received 4 July 2005; published 26 April 2006)

First-principles quantum mechanical calculations show that the exponential-decay law for any metastable state is only an approximation and predict an asymptotically algebraic contribution to the decay for sufficiently long times. In this Letter, we measure the luminescence decays of many dissolved organic materials after pulsed laser excitation over more than 20 lifetimes and obtain the first experimental proof of the turnover into the nonexponential decay regime. As theoretically expected, the strength of the nonexponential contributions scales with the energetic width of the excited state density distribution whereas the slope indicates the broadening mechanism.

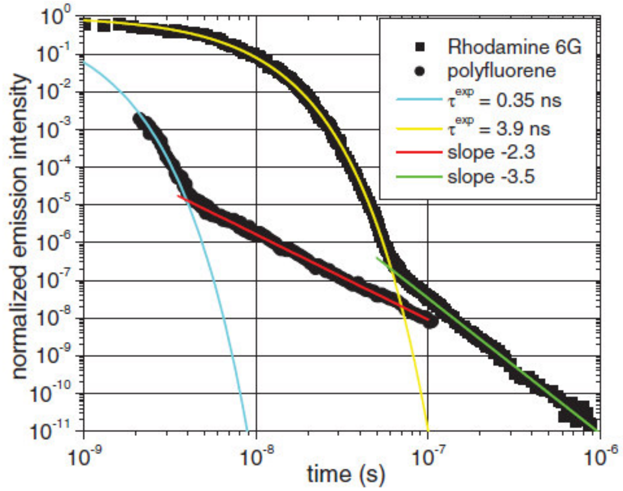


FIG. 2 (color). Corresponding double logarithmic fluorescence decays of the emissions shown in Fig. 1. Exponential and power law regions are indicated by solid lines and the emission intensity at time zero has been normalized.

Part 2: Lee Hamiltonian

Lee Hamiltonian

$$H = H_0 + H_1$$

$$H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk \omega(k) |k\rangle\langle k|$$

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

|S> is the initial unstable state, coupled to an infinity of final states |k>. (Poincare-time is infinite. Irreversible decay). General approach, similar Hamiltonians used in many areas of Physics.

Example/1: spontaneous emission. |S> represents an atom in the excited state, |k> is the ground-state plus photon.

Example/2: pion decay. |S> represents a neutral pion, |k> represents two photons (flying back-to-back)

Propagator and spectral function

$$H = H_0 + H_1 \; ; \; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k) |k\rangle\langle k| \; ; \; H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$G_S(E) = \left\langle S \right| (E - H + i\varepsilon)^{-1} \left| S \right\rangle = (E - M_0 + \Pi(E) + i\varepsilon)^{-1}$$

$$\Pi(E) = -\int_{-\infty}^{+\infty} \frac{dk}{2\pi} \frac{g^2 f(k)^2}{E - \omega(k) + i\varepsilon}$$

$$d_S(E) = \frac{1}{\pi} \operatorname{Im} G_S(E) \; ;$$

$$\boxed{a(t) = \left\langle S \right| e^{-iHt} \left| S \right\rangle = \int_{-\infty}^{+\infty} dE\, d_S(E) e^{-iEt}}$$

It follows:

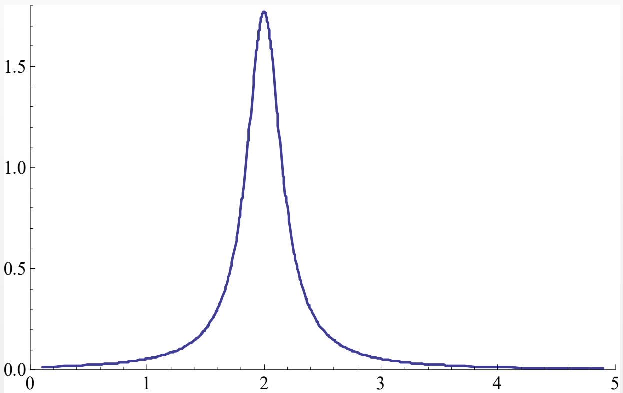
$$\boxed{\int_{-\infty}^{+\infty} dE\, d_S(E) = 1}$$

Fermi golden rule: $\Gamma = \operatorname{Im}[\Pi(M)]/2$.

Exponential limit

$$H = H_0 + H_1 \;;\; H_0 = M_0 |S\rangle\langle S| + \int_{-\infty}^{+\infty} dk\,\omega(k)|k\rangle\langle k| \;;\; H_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dk(g\cdot f(k))(|S\rangle\langle k| + |k\rangle\langle S|)$$

$$\omega(k) = k \;;\quad f(k) = 1 \quad \Rightarrow \quad \Pi(E) = ig^2/2 \;;\; \Gamma = g^2$$



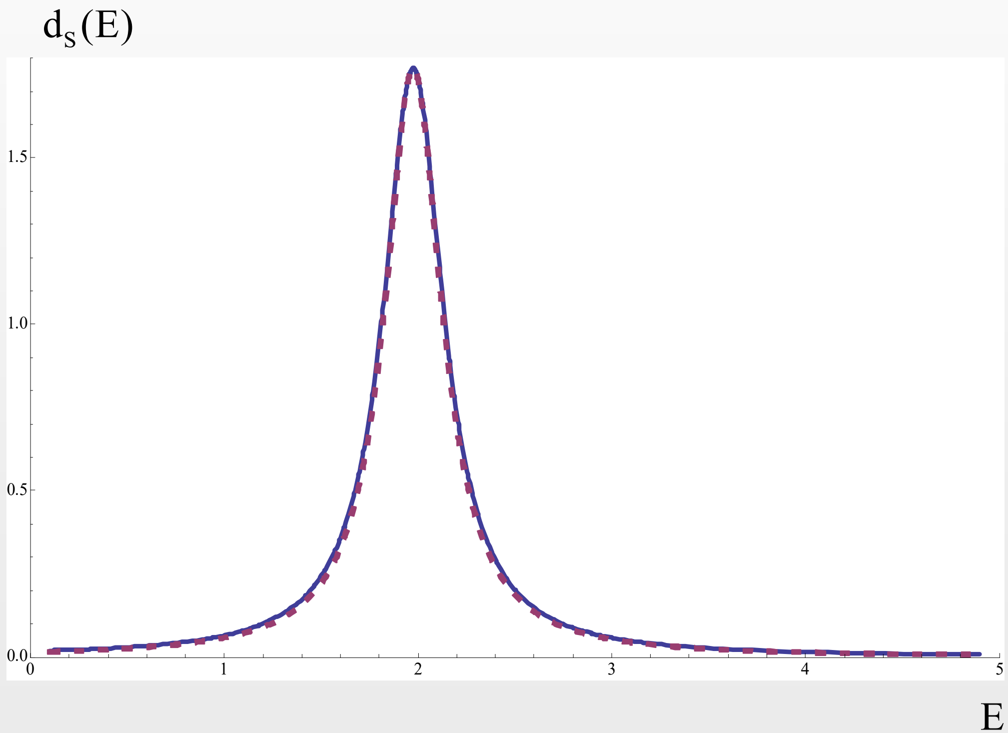
$$d_S(E) = \frac{\Gamma}{2\pi}\frac{1}{(E - M_0)^2 + \Gamma^2/4}$$

$$\Rightarrow a(t) = e^{-i(M_0 - i\Gamma/2)t} \Rightarrow p(t) = e^{-\Gamma t}$$

Non-exponential case (1)

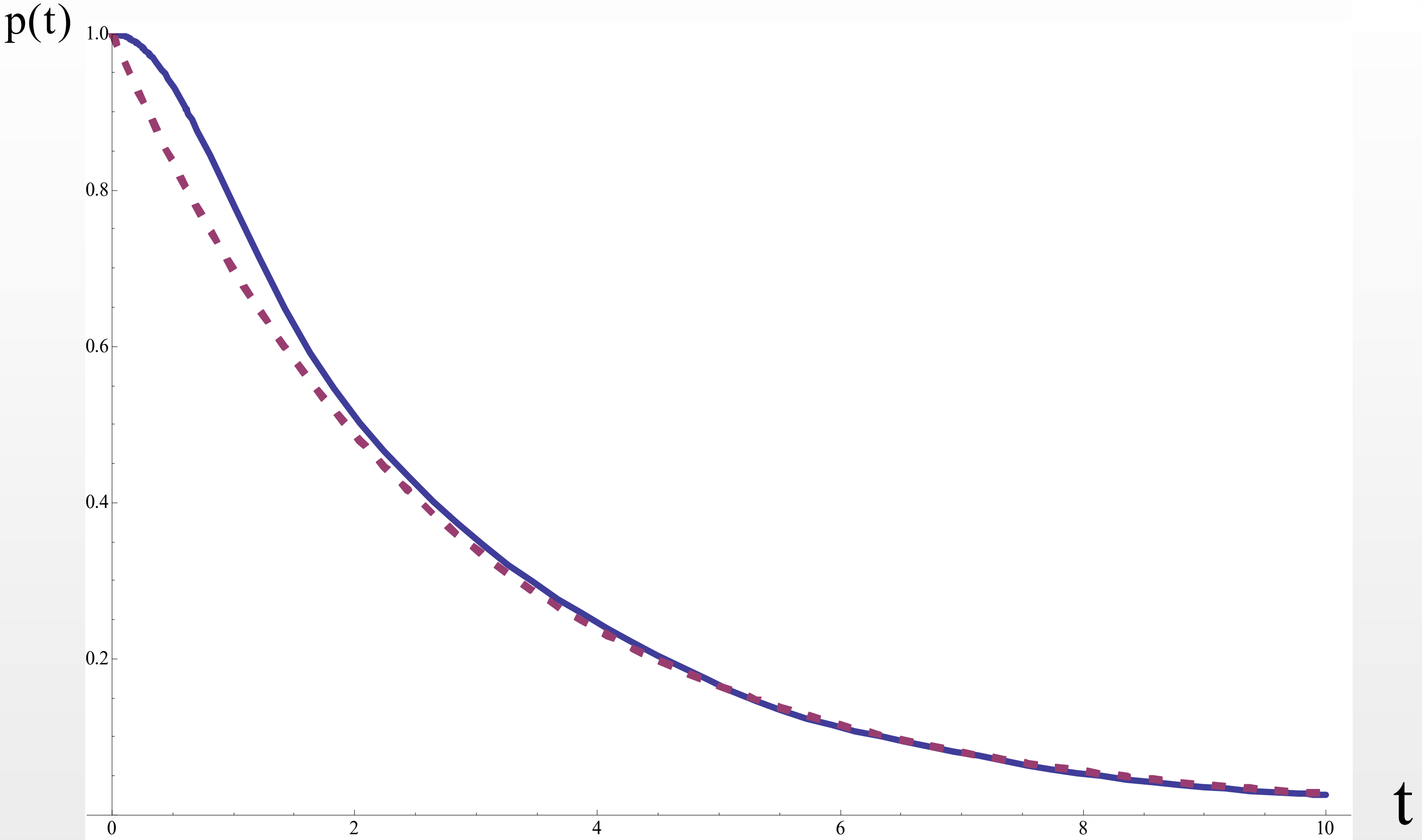
$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g \cdot f(k)) (|S\rangle\langle k| + |k\rangle\langle S|)$$

$$f(k) = \begin{cases} 0 \text{ for } k < E_{min} \\ 1 \text{ for } E_{min} \leq k \leq E_{max} \\ 0 \text{ for } k > E_{max} \end{cases}$$



$$M_0 = 2;\ E_{min} = 0;\ E_{max} = 5;\ g^2 = 0.36 \text{ (all in a.u. of energy)}$$

Non-exponential case (2)

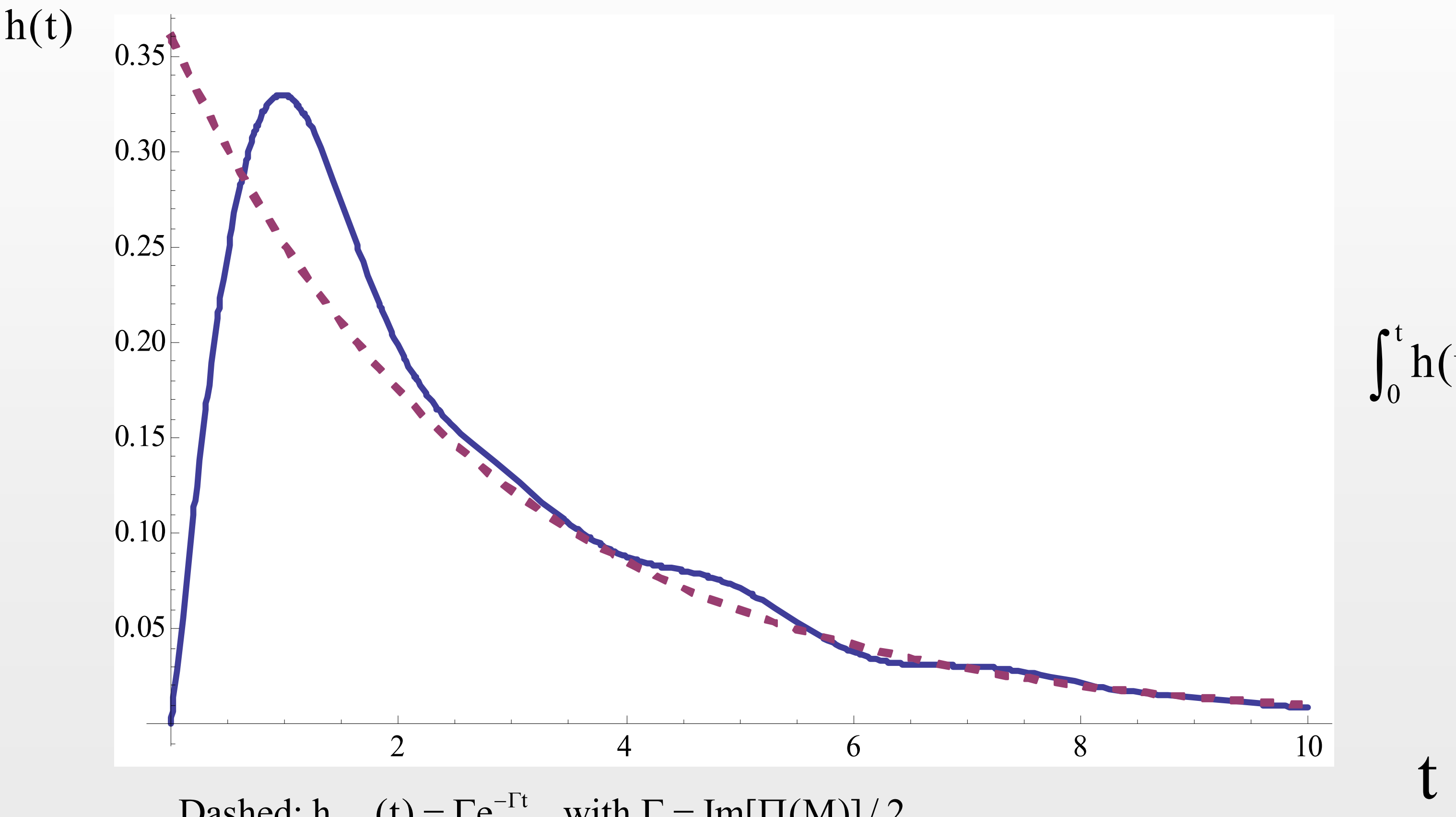


Dashed: $p_{BW}(t) = e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

Non-exponential case (3)

$$\boxed{h(t) = -\frac{dp(t)}{dt}}$$

Namley: $h(t)dt = p(t) - p(t+dt)$ is the probability that the particles decays between t and t+dt



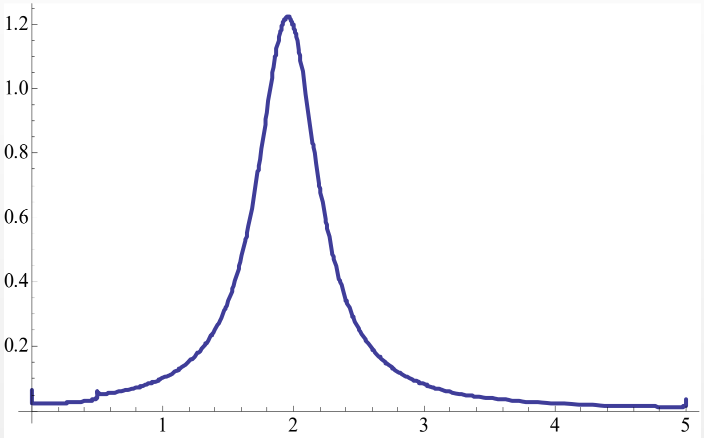
$$\int_0^t h(u)du = 1 - p(t)$$

Dashed: $h_{BW}(t) = \Gamma e^{-\Gamma t}$ with $\Gamma = \mathrm{Im}[\Pi(M)]/2$

Two-channel case (1)

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g_1 \cdot f_1(k)) (|S\rangle\langle k,1| + |k,1\rangle\langle S|) + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk (g_2 \cdot f_2(k)) (|S\rangle\langle k,2| + |k,2\rangle\langle S|)$$

$$f_i(k) = \begin{cases} 0 \text{ for } k < E_{i,\min} \\ 1 \text{ for } E_{i,\min} \leq k \leq E_{i,\max} \\ 0 \text{ for } k > E_{i,\max} \end{cases}$$



$M_0 = 2;\ E_{1,\min} = 0;\ E_{2,\min} = 0\ ;\ E_{1,\max} = E_{2,\max} = 5;$

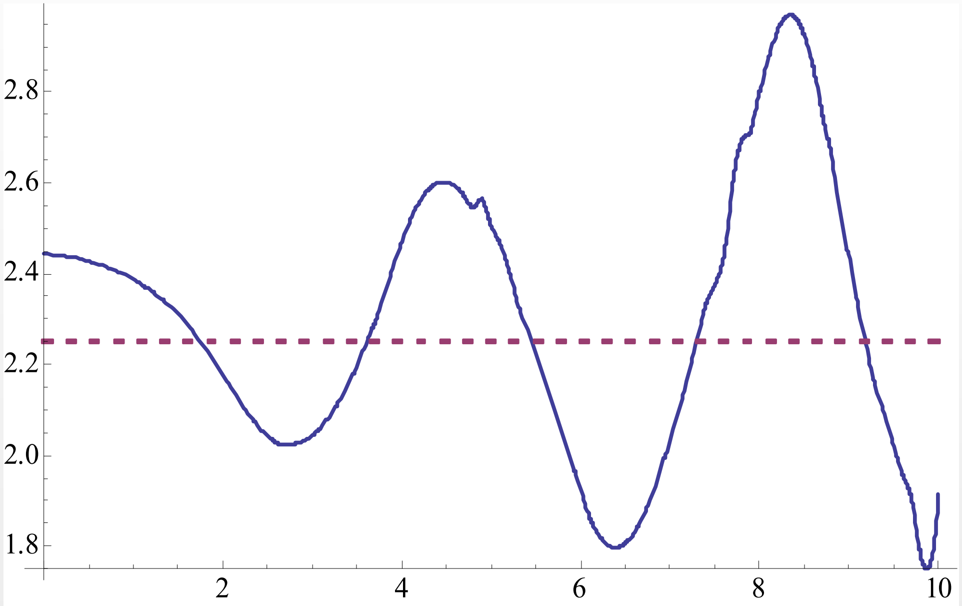
$g_1^2 = 0.36\ ;\ g_2^2 = 0.16$ (all in a.u. of energy)

Two-channel case (2)

$h_1(t)dt =$ probabilty that the state $|S\rangle$ decays in the first channel between (t,t+dt)

$h_2(t)dt =$ probabilty that the state $|S\rangle$ decays in the second channel between (t,t+dt)

$$\frac{h_1(t)}{h_2(t)}$$



t

Dashed: $\frac{h_{1,BW}(t)}{h_{2,BW}(t)} = \frac{\Gamma_1}{\Gamma_2} = \text{const}$

Measurable effect???

Details in:

F. G., Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels, Found. Phys. 42 (2012) 1262 [arXiv:1110.5923].

What about QFT?

The textbook expression for decay

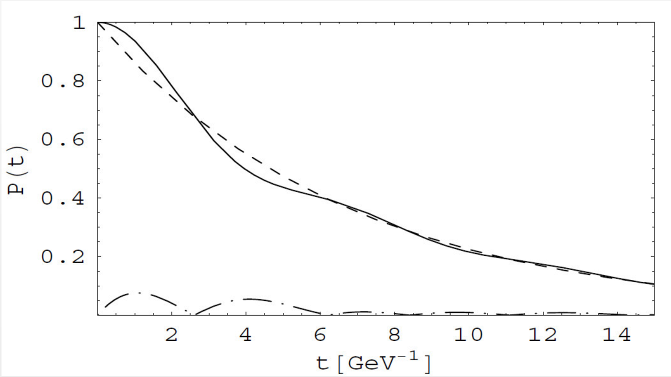
$$d\Gamma = \frac{(2\pi)^4}{2M}\,|\mathcal{M}|^2\,\delta(p-k_1-k_2)\frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}$$

is actually valid only in the exponential limit. (S-matrix formalis, in- and out states).

One can however go beyond!

Also in full QFT deviations exist.

Example: p(t) for the ρ meson



Details in: F. G. and G. Pagliara,
Deviation from the exponential decay law in relativistic quantum field theory: the example of strongly decaying particles,
Mod. Phys. Lett. A 26 (2011) 2247 [arXiv:1005.4817 [hep-ph]].

F. Giacosa,
Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels,'
Found. Phys. **42** (2012) 1262 [arXiv:1110.5923 [nucl-th]].

Quantum field theory: is there a “maximal energy scale“?

$$\int_0^{\Lambda} d_H(m)dm = 1$$

no matter how large is Λ...

but if one tries to do $\Lambda \to \infty$ one encounters problems: normalization, etc.

$$d_H(m) \propto 1/(m \cdot \ln^2 m) \qquad \text{for large m}$$

Finite outcome: even for a renorm. QFT the existence of a maximal energy scale (i.e., a minimal length) is needed.

F. G. and G.Pagliara, Spectral function of a scalar boson coupled to fermions, Phys. Rev. D 88 (2013) 025010 [arXiv:1210.4192].

Part 3: Decay of a moving particle

Unstable particle with momentum p

$$|S,p\rangle \qquad |S,p\rangle = U_p \, |S,0\rangle$$

We **expect** in the exponential limit:

$$P_{nd}(t) = e^{-\frac{\Gamma}{\gamma}t}\ ,\ \tau = \gamma\Gamma^{-1}\ \text{`dilated lifetime'.}$$

Reduction of the decay width

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2+M^2}}$$

Finite momentum vs finite velocity

- Subtle but important point: in the long-life limit, a particle with definite momentum has also definite velocity.
- $$p = \frac{M}{\sqrt{1-v^2}} v$$
- In general, however, there is a difference! For an unstable state a boost is not equivalent to a momentum translation.
- Here, we consider at first definite momentum

$$|S, p\rangle = \int_0^\infty \mathrm{d}m a_S(m) \, |m, p\rangle$$

Non-decay probability

Straightforward calculation

$$a(t,p) = \frac{1}{\delta(p=0)} \langle S,p \left| e^{-iHt} \right| S,p \rangle = \int_{-\infty}^{\infty} \mathrm{dm} d_S(m) e^{-i\sqrt{m^2+p^2}t}$$

$$\simeq \int_{-\infty}^{\infty} \mathrm{dm} d_S^{BW}(m) e^{-i\sqrt{m^2+p^2}t} = e^{-i\sqrt{(M-i\Gamma/2)^2+p^2}t} \; .$$

One obtains:

$$P_{nd}(t) = |a(t,p)|^2 = e^{-\Gamma_p t}$$

$$\Gamma_p = 2\,\mathrm{Im}\left[\sqrt{(M-i\Gamma/2)^2+p^2}\right] \; .$$

This expression does **not** coincide with the usual Einstein expression!

Unstable particle with momentum p: previous works

L. A. Khalfin, Theory of unstable particles and relativity, PDMI Preprint/1997

M. I. Shirokov, JIMR E2 10614 (1977), Int. J. Theor. Phys. 43 (2004) 1541.

E. V. Stefanovich, Int. Jour. Theor. Phys, 35 12 (1996)

K. Urbanowski, Phys. Lett. B 737 (2014) 346.

See also the negative result

S. A. Alavi and C. Giunti, Europhys. Lett. 109 (2015) 6, 6001

My recent paper: F. G., Acta Phys. Pol. B47 (2016) 2135 arXiv:1512.00232 [hep-ph]

Unstable particle with momentum p: unexpected result for the nondecay probability

$$|S,p\rangle = U_p\ |S,0\rangle$$

$$|S,p\rangle = \int_0^\infty dm a_S(m)\,|m,p\rangle$$

The non-decay probability: $P_{nd}(t) = e^{-\Gamma_p t}$

$$\Gamma_p = \sqrt{2}\sqrt{\left[\left(M^2 - \frac{\Gamma^2}{4} + p^2\right)^2 + M^2\Gamma^2\right]^{1/2} - \left(M^2 - \frac{\Gamma^2}{4} + p^2\right)}$$

F. G. arXiv:1512.00232 [hep-ph]

$$\Gamma_p \neq \tilde{\Gamma}_p = \Gamma M/\sqrt{p^2 + M^2}$$

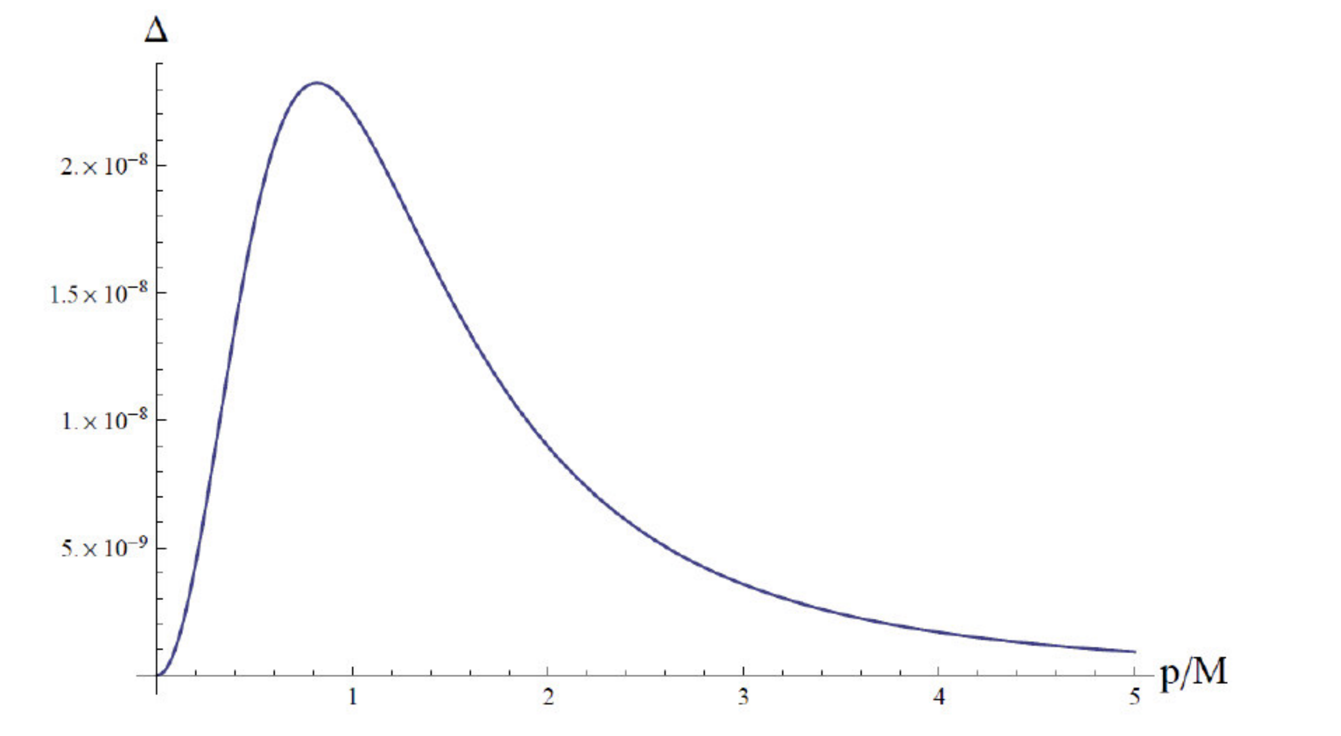
But this is not a breaking of relativity!
It is a different setup.

Unstable particle with momentum p:deviation

$$\Delta = \frac{\Gamma_p - \tilde{\Gamma}_p}{M}$$

$$\Gamma_p = \sqrt{2}\sqrt{\left[\left(M^2 - \frac{\Gamma^2}{4} + p^2\right)^2 + M^2\Gamma^2\right]^{1/2} - \left(M^2 - \frac{\Gamma^2}{4} + p^2\right)}\,.$$

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2 + M^2}}$$



$$\frac{p_{\max}}{M} = \sqrt{\frac{2}{3}} \simeq 0.816$$

$$\Delta_{\max} = \frac{\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}}}{M} \simeq \frac{3}{100}\sqrt{\frac{3}{5}}\left(\frac{\Gamma}{M}\right)^3$$

QFT text-book

Back to QFT. The S-matrix approach

$$d\Gamma = \frac{(2\pi)^4}{2M}\,|\mathcal{M}|^2\,\delta(p-k_1-k_2)\frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}$$

is again justified for very small decay width!
Here, the time-dilatation formula holds exactly.

The full QFT proof of the deviation is strictly speaking still missing.
(Technically, the formalism used above is based on so-called Lee Hamiltonians, which are QFT-like, but care is needed).

Unstable particle with momentum p: some examples of deviations

Muon
M = 105.65 MeV
Γ = $2.99 \cdot 10^{-16}$ MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 5.598 \cdot 10^{-53}\ \mathrm{MeV}$$

Neutral pion
M = 134.98 MeV
Γ = $7.72 \cdot 10^{-6}$ MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 5.81 \cdot 10^{-22}\ \mathrm{MeV}$$

Rho meson
M = 775.26 MeV
Γ = 147.8 MeV

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 0.125\ \mathrm{MeV}$$

Very small deviations!

Boost: state with definite velocity

Point: a velocity translation (i.e. a boost) is not a momentum translation!!!!

$$U_v \left|S, 0\right\rangle \equiv \left|S, v\right\rangle$$

$$\left|\left\langle S, v\right| e^{-iHt} \left| S, v\right\rangle\right|^2 = e^{-i\gamma\Gamma t}$$

The survival probability shows here an absurd Lorentz contraction!

Boost: state with definite velocity revisited

$$U_v \left|S,0\right\rangle \equiv \left|S,v\right\rangle$$

$$\left|S,v\right\rangle = \int_0^\infty dm a_S(m)\sqrt{m}\gamma^{3/2}\left|m,m\gamma v\right\rangle$$

$$P_{nd}(t) = 0$$

A boosted muon consists of an electron and two neutrinos!

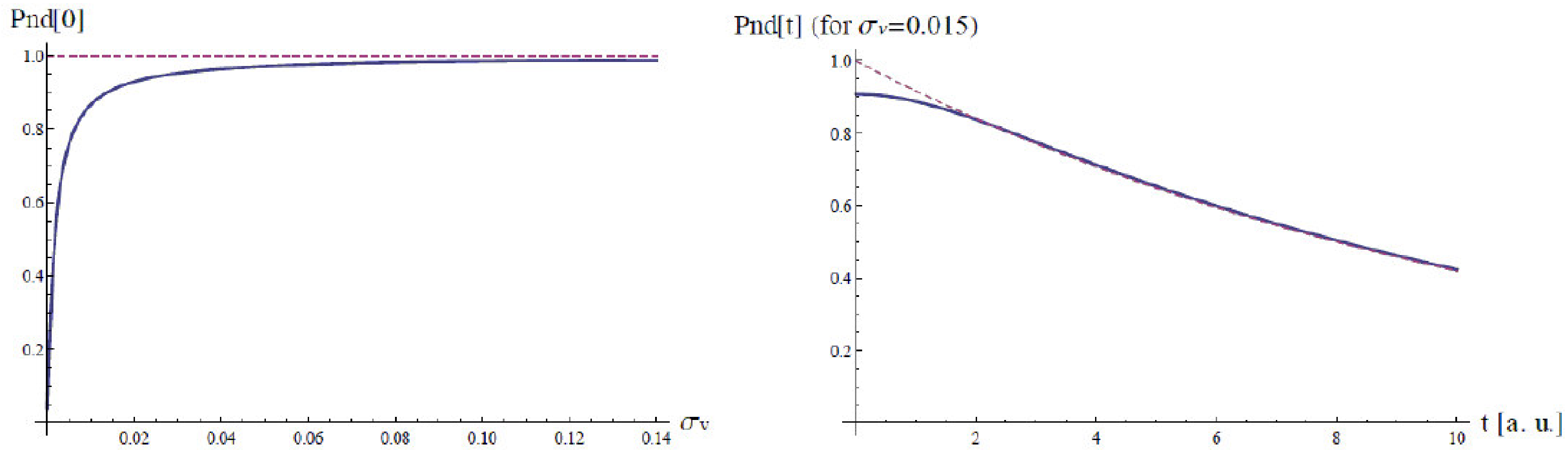
Details in arXiv:1512.00232 [hep-ph]

Boost: wave packet in velocity (is qualitatively different!)

$$|\Phi\rangle = \int_{-1}^{+1} dv C(v) |S, v\rangle$$

$$C(v) = N e^{-(v-v_0)^2/(4\sigma_v^2)}$$

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp \left| \langle S, p | e^{-iHt} | \Phi \rangle \right|^2$$



Summary and outlook

- The decay is never exponential! This is a fact.
- This is so both in QM and QFT.
- Decay of a moving particle: interesting link between relativity and QM and QFT.
- For a particle with definite momentum p (for the measuring observer) there is a different formula. Numerically, the Einstein expression is very good but is not exact.
- A boost is a very subtle operation in QM and QFT.

Thank You

- **When Physicists Attack: Homeless Man Attacks Fellow Transient in Disagreement Over Quantum Physics**
- 1, June 25, 2009 jonathanturley Bizarre, Criminal law, Society
- This week a homeless man in California hit a fellow transient in the face with a skateboard over a disagreement about quantum physics. In San Francisco, Jason Everett Keller, 40, allegedly attacked, Stephan Fava, over a disputed physics question.
- At the time of the attack, Fava was discussing quantum physics with a third homeless man.
- I have been warning for years about the danger of "fighting words" in quantum physics discussions. I confess that I have come close to blows when I hear someone disparage Planck's Action Constant in a bar.

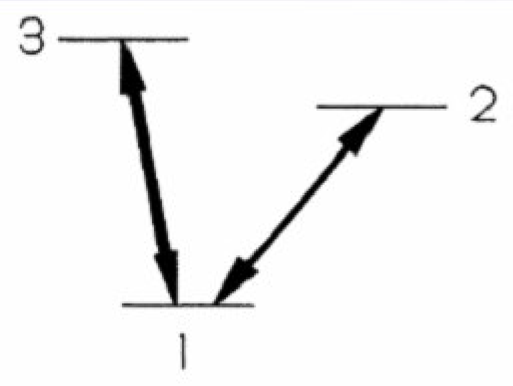
Experimental confirmation of the quantum Zeno effect - Itano et al (1)

PHYSICAL REVIEW A VOLUME 41, NUMBER 5 1 MARCH 1990

Quantum Zeno effect

Wayne M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland
Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80303
(Received 12 October 1989)

The quantum Zeno effect is the inhibition of transitions between quantum states by frequent measurements of the state. The inhibition arises because the measurement causes a collapse (reduction) of the wave function. If the time between measurements is short enough, the wave function usually collapses back to the initial state. We have observed this effect in an rf transition between two $^9Be^+$ ground-state hyperfine levels. The ions were confined in a Penning trap and laser cooled. Short pulses of light, applied at the same time as the rf field, made the measurements. If an ion was in one state, it scattered a few photons; if it was in the other, it scattered no photons. In the latter case the wave-function collapse was due to a null measurement. Good agreement was found with calculations.



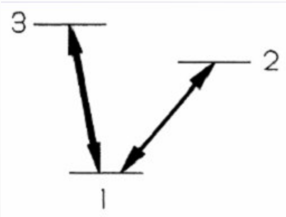
(Undisturbed) survival probability

At $t = 0$, the electron is in $|1\rangle$.

$$p(t) = \cos^2\left(\frac{\Omega t}{2}\right) = 1 - \frac{\Omega^2 t^2}{4} + \ldots$$

$$p(T) = 0 \text{ für } T = \pi/\Omega$$

Experimental confirmation of the quantum Zeno effect - Itano et al (2)



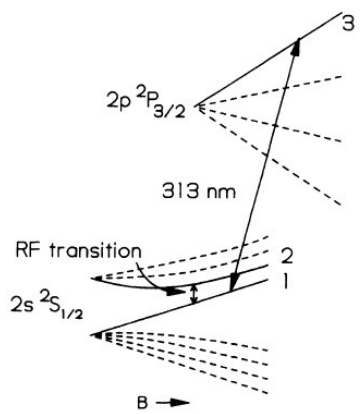


FIG. 2. Diagram of the energy levels of $^9Be^+$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

5000 Ions in a Penning trap

Short laser pulses 1-3 work as measurements.

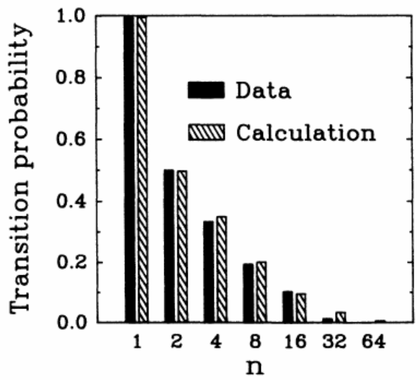


FIG. 3. Graph of the experimental and calculated $1 \rightarrow 2$ transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

$$p(t) = \cos^2\left(\Omega t/2\right) = 1 - \frac{\Omega^2 t^2}{4} + \ldots\,; \qquad p(T) = 0 \text{ für } T = \pi/\Omega$$

(Transition probability (without measuring) at time T) : $\quad 1 - p(T) \; = \; 1$.

Witn n measurements in between the transition probabilty decreases!

The electron stays in state 1.

Other experiments about Zeno

Uniwersytet
Jana Kochanowskiego w Kielcach

ELSEVIER
1 June 2000
OPTICS COMMUNICATIONS
Optics Communications 180 (2000) 115–120
www.elsevier.com/locate/optcom

The quantum Zeno effect – evolution of an atom impeded by measurement

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Optics Communications 211 (2002) 235–241
www.elsevier.com/locate/optcom

A relaxationless demonstration of the Quantum Zeno paradox on an individual atom

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Continuous and Pulsed Quantum Zeno Effect

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(Received 14 June 2006; published 27 December 2006)

PRL 101, 180402 (2008) PHYSICAL REVIEW LETTERS week ending 31 OCTOBER 2008

Freezing Coherent Field Growth in a Cavity by the Quantum Zeno Effect

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(Received 21 July 2008; published 28 October 2008)

Some numbers

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq \quad 5.598 \cdot 10^{-53}\ \text{MeV}$$

$$M = 105.6583\ \text{MeV},\ \Gamma = 2.99 \cdot 10^{-16}\ \text{MeV}$$

Muon

$$M = 134.9766\ \text{MeV},\ \Gamma = 7.72 \cdot 10^{-6}\ \text{MeV}$$

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Neutral pion

$$M = 775.26\ \text{MeV},\ \Gamma = 147.8\ \text{MeV}$$

$$\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}} \simeq 0.125\ \text{MeV}$$

Rho-meson

$$\Gamma_p = \sqrt{2}\sqrt{\left[\left(M^2 - \frac{\Gamma^2}{4} + p^2\right)^2 + M^2\Gamma^2\right]^{1/2} - \left(M^2 - \frac{\Gamma^2}{4} + p^2\right)}\ .$$

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2 + M^2}}$$

Unstable particle with momentum p

We work in the exp. limit

M = rest mass; Γ= decay width in the rest frame.

An unstable particle moves with definite momentum p.

Which is its decay width? The stanard expression is:

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2 + M^2}}$$

Important but sublte point:
in QM and QFT a state with definite momentum has not definite velocity.

Decay width of a general state

$$|\Psi\rangle = \int_{-\infty}^{+\infty} dp B(p)\,|S,p\rangle$$

the quantity $\langle\Psi\,|e^{-iHt}|\,\Psi\rangle$ is *not* what we are looking for.

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp\,\left|\langle S,p\,|e^{-iHt}|\,\Psi\rangle\right|^2$$

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp\,|B(p)|^2\,e^{-\Gamma_p t}$$

Inclusion of spatial wave function is simple. Generalization straightforward.
Details in arXiv:1512.00232.

Exponential limit and final state spectrum (1)

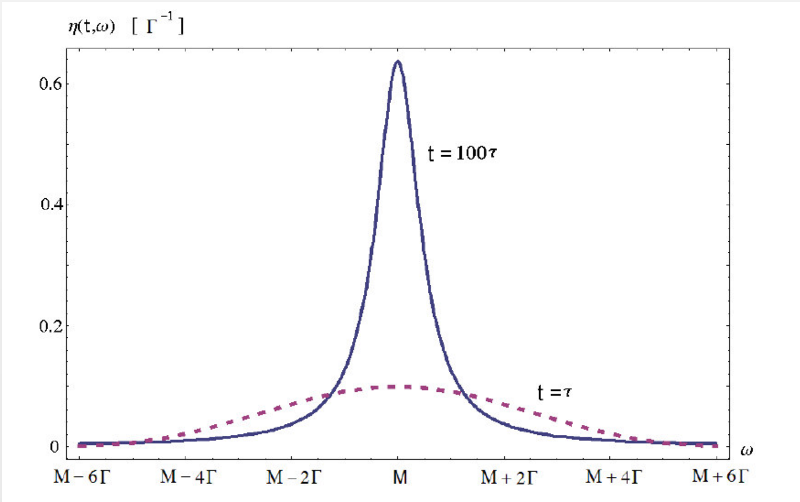
$$\left|\left\langle k \middle| e^{-iHt} \middle| S \right\rangle\right|^2 \text{ is the prob. that } \left|S\right\rangle \text{ transforms into } \left|k\right\rangle$$

Translating into energy:

$$\eta(t,\omega) = \frac{\Gamma}{2\pi}\left|\frac{e^{-i\omega t} - e^{-i(M_0 - i\Gamma/2)t}}{E - M_0 + i\Gamma/2}\right|^2 \quad ;$$

In spont. emission:

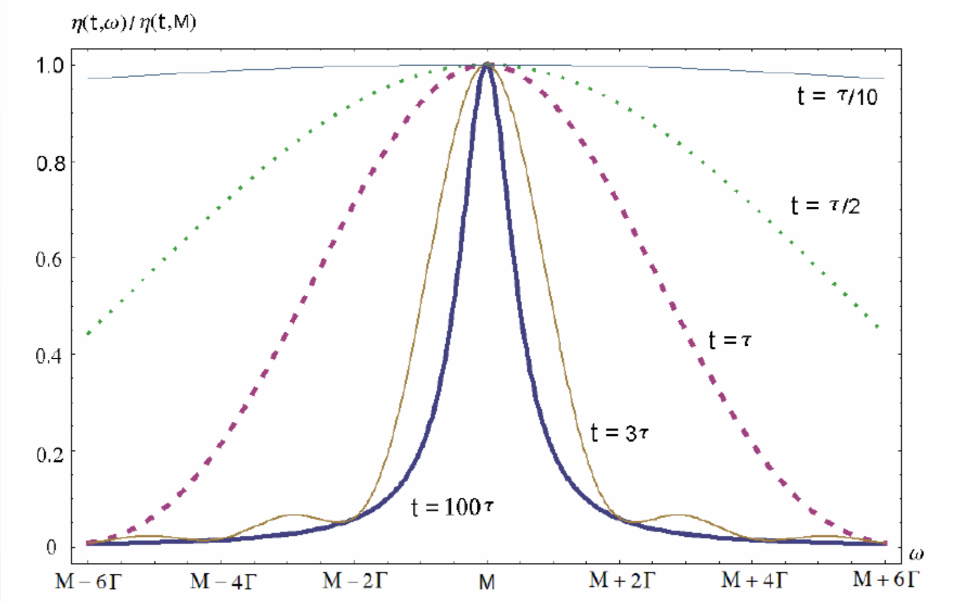
$\eta(t,\omega)d\omega$ is the prob. that the outgoing photon has an energy between ω and $\omega+d\omega$



Details in: F. G.,
Energy uncertainty of the final state of a decay process
arXiv:1305.4467 [quant-ph].

Exponential limit and final state spectrum (2)

$$\eta(t,\omega)=\frac{\Gamma}{2\pi}\left|\frac{e^{-i\omega t}-e^{-i(M_0-i\Gamma/2)t}}{E-M_0+i\Gamma/2}\right|^2$$



Details in: F. G., arXiv:1305.4467 [quant-ph].